# Non-perturbative renormalization factors of bilinear quark operators for Kogut-Susskind fermions and light quark masses in quenched QCD *

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Light quark masses are computed for Kogut-Susskind fermions by evaluating non-perturbatively the renormalization factor for bilinear quark operators. Calculations are carried out in the quenched approximation at $\beta = 6.0$, 6.2, and 6.4. For the average up and down quark mass we find $m_{\overline{\rm MS}}(2{\rm GeV}) = 4.15(27)$MeV in the continuum limit, which is significantly larger than 3.51(20)MeV ($q^* = 1/a$) or 3.40(21)MeV ($q^* = \pi/a$) obtained with the one-loop perturbative renormalization factor.

## 1. Introduction

Light quark masses are important unknown parameters of the standard model, and a number of lattice QCD calculations have been carried out to evaluate quark masses employing the Wilson, clover or Kogut-Susskind (KS) fermion action [1]. Among the results, those with the KS action appear more accurate than others because of small lattice discretization errors and small statistical errors.

A worry with the KS result, however, has been that the employed one-loop renormalization factor takes a large value of $\approx 2$ in the range of $\beta$ studied, calling into question the viability of perturbation theory. In this article we report a study to circumvent this problem: we calculate the renormalization factor of bilinear quark operators for the KS action non-perturbatively using the method of Ref. [2] developed for the Wilson/clover actions. This calculation is carried out in quenched QCD at $\beta = 6.0$, 6.2, and 6.4 on an $32^4$ lattice. The results, combined with our previous calculation of bare quark masses [4], lead to a non-perturbative determination of the light quark mass.

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## 2. Method

The renormalization factor of a bilinear operator $\mathcal{O}$ is obtained from the amputated Green function,

$$\Gamma_{\mathcal{O}}(p) = S(p)^{-1}\langle 0|\phi(p)\mathcal{O}\bar{\phi}(p)|0\rangle S(p)^{-1} \quad (1)$$

where the quark two-point function is defined by $S(p) = \langle 0|\phi(p)\bar{\phi}(p)|0\rangle$. The quark field $\phi(p)$ with momentum $p$ is defined from the original one-component field $\chi(x)$ by $\phi_A(p) = \sum_y \exp(-ip\cdot y)\chi(y+aA)$, where $y_\mu = 2an_\mu$, $p_\mu = 2\pi/(aL)n_\mu$ ($n_\mu = [-L/4, L/4)$) and $A_\mu = [0,1]$.

The renormalization condition imposed on $\Gamma_{\mathcal{O}}(p)$ is given by

$$\Gamma_{\mathcal{O}}(p) = Z_\phi(p) Z_{\mathcal{O}}(p)\Gamma^{(0)}_{\mathcal{O}} \quad (2)$$

where $\Gamma^{(0)}_{\mathcal{O}}$ is the amputated Green function at tree level and $Z_\phi(p)$ is the wave function renormalization factor which can be calculated by the

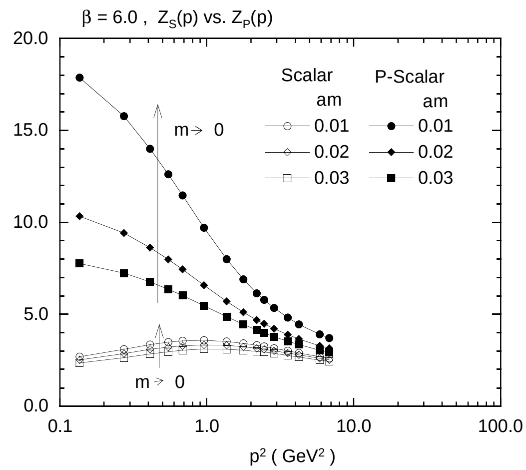


Figure 1. The scalar renormalization factor $Z_S(p)$ and that for the pseudoscalar $Z_P(p)$.

condition $Z_V(p) = 1$ for the conserved vector current.

The relation between the bare operator on the lattice and the renormalized operator in the continuum takes the form,

$$\mathcal{O}_{\overline{\rm MS}}(\mu) = U_{\overline{\rm MS}}(\mu, p) Z_{\overline{\rm MS}}(p)/Z_{\mathcal{O}}(p)\mathcal{O}^{\rm lat.}(a) \qquad (3)$$

where $U_{\overline{\rm MS}}(\mu, p)$ is the renormalization-group running factor, and $Z_{\overline{\rm MS}}(p)$ is the matching factor from the RI scheme defined by (2) to the $\overline{\rm MS}$ scheme, calculated perturbatively in the continuum. For the light quark mass we apply relation (3) in the scalar channel in the chiral limit.

We use a source in momentum eigenstate to evaluate quark propagators. This results in very small statistical errors of $O(0.1\%)$ in the Green functions.

The external momentum $p$ should be taken in the range $\Lambda_{\rm QCD} \ll p \ll O(1/a)$ in order to keep under control higher order effects in continuum perturbation theory, non-perturbative hadronization effect on the lattice, and discretization errors on the lattice. In this work we choose 15 momenta in the range $0.038553 < (ap)^2 < 1.9277$ for all values of $\beta$.

## 3. Result

In Fig. 1 we compare the scalar renormalization factor $Z_S(p)$ with that for pseudoscalar $Z_P(p)$ for three values of bare quark mass $am$ at $\beta = 6.0$. From chiral symmetry of KS fermions, we expect

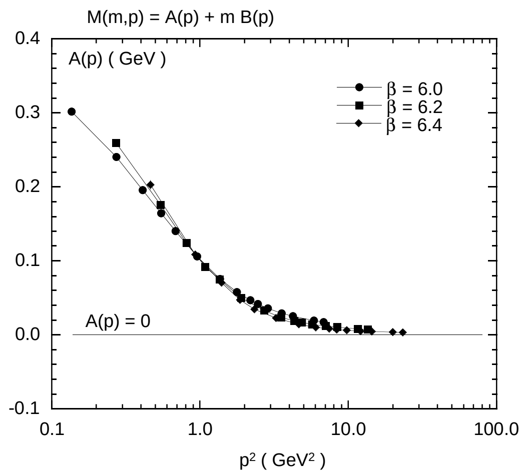


Figure 2. $M(p)$ in the chiral limit.

naively $Z_S(p) = Z_P(p)$ for all momenta $p$ in the chiral limit. Clearly this relation does not hold with our result toward small momenta, where $Z_P(p)$ rapidly increases as $m \to 0$, while $Z_S(p)$ does not show such a trend.

To understand this result, we note that chiral symmetry of KS fermion leads to the following identities :

$$\begin{aligned} &Z_S(p) \cdot Z_\phi(p) = \partial M(p)/\partial m \\ &Z_P(p) \cdot Z_\phi(p) = M(p)/m \end{aligned} \qquad (4)$$

with $M(p) = {\rm Tr}[S(p)^{-1}]$. In Fig. 2 $M(p)$ in the chiral limit obtained by a linear extrapolation in $m$ is plotted. It rapidly dumps for large momenta, but takes large values in the small momentum region. Combined with (4) this implies that $Z_P(p)$ diverges in the chiral limit for small momenta, which is consistent with the result in Fig. 1.

The function $M(p)$ is related to chiral condensate as follows :

$$\langle \phi\bar{\phi} \rangle = \sum_p {\rm Tr}[S(p)] = \sum_p \frac{M(p)}{C_\mu(p)^2 + M(p)^2} \qquad (5)$$

where $C_\mu(p) = -i{\rm Tr}[(\gamma_\mu \otimes I)S(p)^{-1}]/\cos(p_\mu a)$. A non-vanishing value of $M(p)$ for small momenta would lead to a non-zero value of the condensate. Therefore the divergence of $Z_P(p)$ near the chiral limit is a manifestation of spontaneous symmetry breakdown of chiral symmetry; it is a non-perturbative effect arising from the presence of massless Goldstone boson.

We expect this non-perturbative effect to affect the scalar renormalization factor $Z_S(p)$ much less,

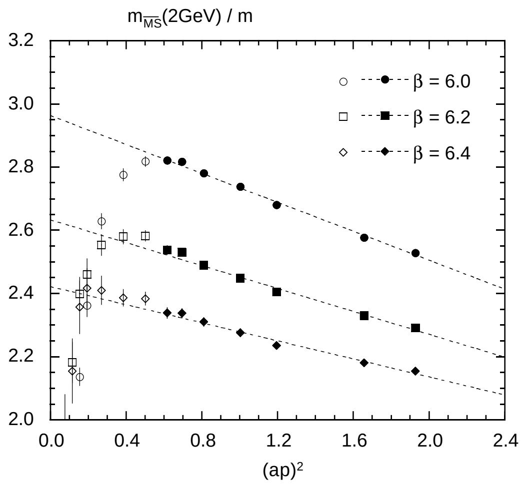


Figure 3. The ratio $m_{\overline{\rm MS}}(\mu)/m$ at $\mu = 2$GeV. For each $\beta$ the filled data points are used for linear extrapolation in $(ap)^2$.

since the scalar operator can not interact directly with the pseudoscalar meson. Indeed the quark mass dependence is quite small as we have seen in Fig. 1.

In Fig. 3 we show the momentum dependence of the ratio $m_{\overline{\rm MS}}(\mu)/m = U_{\overline{\rm MS}}(\mu,p)Z_{\overline{\rm MS}}(p)Z_S(p)$ calculated in the chiral limit where we set $\mu =$ 2GeV and use the three-loop formula [3] for $U_{\overline{\rm MS}}$ and $Z_{\overline{\rm MS}}$. While the ratio should be independent of the quark momentum $p$, our results show a large momentum dependence which is almost linear in $(ap)^2$ for $0.6 < (ap)^2$.

A natural origin of the linear dependence on $(ap)^2$ is the lattice discretization error of the scalar operator, which differs by terms of $O(a^2)$ from that of the continuum for the KS fermion. We then remove this error from the renormalization factor by a linear extrapolation in $(ap)^2$ to $(ap)^2 = 0$. In Fig. 3 the fitting lines are plotted, where filled data points are used for the linear extrapolation. For comparison, the ratio calculated with the one-loop value equals 1.867, 1.877, and 1.871 for $\beta = 6.0$, 6.2 and 6.4 at $q^* = 1/a$. Hence one-loop perturbation theory underestimates the ratio by 40% to 20%.

Our final results for the averaged up and down quark mass at $\mu = 2$GeV are shown in Fig. 4 by filled symbols. Here we use the JLQCD results for bare quark mass [4]. The values are substantially larger than those obtained with one-loop perturbation theory (open circles for $q^* = 1/a$ and squares for $q^* = \pi/a$). Furthermore they exhibit a significant $a^2$ dependence, which we ascribe to the discretization error of the quark mass itself. Making a linear extrapolation in $a^2$, our final result in the continuum limit is given by

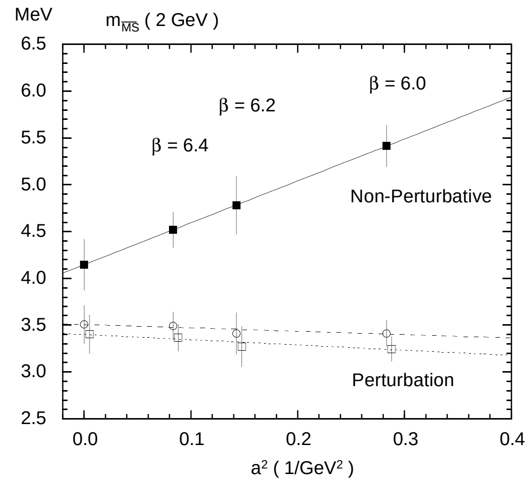


Figure 4. The final results of the light quark mass at $\mu = 2$GeV.

$$m_{\overline{\rm MS}}(2{\rm GeV}) = 4.15(27){\rm MeV}. \quad (6)$$

This value is 20% larger than the perturbative estimates : 3.51(20)MeV for $q^* = 1/a$ and 3.40(21)MeV for $q^* = \pi/a$.

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