Non-Perturbative Determination of $c_{\rm SW}$ in Three-flavor Dynamical QCD*

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We present a fully non-perturbative determination of the $O(a)$ improvement coefficient $c_{\rm SW}$ in three-flavor dynamical QCD for the RG improved as well as the plaquette gauge actions, using the Schrödinger functional scheme. Results are compared with one-loop estimates at weak gauge coupling.

1. Introduction

Realistic simulation of QCD requires treating the light up, down and strange quarks dynamically. Incorporating a degenerate pair of up and down quarks have become almost standard by now, and a first attempt toward the continuum extrapolation has shown that the deviation of the quenched hadron mass spectrum from experiment [1] is sizably reduced[2]. Adding a dynamical strange quark is the next step, which has become possible with the recent algorithmic development for odd number of fermions[3].

The CP-PACS and JLQCD Collaborations have jointly started a 2+1 flavor dynamical QCD, employing the polynomial HMC (PHMC) algorithm for strange quark and the HMC algorithm for up and down quarks. We choose the renormalization-group (RG) improved action for the gauge fields, in order to avoid the lattice artifact present for the plaquette action[4]. We wish to use a fully $O(a)$-improved action for quarks to control lattice spacing errors. Here we report on a non-perturbative determination of $c_{\rm sw}$ for three-flavor QCD by the Schrödinger functional scheme both for the plaquette and RG-improved gauge actions.

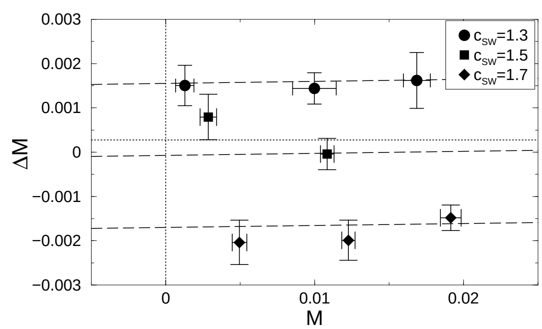

Figure 1. ΔM as a function of M for the RG action with $N_f = 3$ at $\beta = 2.2$.

2. Method and Simulations

For the determination of $c_{\rm SW}$, we basically follow the method of ref.[5], except for the choice B for the boundary weight of the RG-improved gauge action [6]. We refer to ref. [5] and references therein for notations in this report.

We mainly use an $8^3 \times 16$ lattice in our determination of $c_{\rm SW}$ for the RG-improved as well as the plaquette action with $N_f = 3$ dynamical quarks at several values of β. Simulations with $N_f = 4, 2, 0$ are also made for comparison.

*Talk presented by S. Aoki

We measure the modified PCAC quark masses, M and M', and their difference $\Delta M = M - M'$, at several values of $c_{\rm SW}$ and K. We have taken these parameters to realize $M = 0$ by an interpolation, except at strong couplings for the case of $N_f = 3$, where an extrapolation to $M = 0$ is necessary as shown in Fig. 1.

From the linear fit of ΔM as a function of M and $c_{\rm SW}$: $\delta M = a_0 + a_1 M + a_2 c_{\rm SW}$, we obtain the $O(a)$ improvement coefficient $c_{\rm SW} = (\Delta M^{(0)} - a_0)/a_2$, where $\Delta M^{(0)} = 0.000277$, marked by the horizontal dotted line in Fig. 1, is the tree-level value of ΔM on a $8^3 \times 16$ lattice. Note that the dependence of ΔM on $c_{\rm SW}$ becomes weaker at stronger couplings, so that the determination of $c_{\rm SW}$ is more difficult, and hence the error is larger, at stronger couplings.

3. Results

In the upper plot of Fig. 2 we show the non-perturbative value of $c_{\rm SW}$ as a function of the bare gauge coupling g_0^2 for the RG-improved gauge action with $N_f = 3$(circles), 2(diamonds) and 0(squares), together with the one-loop estimate(solid line) and the mean-field(MF) estimate(dashed line) used in ref. [2]. Similarly, results for the plaquette action with $N_f = 3$(circles) and 0(squares) are given in the lower plot of Fig. 2, together with the one-loop estimate(solid line) and the non-perturbative values by the Alpha collaboration with $N_f = 2$(dotted lines)[5] and 0(long-dashed line)[7].

In both cases, the non-perturbative values of $c_{\rm SW}$ are almost N_f independent at weak coupling while they become larger for smaller N_f at strong coupling. This tendency can be clearly seen in Fig. 3, where $c_{\rm SW}$ is plotted as a function of N_f for the RG action(open symbols) and the plaquette action(solid circles).

4. Comparison with perturbative estimates

At first sight, the non-perturbative $c_{\rm SW}$ seems to undershoot the one-loop estimate at weak coupling for the RG action, while it converges smoothly from above for the plaquette action.

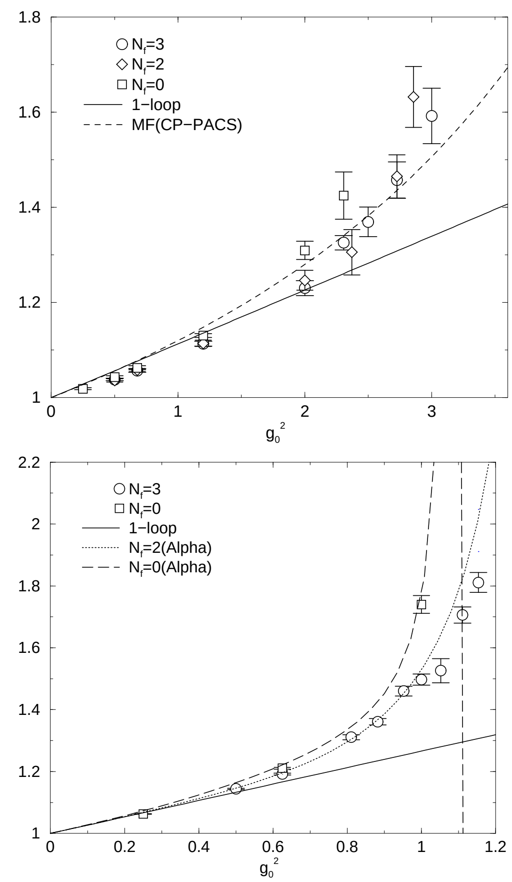

Figure 2. $c_{\rm SW}$ as a function of g_0^2 for the RG action (upper) and for the plaquette action(lower).

We have found that the discrepancy seen for the RG action is caused by the one-loop $O(a/L)$ error in $c_{\rm SW}$ [2], which becomes leading after the $O(a/L)$ error at tree level is removed by requiring $\Delta M = \Delta M^0$. In Fig. 4, the non-perturbative $c_{\rm SW}$ is compared with the one-loop estimate we have calculated on the same lattice size employed in the simulation, $8^3 \times 16$. As seen from the figure the non-perturbative value agrees with the one-loop estimate much better on the $8^3 \times 16$ lattice than in the infinite box. Note that the $O(g_0^2 a/L)$ contribution to $c_{\rm SW}$ slightly depends on N_f through the fermion tadpole in the presence of the background gauge field of the Schrödinger functional scheme. Such an N_f dependence is in-

[2] $O(a)$ errors of $c_{\rm SW}$ in general cause $O(a^2)$ errors in on-shell quantities, which are irrelevant in the $O(a)$ improvement.

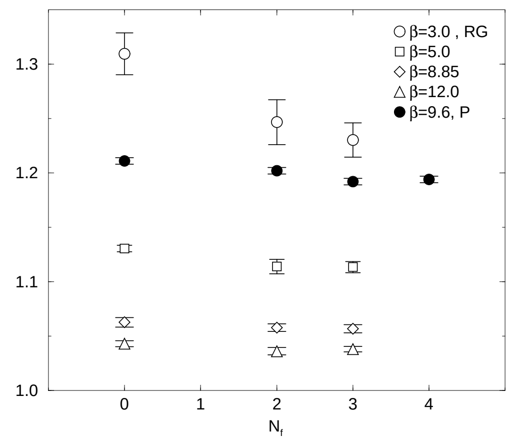

Figure 3. $c_{\rm SW}$ as a function of N_f for RG and P(plaquette) actions.

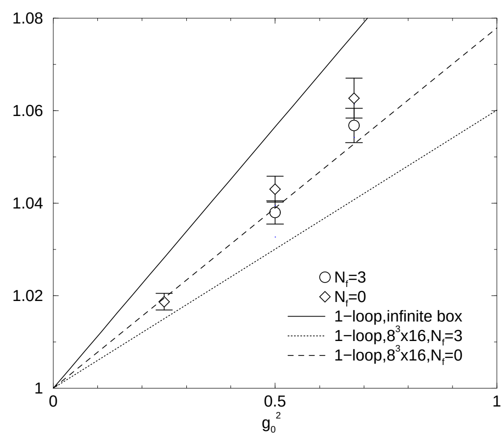

Figure 4. $c_{\rm SW}$ for the RG action at weak coupling, together with the one-loop estimate on the $8^3 \times 16$ lattice for $N_f = 3$(dotted line) and $N_f = 0$(dashed line).

deed seen in the numerical data of Fig. 4.

For the plaquette action we also confirm a presence of the $O(g_0^2 a/L)$ correction, which is small on the $8^3 \times 16$ lattice, as shown in Fig. 5 where $c_{\rm SW}$ is plotted as a function of a/L at $\beta = 24$ with $N_f = 0$. The one-loop estimates(solid circles) reproduce the non-monotonic behaviour of non-perturbative values(open circles) well.

5. Discussion

We have determined the non-perturbative value of $c_{\rm SW}$ for the RG action at several gauge couplings with $N_f = 3, 2, 0$. In order to obtain

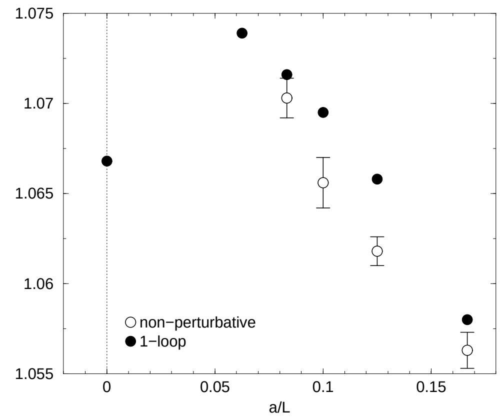

Figure 5. Non-perturbative $c_{\rm SW}$(open circles) and one-loop estimate(solid circles) as a function of a/L at $\beta = 24$ for the plaquette action with $N_f = 0$.

an interpolation formula of $c_{\rm SW}$ as a function of g_0^2, we have to eliminate large $O(g_0^2 a/L)$ corrections to $c_{\rm SW}$ present for the RG action. We are currently investigating this problem.

We are also measuring the hadron spectrum for the RG action at $\beta \equiv 2$ with $N_f = 3$ using the preliminary value of $c_{\rm SW}$, in order to determine the corresponding lattice spacing.

This work is supported in part by Grants-in-Aid of the Ministry of Education (Nos. 11640294, 12304011, 12640253, 12740133, 13135204, 13640259, 13640260, 14046202 14740173). N.Y. is supported by the JSPS Research Fellowship.

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