

Maximum entropy analysis of hadron spectral functions and excited states in quenched lattice QCD*

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Employing the maximum entropy method we extract the spectral functions from meson correlators at four lattice spacings in quenched QCD with the Wilson quark action. We confirm that the masses and decay constants, obtained from the position and the area of peaks, agree well with the results from the conventional exponential fit. For the first excited state, we obtain $m_{\pi_1} = 660(590)$ MeV, $m_{\rho_1} = 1540(570)$ MeV, and $f_{\rho_1} = 0.085(36)$ in the continuum limit.

1. Introduction

The spectral function $f(\omega)$ of hadrons includes information such as masses and decay constants for various bound states and the continuum spectrum for multi-particle states. Recently the maximum entropy method [1] (MEM) has been employed in order to extract the spectral function from lattice QCD data [2,3].

In this article we apply the MEM to the quenched lattice QCD data for pseudoscalar(PS) and vector(V) mesons previously calculated at $\beta = 5.90, 6.10, 6.25,$ and 6.47 [4]. We present the corresponding spectral functions for the case of a point source, and check the reliability of the MEM by comparing the masses and decay constants from the spectral function with those from the exponential fit. We then extract the masses and decay constants for the first excited state in the continuum limit. For details we refer to ref. [5].

2. Maximum entropy method

In the MEM the spectral function is determined

by maximizing the quantity $P[F|DH]$, which is the conditional probability of the spectral function F for a given data D and all prior knowledge H such as $f(\omega) \geq 0$. By Bayes's theorem this probability is replaced with $P[D|FH]P[F|H]$, where $P[D|FH]$ is proportional to $\exp(-\chi^2/2)$ with χ^2 the standard chi-squared, and $P[F|H]$ to $\exp(\alpha S(f))$ with α a real positive parameter. Here the entropy $S(f)$ [1] is defined by $f(\omega)$ and a positive function called model. Combining these two factors, we can determine the most probable spectral function as the solution of the equation $\partial Q_\alpha / \partial f = 0$, where $Q_\alpha = \alpha S(f) - \chi^2/2$. Finally the result is averaged over α with a weight factor $P[\alpha|DH]$.

3. Results

In Fig. 1 a typical result for the spectral functions of ρ meson is presented at $\beta = 5.90$ and 6.47 for three different values of m_π/m_ρ . The parameters of the MEM analysis are compiled in ref. [5]. At each β one observes, as expected, that the peaks move to smaller ωa as quark mass decreases. As β increases, the peaks of the ground and the first excited state also move to smaller

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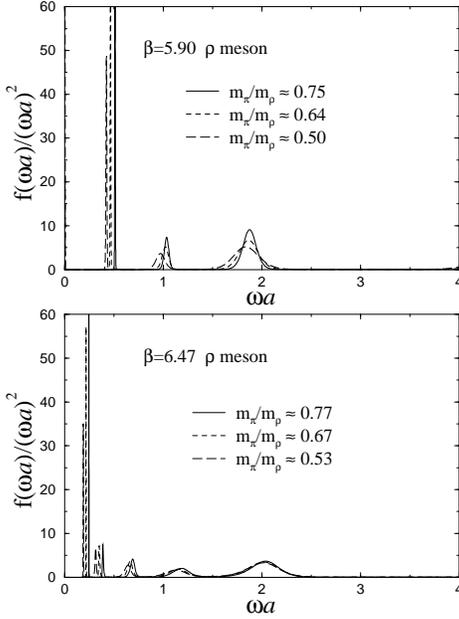


Figure 1. Spectral functions at $\beta = 5.90$ and 6.47 for ρ meson.

ωa as expected. In addition the number of peaks increases, since higher excited states appear below the cutoff π/a at larger β . While the peak of the ground state is very narrow, the peaks for the higher states have larger widths, the reason for which is not understood at present.

3.1. Mass

The mass of a state is determined from the position of the peak in the spectral function. To check the reliability of this analysis, the masses of the ground and the first excited state obtained in this way are compared with those from the double exponential fit of propagators for the point and smeared sources. At $\beta = 5.90$ for ρ meson, the two results for the ground state agree, and those for the first excited state are consistent within errors, as seen in Fig. 2.

The first excited state masses for pseudoscalar and vector mesons in the chiral limit at each β are extrapolated to the continuum limit in Fig. 3. The error for the vector meson at $\beta = 5.90$ is smaller than that obtained by the exponential fit (open square), and our result is consistent with that of ref. [3] (open triangle). In the contin-

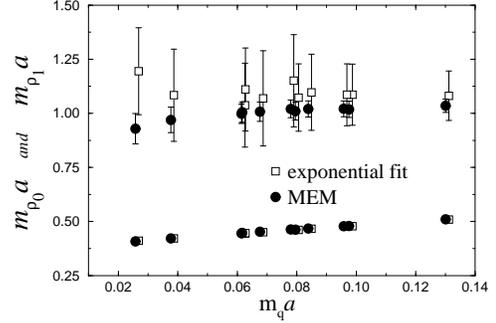


Figure 2. Comparing the results with these from the exponential fit.

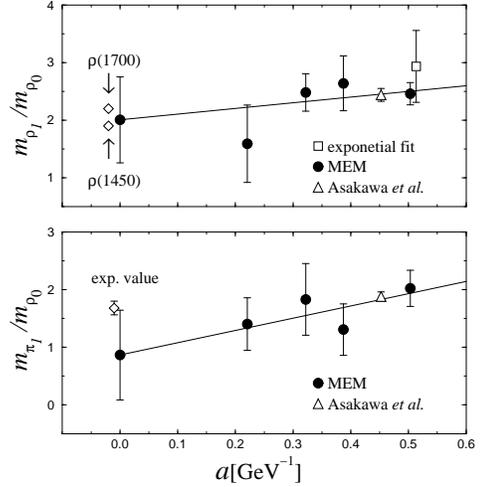


Figure 3. Continuum extrapolation for the first excited state masses for both mesons.

uum limit we obtain $m_{\pi_1} = 660(590)$ MeV and $m_{\rho_1} = 1540(570)$ MeV, which are consistent with the experimental values, though the errors are large.

3.2. Decay constant

The decay constant is related to the area of the peak in the spectral function [3,5], i.e., $f_\pi^2 \propto 2m_q \int_{peak} d\omega f_{PS}(\omega)/m_\pi^3$ for pseudoscalar meson and $f_\rho^2 \propto \int_{peak} d\omega f_V(\omega)/m_\rho^3$ for vector meson.

As shown in Fig. 4, the ground state decay constants for both mesons in the chiral limit are consistent with the results obtained by the exponential fit at each β and in the continuum limit.

The decay constants are determined also for the first excited state. For the excited π meson the

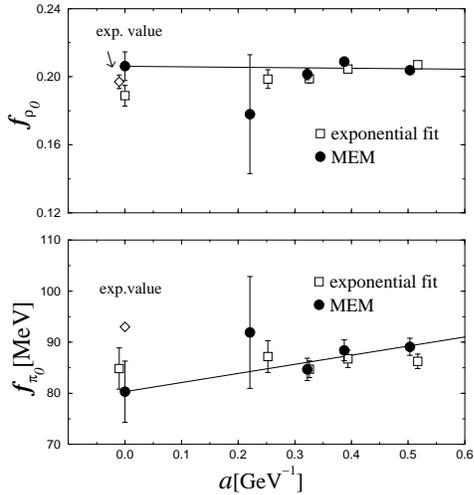


Figure 4. Decay constant for the ground state for both mesons.

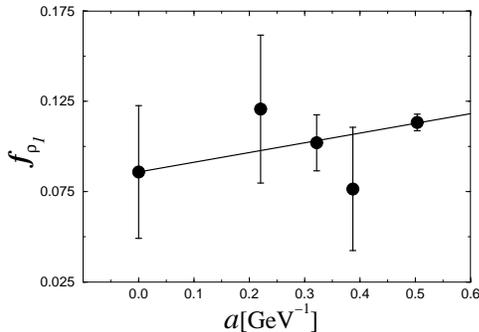


Figure 5. Decay constant for the first excited state of ρ meson.

decay constant vanishes in the chiral limit, since $f_{\pi_1} \propto m_q/m_{\pi_1}^2$ by definition. On the other hand, in Fig. 5, the decay constant for ρ meson is finite in this limit, and we obtain $f_{\rho_1} = 0.085(36)$ in the continuum limit.

3.3. Unphysical state

We observe in Fig. 1 that there is a peak at $\omega a \approx 2$ almost independent of β . A similar peak is also present in the pseudoscalar channel. Apparently the masses of these states in physical units diverge in the continuum limit, as seen in Fig. 6. Here we interpret these states as bound states of two fermion doublers [5]: In free theory the mass of a two doublers system, each with a spatial momentum πa , is $2 \times \log(3) \approx 2.2$, which is interestingly close to $\omega a \approx 2$. The difference may

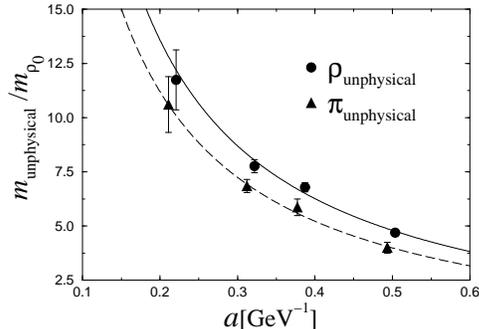


Figure 6. Divergent continuum extrapolation for the unphysical state mass for both mesons.

be ascribed to its binding energy. We note that the bound state of a physical quark and a doubler can not appear at zero spatial momentum.

4. Conclusion

We have applied the maximum entropy method to our precision quenched lattice QCD data [4] to extract the spectral functions for pseudoscalar and vector mesons. The masses and the decay constants extracted from the spectral functions for the ground state are consistent with the ones determined by the exponential fit, while those for the first excited state are new. In future it will be interesting to apply the MEM analysis to full QCD, QCD at finite temperature and a study of decay and scattering.

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