

## Manifestation of Sea Quark Effects in the Strong Coupling Constant in Lattice QCD

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We demonstrate that sea quark effects of a magnitude expected from renormalization group considerations are clearly visible in the strong coupling constant measured in current full QCD simulations. Building on this result an estimate of  $\alpha_{\overline{\text{MS}}}^{(5)}(M_Z)$  ( $\overline{\text{MS}}$  denotes the modified minimal subtraction scheme) is made employing the charmonium  $1S$ - $1P$  mass splitting calculated on full QCD configurations generated with two flavors of dynamical Kogut-Susskind quarks to fix the scale.

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A distinctive feature of QCD that differentiates it from phenomenological quark models of hadrons is the existence of sea quarks built into the theory. Nonetheless, finding a physical manifestation of sea quark effects has been an elusive subject in full lattice QCD simulations. In hadron mass spectrum calculations, for example, full QCD results for flavor nonsinglet hadron masses agree with those of quenched QCD within statistical errors of 5%–10% if the bare coupling constant for the latter is shifted by an appropriate amount [1]. A similar situation holds for the critical coupling of the chiral transition at finite temperatures in full QCD; its value, though largely dependent on the sea quark mass, is reproduced quite well from that of the pure gauge theory by correcting for quark one-loop vacuum polarization effects [2].

A possible interpretation of the matching of full and quenched QCD by a shift of the bare coupling is that the shift represents an adjustment of the renormalized coupling constant at the low energy scale that dominates the behavior of quantities being simulated [3,4]. If this interpretation is valid, one expects that sea quark effects will become manifest in the renormalized coupling constant estimated for a scale sufficiently large compared to the dominant scale, since the full QCD coupling constant decreases more slowly than that of the pure gauge theory. In this Letter we present evidence that this in fact is the case: We find that the full QCD coupling constant extracted from two-flavor full QCD simulations is consistently larger than that of quenched QCD at large momenta ranging over  $\mu \approx 3$ –7 GeV when the scale is determined from the  $\rho$  meson mass; the difference in the magnitude of the full and quenched coupling constants is consistent with the picture that the two couplings merge when evolved down to the low energy scale  $\mu < 1$  GeV via the two-loop renormalization group. We also estimate the physical strong coupling constant for five flavors of  $\alpha_{\overline{\text{MS}}}^{(5)}$  ( $\overline{\text{MS}}$  denotes the modified minimal subtraction scheme) at the  $M_Z$  scale following the work of Refs. [3,4],

employing the  $1S$ - $1P$  mass splitting of charmonium states estimated for two-flavor full QCD.

Calculation of the renormalized value of the coupling for a bare value  $\alpha_0$  taken in a simulation is facilitated by the recent study [5] which has shown that lattice perturbative series is well convergent after lattice gluon tadpole effects are properly taken into account. A proposal for including tadpole effects in the relation between the bare and renormalized coupling is given by [3]

$$\alpha_{\overline{\text{MS}}}^{(N_f)}(\pi/a)^{-1} = P\alpha_0^{-1} + c_g + N_f c_f + O(\alpha_0^2), \quad (1)$$

where  $P$  is the plaquette expectation value,  $c_g = 0.30928$  is the gluon one-loop contribution [6], and the last term represents the contribution of  $N_f$  flavors of quarks with  $c_f = -0.08848$  for the Kogut-Susskind quark action [7] and  $c_f = -0.03491$  for the Wilson action [8]. Alternatively one may use the  $\alpha_V$  coupling defined from the static  $\bar{q}q$  potential, which can be estimated from  $P$  via [5]

$$-\ln P = \frac{4\pi}{3} \alpha_V^{(N_f)}(3.41/a)[1 + (d_g + N_f d_f)\alpha_V^{(N_f)} + O(\alpha_V^2)], \quad (2)$$

with  $d_g = -1.1855$  and  $d_f = -0.0703$  for the Kogut-Susskind quark action and  $d_f = -0.0249$  for the Wilson action. The relation between the two couplings is given by [5,9]

$$\alpha_{\overline{\text{MS}}}^{(N_f)}(\mu)^{-1} = \alpha_V^{(N_f)}(\mu)^{-1} + c_{\overline{\text{MS}}}^V + O(\alpha_V^{(N_f)}(\mu)^2), \quad (3)$$

$$c_{\overline{\text{MS}}}^V = \frac{1}{36\pi} (93 - 10N_f).$$

Equivalently the  $\Lambda$  parameters are related by

$$\Lambda_{\overline{\text{MS}}}^{(N_f)} = \exp\left(-\frac{c_{\overline{\text{MS}}}^V}{8\pi b_0}\right) \Lambda_V^{(N_f)}, \quad (4)$$

with  $b_0 = (11 - 2N_f/3)/(4\pi)^2$ .

The renormalized coupling constant in the  $\overline{\text{MS}}$  scheme extracted from (1) for quenched and two-flavor full QCD is compared in Fig. 1 for Kogut-Susskind and Wilson quark actions as a function of scale  $\mu = \pi/a$  determined from the  $\rho$  meson mass. In full QCD the plaquette data

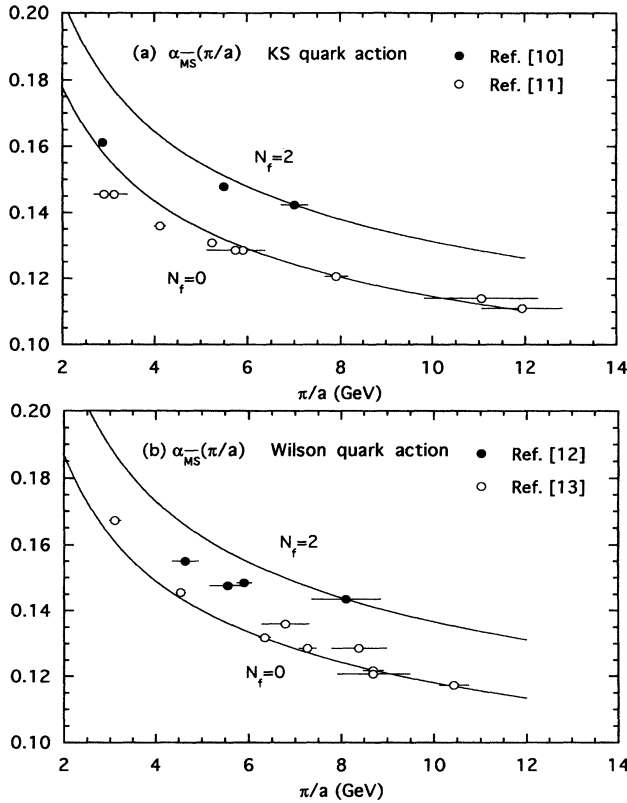


FIG. 1. Comparison of  $\alpha_{\overline{\text{MS}}}^{(N_f)}(\pi/a)$  estimated via (1) for two-flavor full QCD (filled symbols) and quenched QCD (open symbols) with the scale fixed by the  $\rho$  meson mass.

are extrapolated linearly in the sea quark mass to  $m_q = 0$ . In quenched QCD we made a ninth order polynomial fit in  $\beta$  of plaquette values published in the literature [14] in order to calculate the values at  $\beta$  where data are not available. The trend is apparent in Fig. 1 that the full QCD coupling constant is systematically larger than that of the pure gauge theory when compared at the same scale  $\mu$ .

The solid lines in Fig. 1 illustrate the two-loop renormalization group evolution of the coupling constant. Deviation of  $\alpha_{\overline{\text{MS}}}^{(N_f)}(\pi/a)$  from the solid lines toward smaller values of cutoff is in part ascribed to scaling violation effects due to a finite lattice spacing and in part to uncertainties of  $O(\alpha^2)$  in the relation (1). One can estimate the magnitude of the latter through a comparison of the coupling constant extracted from (1) and (2). This analysis shows that the latter estimate yields values for  $\alpha_{\overline{\text{MS}}}^{(N_f)}(\pi/a)$  larger by about 3%–5% at  $\mu \approx 7$  GeV, and by about 5%–10% at  $\mu \approx 3$  GeV. These are taken as uncertainties of our analyses.

In Fig. 2 we compare the two-loop renormalization group evolution of the full and quenched coupling constants toward small momenta  $\mu < 0.5$ –1 GeV. The up-

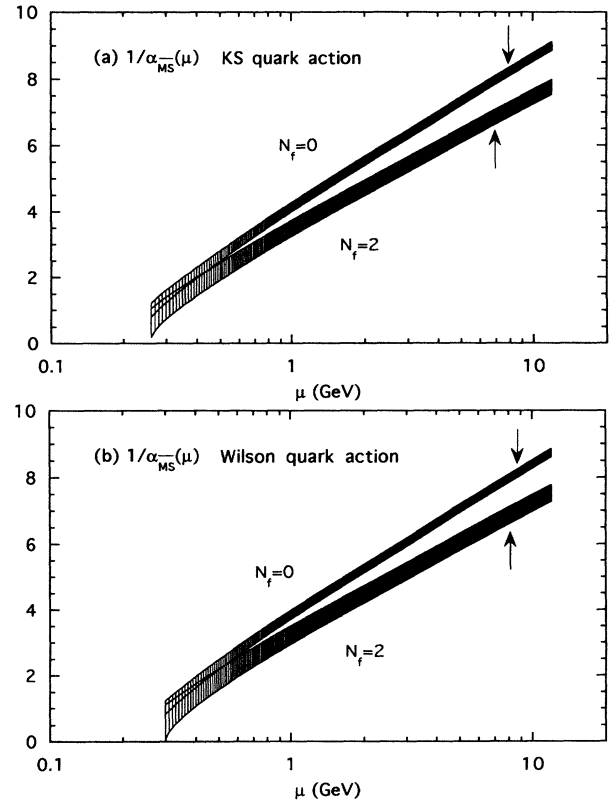


FIG. 2. Evolution of  $\alpha_{\overline{\text{MS}}}^{(N_f)}(\mu)$  for quenched and two-flavor QCD. Bands correspond to uncertainties in the estimate of  $\alpha_{\overline{\text{MS}}}^{(N_f)}(\mu)$ . Arrows indicate the starting value taken from (a) Fukugita *et al.* ( $N_f = 2$ ,  $\beta = 5.7$ ) [10] and Sharpe *et al.* ( $N_f = 0$ ,  $\beta = 6.2$ ) [11], and (b) Gupta *et al.* ( $N_f = 2$ ,  $\beta = 5.6$ ) [12] and Butler *et al.* ( $N_f = 0$ ,  $\beta = 6.17$ ) [13].

per and lower edges of the bands in this figure correspond to  $\alpha_{\overline{\text{MS}}}^{(N_f)}(\pi/a)$  estimated from the relation (1) and that from (2) including scale errors in order to take into consideration the two-loop uncertainty. For full QCD we employ the data taken at the highest  $\beta$  for the starting value, and for quenched QCD the one carried out at a value of  $\beta$  with a nearby value of  $\mu = \pi/a$ . We observe that the evolution of the two coupling constants overlaps below  $\mu \approx 0.4$  GeV, which is the dominant scale relevant for the  $\rho$  meson that is employed for fixing the scale.

The results described above are fully consistent with the view that matching full and quenched results means adjusting the coupling constant at the relevant low energy scale, and that the full QCD couplings estimated for larger momenta should exhibit a slower decrease than the pure gauge coupling with a rate dictated by the renormalization group  $\beta$  function.

Let us note that these findings provide support for the procedure of Refs. [3,4] for estimating the physical strong coupling constant from values measured in simulations with an incomplete spectrum of sea quarks. Namely,

TABLE I. Charmonium spectrum for Wilson valence quarks in full QCD at  $\beta = 5.7$  with two flavors of dynamical Kogut-Susskind quarks with  $m_q a = 0.01$ .

$K$	$m_{\eta_c} a$	$m_{J/\psi} a$		$m_{h_c} a$	$\Delta m_{1P-1S} a$	$\pi/a$ (GeV)
0.130	1.474(16)	1.466(17)	op. 1:	1.692(53)	0.221(49)	6.5(1.5)
			op. 2:	1.672(32)	0.200(32)	7.2(1.2)
0.135	1.285(16)	1.275(15)	op. 1:	1.504(53)	0.221(48)	6.5(1.5)
			op. 2:	1.487(32)	0.205(35)	7.0(1.2)

one initially evolves the measured value at the cutoff scale down to the low energy scale  $\mu_0$  typical of the simulated hadron system using the flavor number of the simulation. The physical coupling constant  $\alpha_s$  at that scale is equated to the evolved value, and  $\alpha_s$  for larger scale is calculated through renormalization group incorporating the full spectrum of quarks active at each scale. A necessary condition for applying this procedure is that the scale  $\mu_0$  is not too small in order not to spoil the two-loop approximation to the  $\beta$  function that breaks down for small momenta. The authors of Ref. [3] proposed to use the  $1S-1P$  charmonium mass splitting for which potential models suggest  $\mu_0 \approx 0.4-0.75$  GeV. The advantage is that heavy quark propagators are easy to calculate and that the  $1S-1P$  mass splitting is empirically insensitive to the quark mass, rendering its fine tuning unnecessary.

We have carried out an analysis along this line employing the full QCD configurations on a  $20^4$  for two flavors of dynamical Kogut-Susskind quarks at  $\beta = 5.7$  with  $m_q a = 0.01$  [15]. For charmonium spectrum measurement we used the Wilson quark action for valence quarks [16], employing Gaussian smeared sources  $\sum_{x,y} \bar{\psi}_x \Gamma_1 \psi_y f(x) f(y)$  and local sinks  $\bar{\psi}_x \Gamma_2 \psi_x$  where  $\Gamma_1 = \Gamma_2 = \gamma^i$ ,  $\gamma^5$ ,  $\sigma^{jk}$  for  $J/\psi$ ,  $\eta_c$ ,  $h_c$  (option 1) with  $f(x) = \exp(-|x|^2/4)$ , and also  $\Gamma_1 = \gamma^5$ ,  $\Gamma_2 = \sigma^{jk}$  with  $f(x) = \sin(2\pi x^i/L) f_s(x)$  for  $h_c$  (option 2). We analyzed 72 configurations at  $K = 0.130$  and 75 configurations at  $K = 0.135$  with the lattice size periodically doubled in the temporal direction.

In Table I we list our result for the charmonium spectrum and the corresponding scale  $\pi/a$  extracted from the experimental value of the  $1P-1S$  mass splitting 457.8(5) MeV [17] as input. The splitting is almost independent of the hopping parameter  $K$  which controls the charm quark mass, though it slightly depends on the choice of the operator for  $h_c$  (options 1 or 2). The splitting yields a value  $\pi/a \approx 7$  GeV, which is consistent with  $\pi/a = 7.01(28)$  [15] estimated from the  $\rho$  meson mass.

Our results for the physical strong coupling constant and the  $\Lambda$  parameter obtained with the scale listed in Table I are summarized in Table II. In Table II(a) the starting value is  $\alpha_{\overline{\text{MS}}}^{(2)}(\pi/a) = 0.142$  estimated from (1), while in Table II(b) we use  $\alpha_V^{(2)}(3.41/a) = 0.169$  obtained from (2). For both cases the actual evolution is made in terms of the  $\alpha_V$  coupling since it is directly related to the heavy quark potential relevant for charmonium. We use  $\mu_0 = 0.4$  and  $0.75$  GeV for the matching scale and take an average of the two choices for the central value for the strong coupling constant. The errors are estimated by allowing  $\mu_0$  to vary over the range  $\mu_0 = 0.4-0.75$  GeV and the scale  $\pi/a$  within the quoted error.

Our results are consistent with the previous lattice estimates carried out in quenched QCD [3,4]. Compared to the world average of phenomenological determinations,  $\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.118(7)$  [18], the results are somewhat small especially for those estimated from the relation (1). We do not view the difference to be alarming at this stage

TABLE II. Strong coupling constant and  $\Lambda$  parameter in the  $\overline{\text{MS}}$  scheme according to the Particle Data Group definition calculated with the  $1S-1P$  charmonium mass splitting with Wilson valence quarks for fixing the scale.

$K$	$\alpha_{\overline{\text{MS}}}^{(4)}(5 \text{ GeV})$	$\Lambda_{\overline{\text{MS}}}^{(4)}$	$\alpha_{\overline{\text{MS}}}^{(5)}(m_Z)$	$\Lambda_{\overline{\text{MS}}}^{(5)}$
(a) Estimate from $\alpha_{\overline{\text{MS}}}^{(2)}(\pi/a)$				
0.130(op. 1)	$0.168^{+0.015}_{-0.010}$	$141^{+57}_{-34}$ MeV	$0.104^{+0.005}_{-0.004}$	$92^{+43}_{-24}$ MeV
0.135(op. 1)	$0.168^{+0.015}_{-0.010}$	$141^{+55}_{-33}$ MeV	$0.104^{+0.005}_{-0.004}$	$92^{+42}_{-24}$ MeV
0.130(op. 2)	$0.173^{+0.012}_{-0.009}$	$159^{+44}_{-31}$ MeV	$0.106^{+0.004}_{-0.003}$	$105^{+34}_{-22}$ MeV
0.135(op. 2)	$0.172^{+0.012}_{-0.009}$	$154^{+46}_{-31}$ MeV	$0.106^{+0.004}_{-0.003}$	$102^{+35}_{-23}$ MeV
(b) Estimate from $\alpha_V^{(2)}(3.41/a)$				
0.130(op. 1)	$0.186^{+0.017}_{-0.013}$	$206^{+74}_{-50}$ MeV	$0.110^{+0.006}_{-0.005}$	$142^{+57}_{-36}$ MeV
0.135(op. 1)	$0.186^{+0.016}_{-0.013}$	$206^{+72}_{-49}$ MeV	$0.110^{+0.006}_{-0.005}$	$142^{+56}_{-38}$ MeV
0.130(op. 2)	$0.192^{+0.013}_{-0.011}$	$231^{+57}_{-44}$ MeV	$0.112^{+0.004}_{-0.004}$	$161^{+45}_{-34}$ MeV
0.135(op. 2)	$0.190^{+0.014}_{-0.011}$	$224^{+60}_{-44}$ MeV	$0.112^{+0.004}_{-0.004}$	$156^{+47}_{-35}$ MeV

since, in addition to errors in the experimental value, there exists a 5% uncertainty in our value of  $\alpha_{\overline{\text{MS}}}^{(5)}(M_Z)$  due to that of the input value at the cutoff scale and an additional 5% that results from the matching procedure. Furthermore, the strong coupling constant estimated at the cutoff scale somewhat deviates from the two-loop renormalization group evolution as we observed in Fig. 1, and hence repeating the calculation for a set of values of lattice spacing will be needed to control scaling violation effects in the estimate of  $\alpha_{\overline{\text{MS}}}^{(5)}(M_Z)$ .

To summarize, our analyses have shown that sea quark effects of a magnitude expected from renormalization group considerations are visible in the strong coupling constant measured in current full QCD simulations incorporating up and down quarks. This indicates a promising prospect for a realistic determination of the strong coupling constant including the full spectrum of sea quarks, since incorporating heavy quarks such as strange and charm is not difficult from the view of the necessary computer power compared to that for light quarks.

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