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Lattice QCD calculation of the proton decay matrix element in the continuum limit

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We present a quenched lattice QCD calculation of the α and β parameters of the proton decay matrix element. The simulation is carried out using the Wilson quark action at three values of the lattice spacing in the range $a \approx 0.1 - 0.064$ fm to study the scaling violation effect. We find only mild scaling violation when the lattice scale is determined by the nucleon mass. We obtain in the continuum limit, $|\alpha(\mathrm{NDR}, 2\ \mathrm{GeV})| = 0.0090(09)(+5 - 19)\ \mathrm{GeV}^3$ and $|\beta(\mathrm{NDR}, 2\ \mathrm{GeV})| = 0.0096(09)(+6 - 20)\ \mathrm{GeV}^3$ with α and β in a relatively opposite sign, where the first error is statistical and the second is due to the uncertainty in the determination of the physical scale.

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Proton decay (or nucleon decay in general) is a characteristic consequence of grand unified theories (GUTs) because of the unification of quarks and leptons into the same gauge multiplet. However, no clear evidence of such decay process has been observed up to now in spite of continual experimental efforts over several decades. Most recent experimental lower bound of the lifetime is given by the Super-Kamiokande experiment: 4.4×10^{33} years for $p \to e^+ + \pi^0$ mode and 1.9×10^{33} years for $p \to \bar{\nu} + K^+$ mode at 90% confidence level[1]. Although some naive GUT models is already ruled out by this experimental bound, we still have several viable GUTs which allow the longer proton lifetime at $O(10^{33-34})$[2]. Further improvement of the experimental bound could give strong constraints on these GUT models.

One of the main sources of uncertainties in the theoretical predictions is the evaluation of the hadronic matrix elements for the nucleon decays $\langle PS|\mathcal{O}|N\rangle$, where PS and N stand for the pseudoscalar meson and the nucleon, respectively, and $\mathcal{O}$ is the three-quark operator violating the baryon number. The conventional procedure of estimating the hadronic matrix element is to invoke current algebra and PCAC and to reduce the three-body matrix element into the two-body transition element $\langle 0|\mathcal{O}|N\rangle$, leaving aside the question as to the validity of PCAC with a long extrapolation. Varieties of models have been employed to estimate this transition elements, but results vary by an order of magnitude; see [3].

A promising method to reduce the uncertainty is to resort to lattice QCD, which allows direct evaluation of nonperturbative effects and has been successfully used in giving various weak interaction matrix elements. There are already a few calculations to evaluate the two-body transition element [4,5], and even a few attempts to evaluate directly the three-body amplitude $\langle PS|\mathcal{O}|N\rangle$ [6,7]. Gavela *et al.* [6] argued that the three-body amplitude gives proton decay lifetime that differs largely from the one derived from a two-body calculation with the use of PCAC. The JLQCD calculation [7], however, showed that their results are due to a neglect of one of the two form factors and that the three-body and two-body calculations yield the results that agree at a reasonable accuracy, say 20–30%.

Lattice QCD calculations, being carried out today, however, contain a number of sources that lead to systematic errors, such as the quenching approximation, finite lattice spacing, finite lattice size, chiral extrapolation and so forth. Particularly worrisome are the finite lattice spacing effects that could modify the continuum results even by a factor two if scaling violation is substantial in the relevant quantity. In fact, the recent preliminary result of the RBC Collaboration [8] gives the matrix elements that differs from those by JLQCD [7] by $\approx 50\%$, which urges us to study the issue of systematic errors. As for the quenching effects, a large-scale simulation of CP-PACS Collaboration for quenched light hadrons shows deviation from the experiment by at most 10% [9], which suggests that the systematic error due to quenching in the matrix elements would be less significant than the scaling violation effects and 100 times smaller compared to the uncertainties of the QCD model predictions.

In this paper we focus on the issue of the lattice spacing effects, by carrying out simulations at three different values of bare coupling constant, adopting the lattices that are large enough so that finite lattice effects are negligible even for baryons, and borrowing the results

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of CP-PACS Collaboration for quenched hadrons [9]. We consider two-body matrix elements

$$\langle 0|\epsilon_{ijk}(u^{iT}CP_Rd^j)P_Lu^k|p(\vec{k}=\vec{0})\rangle = \alpha P_L u_p, \quad (1)$$

$$\langle 0|\epsilon_{ijk}(u^{iT}CP_Ld^j)P_Lu^k|p(\vec{k}=\vec{0})\rangle = \beta P_L u_p, \quad (2)$$

expressed by α and β parameters, where i, j, and k are color indices, C is the charge conjugation matrix, $P_{R/L}$ is chiral projection operator and u_p denotes the proton spinor with the zero spatial momentum. We deal with the two-body matrix elements in view of the feasibility on current computers, rather than three-body matrix elements, which need three-point correlators with finite spatial momenta injected to disentangle the relevant and irrelevant form factors [7].

The continuum operators relevant to the α and β parameters are connected with the lattice operators as

$$\mathcal{O}^{\rm cont}_{R/L,L}(\mu) = Z(\alpha_s, \mu a)\mathcal{O}^{\rm latt}_{R/L,L}(a) + \frac{\alpha_s}{4\pi}Z_{\rm mix}\mathcal{O}^{\rm latt}_{L/R,L}(a) \mp \frac{\alpha_s}{4\pi}Z'_{\rm mix}\mathcal{O}^{\rm latt}_{\gamma_\mu L}(a), \quad (3)$$

where

$$\mathcal{O}_{R/L,L} = \epsilon_{ijk}(u^{iT}CP_{R/L}d^j)P_Lu^k, \quad (4)$$

and the mixing operator

$$\mathcal{O}_{\gamma_\mu L} = \epsilon_{ijk}(u^{iT}C\gamma_\mu\gamma_5 d^j)P_L\gamma_\mu u^k \quad (5)$$

appears due to explicit chiral symmetry breaking of the Wilson quark action. The renormalization constants Z, $Z_{\rm mix}$, and $Z'_{\rm mix}$ are evaluated perturbatively at one-loop order [7,10]. The continuum operators are defined in naive dimensional regularization (NDR) with the $\overline{\rm MS}$ subtraction scheme. The matrix elements defined on the lattice are converted to those in the continuum at $\mu = 1/a$ and are evolved to $\mu = 2$ GeV using the two-loop renormalization group in the continuum [11].

To obtain the matrix elements, we consider the ratio

$$R_{R/L}(t) = \frac{\sum_{\vec{x}}\langle \mathcal{O}_{R/L,L}(\vec{x},t)\bar{J}'_{p,s}(0)\rangle}{\sum_{\vec{x}}\langle J_{p,s}(\vec{x},t)\bar{J}'_{p,s}(0)\rangle}\sqrt{Z_p} \overset{\text{large } t}{\longrightarrow} \langle 0|\epsilon_{ijk}(u^{iT}CP_{R/L}d^j)P_Lu^k|p^{(s)}\rangle, \quad (6)$$

where $J_p(\vec{x},t)$ is a local sink operator for the proton with spin s and $\bar{J}'_{p,s}(0)$ is the smeared source,

$$J_{p,s}(\vec{x},t) = \epsilon_{ijk}[u^{iT}(\vec{x},t)C\gamma_5 d^j(\vec{x},t)]u^k_s(\vec{x},t), \quad (7)$$

$$J'_{p,s}(t) = \sum_{\vec{x},\vec{y},\vec{z}}\Psi(\vec{x})\Psi(\vec{y})\Psi(\vec{z}) \times \epsilon_{ijk}[u^{iT}(\vec{x},t)C\gamma_5 d^j(\vec{y},t)]u^k_s(\vec{z},t), \quad (8)$$

with the smearing function Ψ. The factor $\sqrt{Z_p}$ defined by

$$\langle 0|J_{p,s}(\vec{0},0)|p^{(s')}(\vec{k}=\vec{0})\rangle = \sqrt{Z_p}u^{(s')}_s \quad (9)$$

is obtained from the proton correlator with the local source and local sink. It is recognized that the precise determination of $\sqrt{Z_p}$ is not easy because of large statistical fluctuations of the local-local correlator (see, e.g., Fig. 16 of Ref. [9]). On the other hand, the ratio of two-point functions in Eq. (6), calculated using the smeared-local proton correlator, is determined well with small statistical errors.

Under this circumstance we calculate $\sqrt{Z_p}$ from the proton correlators generated in high statistics calculations of the quenched light hadron spectrum performed by the CP-PACS Collaboration [9]. We then carry out new simulations with the same parameters to obtain the ratio of two-point functions including the mixing operator, for which we do not necessarily need very high statistics. We attain a few percent statistical accuracy for the latter, while the overall accuracy is still limited by the error of $\sqrt{Z_p}$.

Our simulation generates quenched gauge configurations at $\beta = 5.90, 6.10$ and 6.25, which correspond to lattice spacings in the range $a \approx 0.1 - 0.064$ fm when determined from the ρ meson mass $m_\rho = 0.7684$ GeV. The spatial lattice size is kept at about 3 fm to avoid finite size effects. We take four quark masses corresponding to $m_{PS}/m_V \approx 0.75 - 0.5$ for each β. These parameters are the same as those of the CP-PACS spectrum calculations [9], except that we drop the finest lattice and the lightest quark mass at each β for the computational cost. The simulation parameters are presented in Table I. The number of configurations in our simulation is about 1/3 that of the CP-PACS calculation. We employ for Ψ in Eq. (8) the pion quark wave function, which is measured for each hopping parameter on 30 gauge configurations fixed to

TABLE I. Simulation parameters and results. The lattice spacing a [fm] in the third column is determined from m_ρ. Results are given with three different input quantities for the lattice spacing, i.e., m_N, m_ρ, and f_π.

			Number of Configurations	$\lvert\alpha$(NDR, 2 GeV)$\rvert$[GeV^3]			$\lvert\beta$(NDR, 2 GeV)$\rvert$[GeV^3]		
β	$L^3\times T$	a [fm]	this work/CP-PACS [9]	m_N input	m_ρ input	f_π input	m_N input	m_ρ input	f_π input
5.90	$32^3\times 56$	0.1020(8)	300/800	0.01026(31)	0.01265(43)	0.01563(72)	0.01064(32)	0.01312(44)	0.01621(76)
6.10	$40^3\times 70$	0.0777(7)	200/600	0.01041(35)	0.01152(41)	0.01398(81)	0.01092(37)	0.01209(44)	0.01467(85)
6.25	$48^3\times 84$	0.0642(7)	140/420	0.00956(35)	0.01012(45)	0.01321(75)	0.01004(35)	0.01063(47)	0.01388(78)
		$a=0$		0.0090(09)	0.0063(13)	0.0092(22)	0.0096(09)	0.0069(14)	0.0100(23)

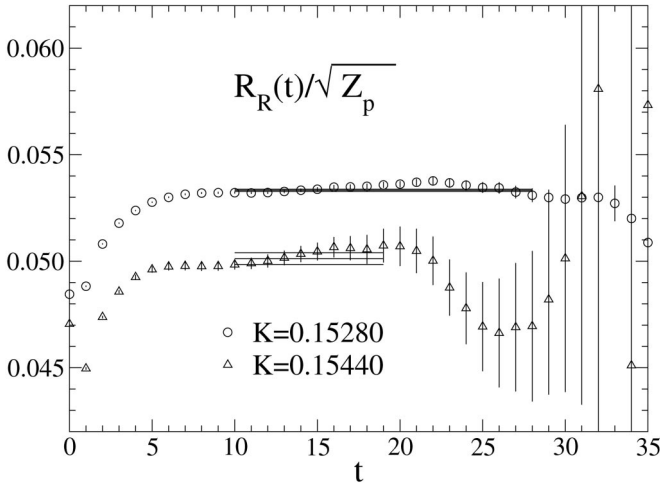

FIG. 1. Time dependence of $R_R(t)$ (the factor $\sqrt{Z_p}$ being removed) for the heaviest (circle) and lightest (triangle) quark masses at $\beta = 6.10$.

the Coulomb gauge except for the $t = 0$ time slice where the wall source is placed[12].

To estimate $\sqrt{Z_p}$, we fit the smeared-local proton correlator to a single exponential $Z_p' \exp(-m_p t)$, and then fit the local-local proton correlator to $Z_p \exp(-m_p t)$ with m_p fixed to the value determined from the smeared-local correlator, which is borrowed from the CP-PACS simulation [9]. Figure 1 shows the ratio of two-point functions in Eq. (6) with the $\sqrt{Z_p}$ factor removed for the heaviest ($K = 0.15280$) and the lightest ($K = 0.15440$) quark masses at $\beta = 6.10$. The horizontal lines represent the fits together with 1 standard deviation errors, which are smaller than 1%. In Fig. 2 we plot the quark mass dependence of the α parameter at $\beta = 6.10$, which is well described by a linear function. A similar quark mass dependence is observed for the β parameter. We find that linear plus quadratic extrapolations yield results consistent within

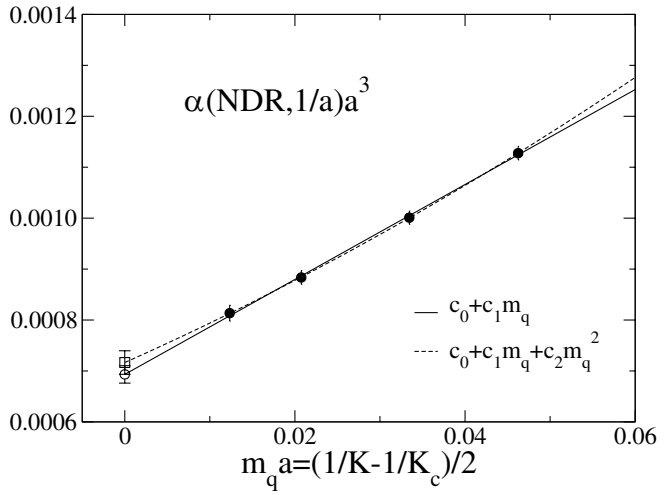

FIG. 2. Chiral extrapolation of the α parameter with linear (solid) and quadratic (dotted) functions at $\beta = 6.10$.

error bars in the chiral limit at all lattice spacings. The α and β parameters in the chiral limit obtained by linear extrapolations are summarized in Table I. The errors are at most a few percent. The contribution of the mixing operator in Eq. (3) is smaller than 10%.

We present in Fig. 3 α and β in physical units as a function of a. We examine three choices, the nucleon mass m_N, the ρ mass m_ρ, and the pion decay constant f_π to determine the physical scale of the lattice. We take these physical parameters given by CP-PACS [9], because α and β have dimension three and the error of mass scale is magnified by a factor of 3, so that high statistics results are essential. Figure 3 indicates that scaling violation α and β is minimized if nucleon mass is used as input. The use of mesonic quantities, m_ρ or f_π, on the other hand, leads to substantial scaling violation. A simple linear extrapolation to the continuum limit results in α and β that vary up to 30% depending on the input physical scale as found in Table I.

We adopt the α and β, extrapolated to $a = 0$, using the m_N input as our central value, since small scaling violation would minimize the error associated with the continuum extrapolation, and include the uncertainty in the physical scale as systematic error. We obtain

$$|\alpha(\mathrm{NDR}, 2\ \mathrm{GeV})| = 0.0090(09)(^{+5}_{-19})\ \mathrm{GeV}^3, \qquad (10)$$

$$|\beta(\mathrm{NDR}, 2\ \mathrm{GeV})| = 0.0096(09)(^{+6}_{-20})\ \mathrm{GeV}^3, \qquad (11)$$

where the first error is statistical and the second one is systematic. Since the CP-PACS spectrum calculation [9] is superior to this work in controlling the systematic errors using finer lattices and lighter quark masses than this simulation, we estimate the ambiguity due to scale setting from their results of quenched light hadron mass

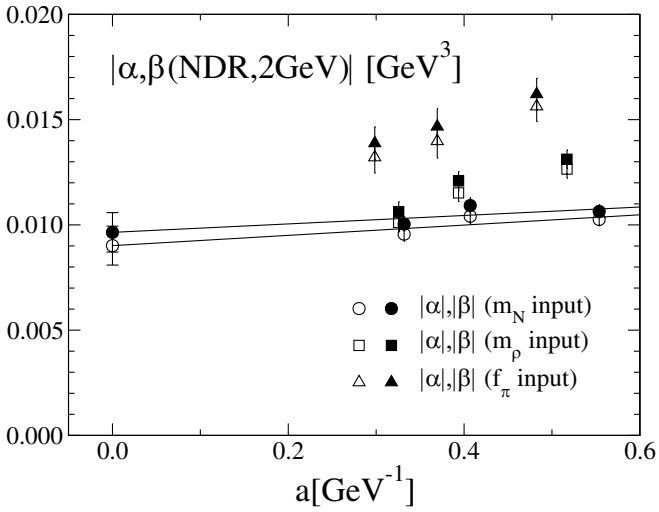

FIG. 3. Continuum extrapolation of α (open) and β (filled) parameters. The lattice scale is determined from m_N (circle), m_ρ (square) and f_π (triangle) at each β. The errors in the continuum limit are statistical only.

spectrum. They show that the values of m_ρ and f_π in quenched QCD deviate from the experiment by $+7\%$ and -2% respectively in the continuum limit, once we set the lattice spacing by m_N. The errors of mass scale in α and β are magnified by a factor of 3 and found to be comparable with the variation of the results at the continuum limit in Table I. This implies that the systematics from the physical scale dependence are mostly ascribed to the quenching effects. We note that the sign of α and β are relatively opposite, while the overall sign is a convention. Our results are about 3 times larger than the smallest estimate among various QCD model predictions, $|\alpha| = |\beta| = 0.003\ \mathrm{GeV}^3$ [13], which is often used in phenomenology of GUTs to derive "conservative" estimates of proton lifetime.

Our present α and β are smaller than the previous results using the same gauge and quark actions, $|\alpha(\mathrm{NDR}, 1/a)| = 0.015(1)\ \mathrm{GeV}^3$ and $|\beta(\mathrm{NDR}, 1/a)| = 0.014(1)\ \mathrm{GeV}^3$ at $1/a = 2.30(4)$ GeV [7], beyond what is expected from scaling violation obtained in this work. We suspect that $\sqrt{Z_p}$ and the lattice scale determined from m_ρ are overestimated while their errors are not properly estimated, probably due to large fluctuations in $\sqrt{Z_p}$ for which only 100 configurations were used. We also compare our α and β with those of the preliminary results of RBC Collaboration using quenched domain wall QCD with the DBW2 gauge action on an $L^3 \times T \times N_5 = 16^3 \times 32 \times 12$ lattice at $1/a = 1.23(5)$ GeV: $|\alpha(\mathrm{NDR}, 1/a)| = 0.006(1)\ \mathrm{GeV}^3$ and $|\beta(\mathrm{NDR}, 1/a)| = 0.007(1)\ \mathrm{GeV}^3$ [8], which are smaller by 30% than our values. This is of the order of scaling violation that is expected when the physical scale is set by mesonic quantities, but a quantitative comparison awaits their calculation of the continuum limit. It should be noted that the effect of the change of renormalization scale from $1/a = 1.23$ GeV to 2 GeV is about 3.5% and negligibly small.

In conclusion, we have studied scaling violation in the proton decay α and β parameters in quenched lattice QCD. Scaling violation is mild if the physical lattice scale is set by the nucleon mass, whereas a 30% systematic errors may arise from the physical scale, reflecting a part of the quenching error. Our estimate of α and β is larger by about 3 times than the smallest prediction among various QCD models. This implies stronger constraints on GUT models. We can eliminate a remaining major uncertainty due to the quenched approximation by repeating the calculation on the full QCD gauge configurations which we already have [14]. This should be a next task.

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