Kaon B Parameter from Quenched Lattice QCD

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(Received 22 October 1997)

We present results of a large-scale simulation for the kaon B parameter B_K in quenched lattice QCD with the Kogut-Susskind quark action. Calculating B_K at 1% statistical accuracy for seven values of lattice spacing in the range $a \approx 0.24-0.04$ fm on lattices up to $56^3 \times 96$, we verify the theoretically predicted quadratic a dependence. Strong indications are found that, with our level of accuracy, $\alpha_{\overline{\rm MS}}(1/a)^2$ terms arising from our one-loop matching procedure have to be included in the continuum extrapolation. We present $B_K(\text{NDR, 2 GeV}) = 0.628(42)$ as our final value, where NDR indicates naive dimensional regularization. [S0031-9007(98)06280-2]

PACS numbers: 12.38.Gc, 14.40.Aq

The knowledge of the kaon B parameter B_K

$$B_K = \frac{\langle \bar{K}^0 | \bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle}{\frac{8}{3}\langle \bar{K}^0|\bar{s}\gamma_\mu\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K^0\rangle}, \quad (1)$$

is imperative to extract the CP violation parameter of the Cabibbo-Kobayashi-Maskawa matrix from experiment. Work has been continued for a decade to determine this parameter with lattice QCD [1] using both Wilson and Kogut-Susskind quark actions. Calculations with the latter have the advantage [2] that the correct chiral behavior of the matrix element is ensured by U(1) chiral symmetry. Nonetheless, previous studies with this action [2–5] have not yielded a definitive result for the matrix element.

A major difficulty, uncovered in Ref. [2], is the presence of a large scaling violation in B_K, which renders a reliable extrapolation to the continuum limit nontrivial. Whereas the scaling violation is theoretically expected to be $O(a^2)$ [6] with the Kogut-Susskind action, simulations so far [2,3,5] could not confirm it due to large statistical errors.

Another problem concerns systematic uncertainties in the renormalization factors needed to match the lattice result to that in the continuum. While an earlier study [3] found that one-loop perturbation theory is reasonably accurate, the problem of the systematic error associated with renormalization has not been fully explored yet.

In order to resolve these problems, we have carried out a large-scale simulation for B_K with the Kogut-Susskind quark action in quenched lattice QCD. In this Letter we report on the continuum limit of B_K, expounding the crucial points of our simulations and analysis.

The parameters employed in our simulations are summarized in Table I. In order to study the continuum limit, seven values of the inverse gauge coupling constant $\beta = 6/g^2$ spanning the range $\beta = 5.7-6.65$ are chosen for the simulations, corresponding to the lattice spacing $a \approx 0.24-0.04$ fm. We set the physical scale of lattice spacing by ρ meson mass in the VT channel. The physical lattice size is kept approximately constant at La $\approx 2.3-2.5$ fm in order to distinguish scaling violation effects from those of finite lattice. Finite-size effects are examined separately at $\beta = 6.0$ and 6.4, varying the lattice size over the range La $\approx 1.8-3.1$ fm. Numerical simulations have been carried out on the Fujitsu VPP500/80 supercomputer at KEK.

We employ both gauge-invariant and noninvariant four-quark operators [3], which differ by an insertion of gauge link factors connecting the quark fields spread over a 2^4 hypercube. The bare lattice operators are mean-field improved through a replacement $\chi \to \sqrt{u_0}\,\chi$ for the quark field and $U_\mu \to u_0^{-1}U_\mu$ for the gluon field, where $u_0 = P^{1/4}$ [7], P being the average value of the plaquette.

The matching of B_K between lattice and continuum is made in the following way. We first correct lattice values of B_K by the one-loop renormalization factor [8,9] evaluated with the $\overline{\rm MS}$ coupling $\alpha_{\overline{\rm MS}}(q^*)$ at a matching scale $q^* = 1/a$ [10,11] to obtain the continuum operator $B_K(\text{NDR}, q^*)$ renormalized in the $\overline{\rm MS}$ scheme with the naive dimensional regularization (NDR). The continuum value at a physical scale $\mu = 2$ GeV is then obtained via a two-loop running of the continuum renormalization group starting from $B_K(\text{NDR}, q^*)$,

$$\begin{aligned} B_K(\text{NDR}, \mu) = &\left[1 - \frac{\alpha_{\overline{\rm MS}}(\mu)}{4\pi}\frac{\gamma_1\beta_0 - \gamma_0\beta_1}{2\beta_0^2}\right]^{-1} \\ &\times \left[1 - \frac{\alpha_{\overline{\rm MS}}(q^*)}{4\pi}\frac{\gamma_1\beta_0 - \gamma_0\beta_1}{2\beta_0^2}\right] \\ &\times \left[\frac{\alpha_{\overline{\rm MS}}(q^*)}{\alpha_{\overline{\rm MS}}(\mu)}\right]^{-\gamma_0/2\beta_0} B_K(\text{NDR}, q^*), \end{aligned} \quad (2)$$

TABLE I. Run parameters.

β	$m_q a$	$L^3 T$	No conf.	m_ρ	a^{-1} (GeV)	La (fm)	$t_{\min} - t_{\max}$	$m_s a/2$	P
5.7	0.02–0.08	$12^3 24$	150	0.9120(7)	0.837(6)	2.83	6–16	0.0519(8)	0.549 00
5.85	0.01–0.04	$16^3 32$	60	0.567(13)	1.36(3)	2.32	10–20	0.0201(9)	0.575 06
5.93	0.01–0.04	$20^3 40$	50	0.484(10)	1.59(3)	2.48	12–26	0.0160(6)	0.585 64
6.0	0.01–0.04	$24^3 64$	50	0.410(10)	1.88(4)	2.52	15–47	0.0125(5)	0.593 74
		$18^3 64$	50	0.413(12)	1.87(6)	1.90	15–47	0.0127(7)	
		$32^3 48$	40	0.383(3)	2.01(2)	3.14	15–31	0.0109(2)	
6.2	0.005–0.02	$32^3 64$	40	0.291(10)	2.65(9)	2.39	20–42	0.008 84(57)	0.613 65
6.4	0.005–0.02	$40^3 96$	40	0.222(5)	3.47(7)	2.28	25–69	0.006 92(29)	0.630 65
		$32^3 96$	40	0.216(7)	3.57(11)	1.77	25–69	0.006 59(41)	
		$48^3 96$	20	0.219(4)	3.52(7)	2.69	25–69	0.006 81(26)	
6.65	0.004–0.016	$56^3 96$	40	0.158(4)	4.87(12)	2.27	36–58	0.005 11(25)	0.649 12

where $\beta_0 = 11$, $\beta_1 = 102$, $\gamma_0 = 4$, and $\gamma_1 = -7$ [12] are the $N_f = 0$ quenched values for the renormalization group coefficients. This procedure leaves an uncertainty of $O(\alpha_{\overline{\rm MS}}(q^*)^2)$ in $B(\mathrm{NDR}, q^*)$ arising from the use of one-loop renormalization factors [5,13].

The coupling constant $\alpha_{\overline{\rm MS}}(q^*)$ needed in the matching factor is obtained once the $\Lambda_{\overline{\rm MS}}$ is specified. To estimate this, we start from α_P [14] defined by $-\ln P = 4\pi/3\alpha_P(3.40/a)(1 - 1.19\alpha_P)$, and calculate $\Lambda_{\overline{\rm MS}} = 0.625\Lambda_P$ from $\alpha_P(3.40/a)$, where the three-loop correction term is included. The value of $\Lambda_{\overline{\rm MS}}$ estimated in this way, however, suffers from scaling violation. We therefore extrapolate the results at our seven values of β quadratically in $m_\rho a$ to the continuum limit, finding $\Lambda_{\overline{\rm MS}} = 232(4)$ MeV. We then take $\Lambda_{\overline{\rm MS}} = 230$ MeV, and calculate the $\overline{\rm MS}$ running coupling to three-loop accuracy, which is used throughout our analyses to minimize additional scaling violation entering into the B_K calculation.

In our simulations gauge configurations are generated with the 5-hit heat bath algorithm, and B_K is calculated at every 1000 ($\beta = 5.7$), 2000 ($\beta \le 6.0$), or 5000 ($\beta \ge 6.2$) sweep intervals. Our main results are based on calculations at four values of degenerate strange and down quark mass $m_q a$, equally spaced in the interval given in Table I.

Lattice values of B_K are calculated from the three-point Green function of the four-quark operator at time t with two kaons created at the temporal edges of the lattice, divided by the vacuum saturation of the same operator. Eight wall sources corresponding to the corners of a spatial cube are employed to construct a quark-antiquark propagator combination such that only the pseudoscalar meson in the Nambu-Goldstone channel propagates [15]. Quark propagators are calculated with the Dirichlet boundary condition in time and the periodic boundary condition in space. Gauge configurations are fixed to the Landau gauge.

The fitting interval to extract B_K from the Green function is chosen so that the minimum time $t_{\min}a$ is approximately constant at $t_{\min}a \approx 1.4$–1.5 fm for all values of β. The resulting B_K changes less than $\pm 0.3\%$ for all $m_q a$ and β under a variation of $t_{\min}$ by ± 2.

At each value of β lattice results are interpolated in $m_q a$ with the formula suggested by chiral perturbation theory [16],

$$B_K = B[1 - 3cm_q a \ln(m_q a) + bm_q a]. \quad (3)$$

The physical value of B_K is obtained at half the strange quark mass $m_s a/2$, estimated from experiment, $m_K/m_\rho = 0.498/0.770$.

We present our results in Table II. The errors are estimated by a single elimination jackknife procedure. Our statistical error is small, being 0.1% at $\beta = 5.7$ and gradually increasing to 1.2% at $\beta = 6.65$. At $\beta = 6.0$ and 6.4, three spatial sizes are examined for a finite-size study. Some size dependence of order 2% is seen below the spatial size La ≈ 2.0 fm at $\beta = 6.4$, but the magnitude decreases to less than 0.5% for La $\gtrsim 2.2$ fm at both values of β. We have made our main runs with a spatial size larger than La ≈ 2.3 fm, thus expecting finite-lattice corrections being smaller than the statistical error.

We present B_K(NDR, 2 GeV) as a function of $m_\rho a$ in Fig. 1 for both gauge noninvariant (circles) and invariant

TABLE II. Results for B_K(NDR, 2 GeV) at each β calculated with the matching scale $q^* = 1/a$.

		B_K(NDR, 2 GeV)		
β	$L^2 T$	Noninvariant	Invariant	ΔB_K
5.7	$12^3 24$	0.8464(7)	0.8224(7)	0.0240(3)
5.85	$16^3 32$	0.7798(25)	0.7562(25)	0.0236(11)
5.93	$20^3 40$	0.7522(23)	0.7229(22)	0.0292(8)
6.0	$24^3 64$	0.7154(23)	0.6826(24)	0.0328(5)
	$18^3 64$	0.7174(68)	0.6787(68)	0.0388(12)
	$32^3 48$	0.7128(14)	0.6790(16)	0.0339(8)
6.2	$32^3 64$	0.6619(48)	0.6243(45)	0.0376(14)
6.4	$40^3 96$	0.6428(67)	0.6069(69)	0.0359(10)
	$32^3 96$	0.6577(122)	0.6126(112)	0.0451(24)
	$48^3 96$	0.6415(48)	0.6072(51)	0.0343(11)
6.65	$56^3 96$	0.6350(70)	0.6055(72)	0.0295(10)

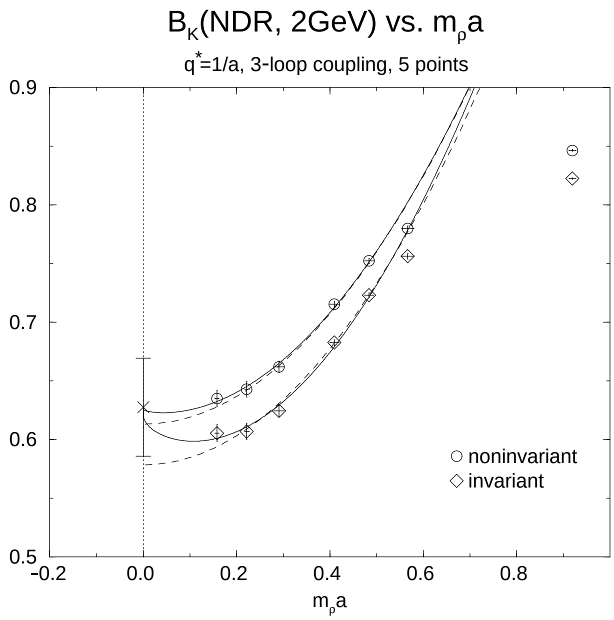

FIG. 1. Gauge noninvariant (circles) and invariant (diamonds) B_K(NDR, 2 GeV) as a function of $m_\rho a$, together with a simultaneous fit for the two operators including α^2 term (solid lines) and separate fits quadratic in a (dashed lines) to the five pairs of data points for $\beta \geq 5.93$.

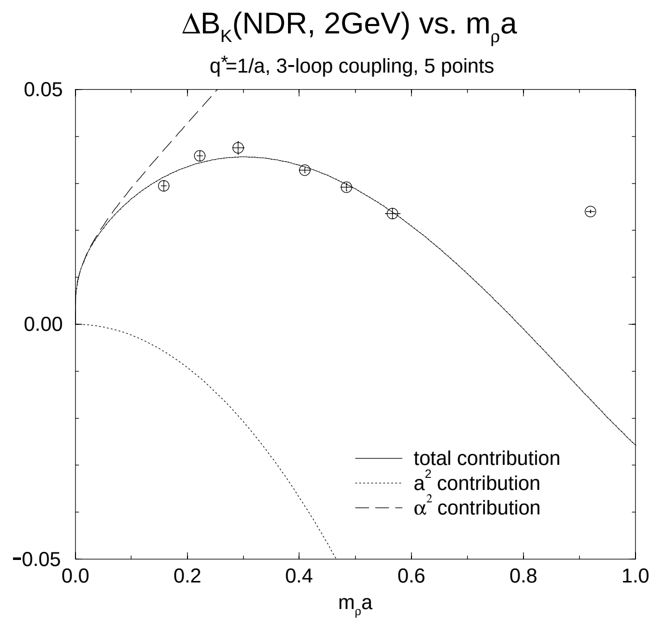

FIG. 2. Difference of B_K(NDR, 2 GeV) between gauge noninvariant and invariant operators as a function of $m_\rho a$. The solid line represents a fit with a^2 and α^2 terms, while the dotted (dashed) line is the contribution from the a^2 (α^2) term.

(diamonds) operators. The five points below $m_\rho a \approx 0.6(\beta \geq 5.93)$ are consistent with the $O(a^2)$ scaling behavior theoretically expected [6]. Toward large lattice spacings, however, we observe a change of curvature from a positive to a negative sign. At an intermediate range $m_\rho a \approx 0.6$–0.3 ($\beta = 5.85$–6.2) a cancellation among the a^2 and higher order terms conspires to yield an apparently linear dependence of B_K. This is the linear behavior we observed at an early stage of our work [17]. The later result at a smaller lattice spacing $m_\rho a \approx 0.22(\beta = 6.4)$ gave a first indication of an $O(a^2)$ behavior [18]; this is now confirmed by the calculation at a yet smaller lattice spacing $m_\rho a \approx 0.16(\beta = 6.65)$ given in this paper.

In our preliminary report [18] we took a naive approach to estimate the continuum B_K, simply by applying a polynomial fit assuming $O(a^2)$ dependence. A fit of the five points above $\beta = 5.93$ with the form $B_K = c_0 + c_1(m_\rho a)^2$, shown by the dashed lines in Fig. 1, gives a value at the continuum B_K(NDR, 2 GeV) $= 0.616(5)$ for the gauge noninvariant operator, and 0.580(5) for the invariant one, the average of the two being 0.598(5).

An obvious problem with this analysis is that the two operators yield different values. We recall that B_K for the two operators, and hence also their difference, should receive not only $O(a^2)$ scaling violation but also $\alpha_{\overline{\rm MS}}(q^*)^2$ errors from the matching procedure. Figure 2 plots the difference as a function of $m_\rho a$ (numerical values given in Table II). Errors, as calculated with the jackknife procedure, are only 3%–4%, as a result from a strong correlation between the matrix elements of the two operators. We find that the difference can be fitted by the form $b_1(m_\rho a)^2 + b_2\alpha_{\overline{\rm MS}}(q^*)^2$: employing five data points for $m_\rho a \lesssim 0.5$ we obtain $b_1 = -0.23(2)$ and $b_2 = 1.73(5)$ for $\chi^2/\text{d.o.f} = 2.2$. The solid line indicates the fit, and the others show the breakdown into the a^2 (dotted line) and α^2 (dashed line) contributions. A fit allowing a constant b_0 yields a value of b_0 vanishing within 2σ: $b_0 = -0.032(16)$, $b_1 = -0.44(11)$, and $b_2 = 3.4(8)$. These results strongly indicate that the decrease of the difference toward small lattice spacings seen in Fig. 2 is actually an $\alpha_{\overline{\rm MS}}(q^*)^2$ effect.

Encouraged by this analysis we attempt to fit the five points at $\beta \geq 5.93$ simultaneously for both operators including their correlations with the form $B_K^{\rm noninv} = c_0 + c_1a^2 + c_2\alpha_{\overline{\rm MS}}(q^*)^2$ and $B_K^{\rm inv} = d_0 + d_1a^2 + d_2\alpha_{\overline{\rm MS}}(q^*)^2$. This yields $c_0 = 0.67(6)$ and $d_0 = 0.71(7)$, and hence we impose the constraint $c_0 = d_0$ in our final fit. In the continuum limit the fit (solid lines in Fig. 1) gives B_K(NDR, 2 GeV) $= 0.628(42)$ with $\chi^2/\text{d.o.f} = 1.37$. The error is roughly 10 times the one from the naive quadratic fit. This large error reflects uncertainties of the coefficient of the α^2 terms: $c_2 = -0.5(2.0)$ and $d_2 = -2.2(2.0)$. The difference, however, is well constrained; $c_2 - d_2 = 1.7$ agrees well with $b_2 = 1.73$ obtained above.

We find larger coefficients $c_2 = -1.0(4.2)$ and $d_2 = -4.3(4.2)$ when $q^* = \pi/a$ is used, or $c_2 = 1.6(1.5)$ and $d_2 = -3.2(1.5)$ if mean-field improvement is not made for the operators. This supports the tadpole argument of Ref. [7].

The final value depends only weakly on our choice of $\Lambda_{\overline{\rm MS}} = 230$ MeV: e.g., B_K(NDR, 2 GeV) = 0.627(42) for $\Lambda_{\overline{\rm MS}} = 220$ MeV and 0.628(41) for 240 MeV.

As our final value of B_K in the continuum limit, we adopt the fit including the α^2 term,

$$B_K(\text{NDR, 2 GeV}) = 0.628 \pm 0.042\,, \tag{4}$$

which includes a systematic error from the two-loop uncertainty. The size of the quoted error is 6.6%, which roughly equals $3 \times \alpha_{\overline{\rm MS}}(q^* = 1/a)^2$ at our smallest lattice spacing $1/a = 4.87$ GeV at $\beta = 6.65$ where $\alpha_{\overline{\rm MS}}(4.87\text{ GeV}) = 0.147$. This magnitude of error is unavoidable, even with 1% statistical accuracy at each β achieved in our simulation, unless a two-loop calculation is carried out for the lattice renormalization.

Let us compare our final result with the JLQCD value obtained using the Wilson quark action,

$$B_K(\text{NDR, 2 GeV}) = 0.62 \pm 0.10\,, \tag{5}$$

in which the operator mixing problem is solved nonperturbatively with the aid of chiral Ward identities [19]. The error, which is either statistical or systematic depending on the method of the continuum extrapolation, is substantially larger than that of the present work with the Kogut-Susskind action. Thus, while the two results are consistent, reducing the uncertainty of the Wilson result is needed to verify an agreement of the value of B_K at the level of precision achieved for our Kogut-Susskind result (4).

One of the systematic errors not taken into account in our final result (4) is the effect of nondegenerate strange and down quark masses $m_s \neq m_d$. Analyzing this problem is difficult within quenched QCD since the chiral limit $m_d \to 0$ with $m_s \neq 0$ is expected to diverge due to a quenched chiral logarithm [20]. Our attempt at a verification of the logarithmic divergence is also inconclusive: our results for nondegenerate quarks can be fitted quite well either with or without the singular term. At this stage we are not able to quote the magnitude of error due to the use of degenerate quark masses.

Finally, our quoted error does not include effects of sea quarks. Preliminary attempts suggest that the quenching error may not exceed 5% or so [3,4,21]. More extensive efforts, however, are clearly needed to estimate dynamical quark contributions to the B_K parameter. Full QCD simulations should also enable us to answer the issues with nondegenerate quark masses. Carrying out such calculations represents the final step toward the first-principle determination of the kaon B parameter.

This work is supported by the Supercomputer Project (No. 97-15) of High Energy Accelerator Research Organization (KEK), and also in part by the Grants-in-Aid of the Ministry of Education (No. 08640349, No. 08640350, No. 08640404, No. 09246206, No. 09304029, and No. 09740226).

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