$K^+ \to \pi^+\pi^0$ Decay Amplitude in Quenched Lattice QCD*

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A new study is reported of a lattice QCD calculation of the $K^+ \to \pi^+\pi^0$ decay amplitude with the Wilson quark action in the quenched approximation at $\beta = 6.1$. The amplitude is extracted from the $K \to \pi\pi$ Green function, and a conversion to the continuum value is made employing a recent one-loop calculation of chiral perturbation theory. The result is consistent with the experimental value if extrapolated to the chiral limit.

1. Introduction

It has long been a problem that the $\Delta I = 3/2$ $K^+ \to \pi^+\pi^0$ amplitude calculated in quenched lattice QCD is about a factor two too large compared to the experimental value[1]. In this article we report a new study of this problem, incorporating various theoretical and technical advances in recent years for analysis. In particular we discuss in detail how one-loop corrections of chiral perturbation theory (CHPT) recently calculated[3] affect physical predictions for the decay amplitude from lattice QCD simulations.

Our simulation is carried out in quenched lattice QCD employing the standard plaquette action for gluons at $\beta = 6.1$ and the Wilson action for quarks. Two lattice sizes, $24^3 \times 64$ and $32^3 \times 64$, are employed. We take up, down and strange quarks to be degenerate, and make measurements at four values of the common hopping parameter, $\kappa = 0.1520$, 0.1530, 0.1540 and 0.1543, which correspond to $m_\pi/m_\rho = 0.797$, 0.734, 0.586 and 0.515.

2. Extraction of decay amplitude

The 4-quark operator most relevant for the $\Delta I = 3/2$ $K \to \pi\pi$ decay is $Q_+ = [(\bar{s}d)_L(\bar{u}u)_L + (\bar{s}u)_L(\bar{u}d)_L]/2$. We extract the decay amplitude from the 4-point correlation function defined by $M_Q = \langle 0|W_+ W_0 Q_+(t) W_K|0\rangle$ where $W_{0,+,K}$ are wall sources for π^0, π^+ and K^+. In our calculations on a lattice of a temporal size $T = 64$, the walls are placed at the time slices $t_{K^+} = 4$, $t_{\pi^+} = 59$ and $t_{\pi^0} = 60$. The mesons are all created at rest, and the 4-quark operator Q_+ is projected to zero spatial momentum.

We can use two types of factors to remove the normalization factors in M_Q. If we define

$$M_W = \langle 0|W_0\pi^0(t)|0\rangle\langle 0|W_+\pi^+(t)|0\rangle\langle 0|K(t)W_K|0\rangle,$$
$$M_P = \langle 0|K(t)W_K|0\rangle\langle 0|W_+W_0\pi^+(t)\pi^0(t)|0\rangle, \quad (1)$$

we find $R_W \equiv M_Q/M_W \sim \langle\pi^+\pi^0|Q_+|K^+\rangle / \langle\pi|\pi|0\rangle^3 \cdot \exp(t - t_+)\Delta$ and $R_P \equiv M_Q/M_P \sim \langle\pi^+\pi^0|Q_+|K^+\rangle/\langle\pi|\pi|0\rangle^3$ for $t_K \ll t \ll t_+, t_0$, where $\Delta = m_{\pi\pi} - 2m_\pi$ is the mass shift of the 2-pion state due to finite lattice size effects [4].

In Fig. 1 we plot $\langle\pi|\pi|0\rangle^3 \cdot R_W$ and $\langle\pi|\pi|0\rangle^3 \cdot R_P$ at $\kappa = 0.1530$ as a function of time t of the weak

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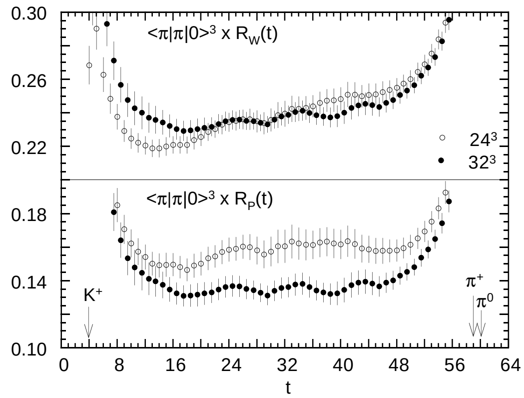

Figure 1. $R_W \cdot \langle\pi|\pi|0\rangle^3$ and $R_P\langle\pi|\pi|0\rangle^3$ at $\kappa = 0.153$. Open and filled circles refer to data for 24^3 and 32^3 lattices.

operator. We clearly observe a non-vanishing slope for R_W, while R_P exhibits a plateau as expected. The decay amplitude can be obtained by fitting R_W to a single exponential or R_P to a constant. We find the results to be mutually consistent within the statistical error.

The one-loop renormalization factor relating the operator Q_+ on the lattice and that in the continuum was obtained in Refs. [5,6]. We set $q^* = 1/a$ or π/a as the matching point and employ tadpole-improved perturbation theory with the mean-field improved $\overline{\rm MS}$ coupling constant[7]. Quark fields are normalized with the KLM factor [8], $\sqrt{1-3\kappa/4\kappa_c}$.

3. Result

In earlier calculations tree-level chiral perturbation theory (CHPT) formula was used to obtain the physical amplitude A^{phys} from the lattice counterpart A^{lat}. The formula takes the form $A^{phys} = (m_K^2 - m_\pi^2)/(2M_\pi^2) \cdot A^{lat}$ where $m_K = 497$MeV and $m_\pi = 136$MeV are physical masses, and M_π is the degenerate mass of K and π mesons on the lattice. Clearly the physical amplitude should be independent of the lattice mass M_π if the matching formula is exact.

In Fig. 2 we compare results for the physical decay amplitude from a previous work[2] carried out at $\beta = 6.0$ on a 24^3 spatial lattice with those of our simulation at $\beta = 6.1$. Our values obtained with the conventional quark normalization $\sqrt{2\kappa}$

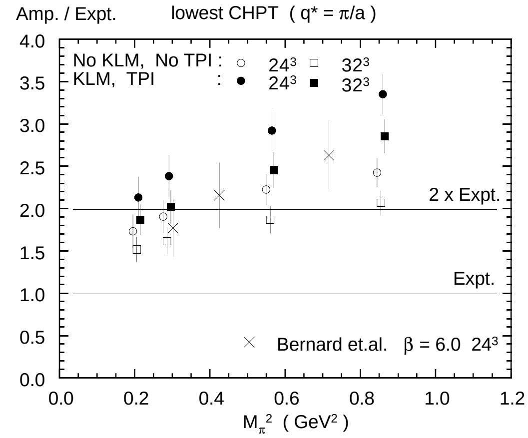

Figure 2. Comparison of our results normalized by the experimental value obtained with tree-level CHPT relation for $q^* = \pi/a$ at $\beta = 6.1$ with those of previous work[2] at $\beta = 6.0$ (crosses). Open symbols refer to results with the traditional normalization $\sqrt{2\kappa}$ and no tadpole improvement, and filled ones are those with the KLM normalization and tadpole improvement.

(crosses and open squares) are consistent with their results, but are larger than the experimental result roughly by a factor two.

Let us also note with our results (circles and squares) that (i) the KLM normalization and tadpole improvement of the operator have a significant effect on the amplitude, (ii) there is a clear dependence of the amplitude on the lattice meson mass M_π, and (iii) a significant finite-size effect is observed between the two lattice sizes. These features show that the tree-level CHPT is inadequate to extract the physical amplitude from lattice results.

In Fig. 3 we show how predictions for the physical amplitude change if we apply the one-loop formula of CHPT [3] to our results. Here Λ^q and Λ^{cont} are the cutoffs of CHPT for quenched and full theory. In converting to physical values we ignore effects of $O(p^4)$ contact terms of the CHPT Lagrangian since their values are not well known. An interesting point is that a size dependence seen with the tree-level analysis in Fig. 2 is absent. Another important point is that the magnitude of the amplitude decreases by $30-40\%$ over the range of meson mass covered in our simulation. The amplitude depends significantly on the

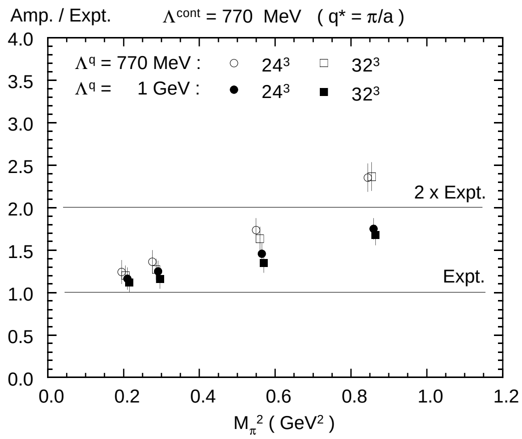

Figure 3. Decay amplitude normalized by experimental value for $q^* = \pi/a$ obtained with one-loop CHPT for $\Lambda^{cont} = 770$MeV. Circles and squares refer to data for 24^3 and 32^3 spatial sizes. Open symbols are for $\Lambda^q = 770$MeV and filled ones for 1 GeV.

quenched lattice cutoff Λ^q, in particular toward larger values of M_π.

Our results in Fig. 3 show that a sizable meson mass dependence still remains. This may be attributed to $O(p^4)$ contact terms which were not taken into account. If we assume that the contact terms of the full theory are as small as suggested by the phenomenological analysis [9], we can estimate the physical amplitude by a linear extrapolation of M_π^2 to the chiral limit. In Table 1 we list results for several choices of the cutoff and the operator matching point q^*. As is expected from Fig. 3, variation of results on the choice of the cutoff parameters is small, being of the order of 10%. Allowing for this uncertainty, the values in Table 1 are consistent with the experimental value of $10.4 \times 10^{-3}\text{GeV}^3$.

4. Conclusions

The present study has shown that the quenched result for the $K^+ \to \pi^+\pi^0$ decay amplitude agrees with experiment much better than previously thought, especially if the chiral limit is taken for the lattice meson mass. Away from the chiral limit, the amplitude depends significantly on the cutoff scales of CHPT, however. This problem is largely ascribed to a mismatch of chiral logarithms between the quenched and full theories[3]. In order to obtain $K^+ \to \pi^+\pi^0$ decay amplitude free from this uncertainty, we need simulations in full QCD.

		$C_+\langle\pi\pi\vert Q_+\vert K\rangle(\times 10^{-3}\text{GeV}^3)$			
Λ^{cont}	Λ^q	24^3		32^3	
(GeV)	(GeV)	$q^* = \frac{1}{a}$	$q^* = \frac{\pi}{a}$	$q^* = \frac{1}{a}$	$q^* = \frac{\pi}{a}$
0.77	0.77	9.3(19)	10.2(21)	8.9(17)	9.7(19)
0.77	1.0	9.4(13)	10.3(14)	8.8(11)	9.6(12)
1.0	0.77	10.3(21)	11.3(23)	9.8(19)	10.7(21)
1.0	1.0	10.4(14)	11.4(15)	9.7(12)	10.6(13)

Table 1
Results of linear extrapolation of the decay amplitude to $M_\pi^2 = 0$. Statistical and extrapolation errors are combined. The experimental value is $10.4 \times 10^{-3}\text{GeV}^3$.

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