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$I = 2$ pion-pion scattering phase shift in the continuum limit calculated with two-flavor full QCD *

CP-PACS Collaboration : T. Yamazaki[a], S. Aoki[a], M. Fukugita[b], K-I. Ishikawa[c], N. Ishizuka[a,d], Y. Iwasaki[a,d], K. Kanaya[a], T. Kaneko[e], Y. Kuramashi[e], V. Lesk[d], M. Okawa[c], Y. Taniguchi[a], A. Ukawa[a,d] and T. Yoshié[a,d]

[a]Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan

[b]Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba, 277-8582, Japan

[c]Department of Physics, Hiroshima University, Higashi-Hiroshima, Hiroshima 739-8526, Japan

[d]Center for Computational Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan

[e]High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan

We present a calculation of the scattering phase shift for the $I = 2$ S-wave pion-pion system in the continuum limit with two-flavor full QCD. Calculations are made at three lattice spacings, using the finite volume method of Lüscher in the center of mass frame, and its extension to the laboratory frame.

1. Introduction

Pilot studies of the scattering phase shift were carried out for the I=2 S-wave two-pion system in Ref. [1,2]. These studies were made only at one lattice spacing employing the quenched approximation. In this report, we present a calculation including two flavors of dynamical quarks, so that we can get rid of errors from unitarity violation, at three different lattice spacings to carry out extrapolation to the continuum limit.

Our study is based on the finite volume method of Lüscher formulated for the center of mass (CM) frame [4], and its extension to the laboratory frame by Rummukainen and Gottlieb [5]. While the inclusion of the lab frame adds little extra computational cost, the use of the lab frame allows significantly more dense sampling of the energy states.

We use the full QCD configurations [3] previously generated with an RG-improved gauge action and a mean-field improved clover quark action at three gauge couplings $\beta = 1.80, 1.95$ and 2.10, corresponding to $a^{-1} \approx 0.92, 1.3$ and 1.8 GeV, on $12^3 \times 24, 16^3 \times 32$ and $24^3 \times 48$ lattices. The physical volumes are about 2.5 fm. Four hopping parameters corresponding to $m_\pi \approx 1.0, 0.9, 0.75$ and 0.55 GeV are taken at each β for chiral extrapolation. The numbers of the configurations vary from 380 to 720.

2. Methods

On a finite volume of L^3 the interaction between two pions shifts the two-pion energy E. In the CM frame, writing $E = 2\sqrt{m_\pi^2 + p^2}$ with $p^2 = (2\pi/L)^2\overline{n}$ ($\overline{n}$ being non-integer), the phase shift $\delta(p)$ is given by the relation $\tan(\delta(p)) = \pi^{3/2}\sqrt{\overline{n}}/Z_{00}(1;\overline{n})$ where $Z_{00}(i;\overline{n}) = (1/\sqrt{4\pi})\sum_{\vec{l}}(l^2 - \overline{n})^{-i}$, $\vec{l}$ being integer vectors.

In the laboratory frame, the two-pion energy E_L and the sum of the two pion momenta $P = p_1 + p_2$ are given by $p^2 = (2\pi/L)^2\overline{m} = (E_L^2 - P^2)/4 - m_\pi^2$. The phase shift is now $\tan(\delta(p)) = \gamma\pi^{3/2}\sqrt{\overline{m}}/Z_{00}^P(1;\overline{m})$ where $\gamma = E_L/\sqrt{E_L^2 - P^2}$ and $Z_{00}^P(i;\overline{m}) = (1/\sqrt{4\pi})\sum_{\vec{r}}(r^2 - \overline{m})^{-i}$. Here the summation is taken over the vectors $\vec{r}$ that are constructed as $\vec{r} = \gamma^{-1}(\vec{l} + \vec{d}/2)$ where $\vec{d} = L\vec{P}/2\pi$.

The pion four-point function behaves as a sum of exponentials due to the presence of a num-

*Talk presented by T. Yamazaki

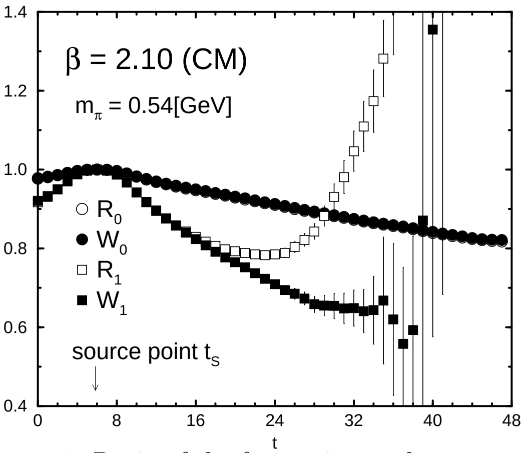

Figure 1. Ratio of the four-point to the square of two-point functions. The values before diaginalization $R_n(t)$ are compared to those after diaginalization $W_n(t)$. Note that $W_0(t)$ is completely overlayed on $R_0(t)$.

ber of states having the same quantum numbers. To resolve the energy eigenvalues, we construct in each frame the pion four-point function matrix $C_{nm}(t) = \langle 0|\Omega_n(t)\Omega_m(t_S)|0\rangle$, where t_S is the source point and $\Omega_n(t) = \pi(p_1)\pi(p_2)$ is the two-pion operator with momenta p_i that depends on the state n. The energy eigenvalues $\lambda_n(t) = \exp[-E_n(t-t_0)]$ are obtained by diagonalizing the matrix $C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)$ with t_0 a reference time [6]. A cut-off N is introduced in the number of the energy states considered.

We work with the CM and two laboratory frames, L1 and L2. The sums of the two pion momenta $\vec{P}$ in L1 and L2 are $\vec{P}_{\rm L1} = (1,0,0)\times(2\pi/L)$ and $\vec{P}_{\rm L2} = (1,1,0)\times(2\pi/L)$. We choose $N = 3$ in the CM frame and 4 in the lab frame. In each frame, 3×3 or 4×4 matrices are diagonalized and the phase shift is determined for the energies of the ground ($n = 0$) and the first excited ($n = 1$) states.

Figure 1 illustrates the effect of diagonalization. We compare the ratio $R_n(t) = C_{nn}(t)/G_n^2(t)$, where $G_n(t) = \langle 0|\pi(p_n,t)\pi(-p_n,t_S)|0\rangle$ with $p_n^2 = (2\pi/L)^2 n$, before the diagonalization (denoted by open symbols) to $W_n(t) = \lambda_n(t)\cdot G_n^2(t_0)/G_n^2(t)$ after the diagonalization (closed symbols) normalized at $t = t_S$. For the ground state, $W_0(t) \simeq R_0(t)$ so that the diagonalization is unnecessary.

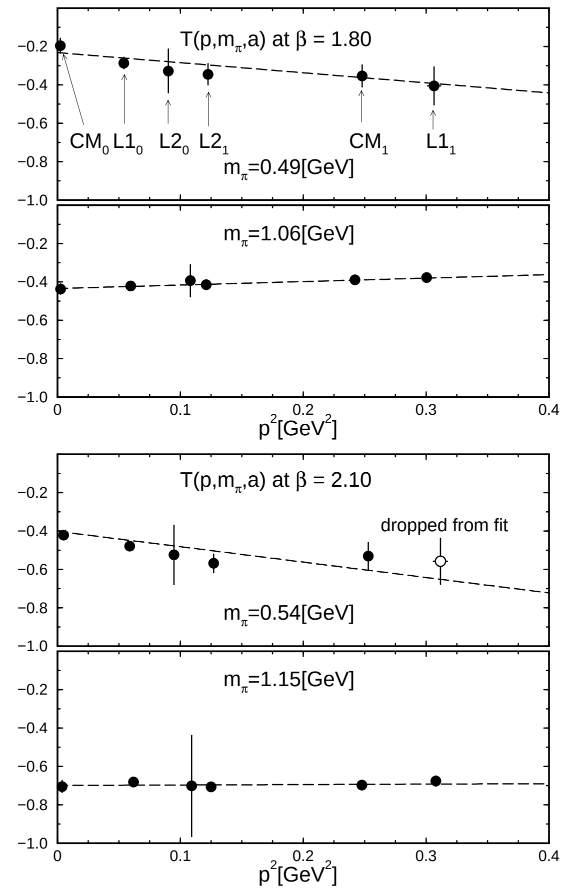

Figure 2. Scattering amplitudes defined by (1) for the heviest and lightest pion masses at $\beta = 1.80$ (top panels) and 2.10 (bottom panels). The dashed lines show the global polynomial fit.

For the $n = 1$ state, however, the ratio shows an exponential fall-off only after diagonalization: $R_1(t)$ blows up at large t. We apply an exponential fit to $W_1(t)$. The energy eigenvalues for the $n = 0$ and 1 states change very little even if we adopt the cut-off $N = 2$ instead of 3 and 4.

3. Phase shifts

We define the 'scattering amplitude'

$$T(p, m_\pi, a) = \tan(\delta(p))E_\pi/p \qquad (1)$$

where $E_\pi = \sqrt{m_\pi^2 + p^2}$. This gives the scattering length a_0 in the zero-momentum limit: $\lim_{p\to 0} T(p, m_\pi, a) = a_0 m_\pi$.

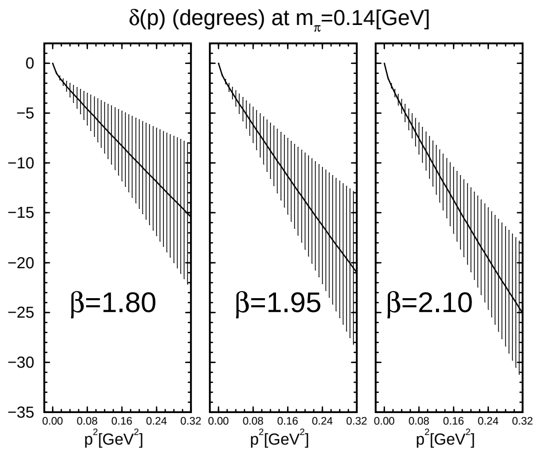

Figure 3. Phase shifts at the physical pion mass for the three values of β.

Figure 2 shows the scattering amplitude at $\beta = 1.80$ and 2.10 for the lightest and heaviest pion masses. The CM_n refers to the amplitude obtained from the n-th state in the CM frame, and the $\mathrm{L1}_n$ and $\mathrm{L2}_n$ to those in the lab frames. Note that we have three points between the two CM data, showing the improvement of the momentum sampling.

We carry out a global fit to the T at each β with a polynomial $T(p, m_\pi, a) = A_{10}m_\pi^2 + A_{20}m_\pi^4 + A_{30}m_\pi^6 + A_{01}p^2 + A_{11}m_\pi^2 p^2 + A_{21}m_\pi^4 p^2$. One data point (shown with open symbol) is excluded from the fit, since the $W_n(t)$ does not show a good exponential behavior. We find a reasonable fit to the data for both CM and lab frames.

In Fig.3 the phase shift at the physical pion mass $m_\pi = 0.14$ GeV, as calculated from the fit, decreases with β, showing the presence of an $O(a)$ effect.

The continuum limit is taken by extrapolating the fits linear in a to the scattering amplitude at the physical pion mass for each momentum, $T(p, m_\pi, a) = T(p, m_\pi) + aT^a(p, m_\pi)$. The phase shift in the continuum limit is presented in Fig.4 with the dashed line, associated by a band of error bars. This is compared to the experimental data.

The scattering length obtained by taking the zero-momentum limit of $T(p, m_\pi)$ in the continuum, $a_0 m_\pi = -0.0488(49)$, may be compared with the prediction of chiral perturbation theory: $a_0 m_\pi = -0.0444(10)$ [7].

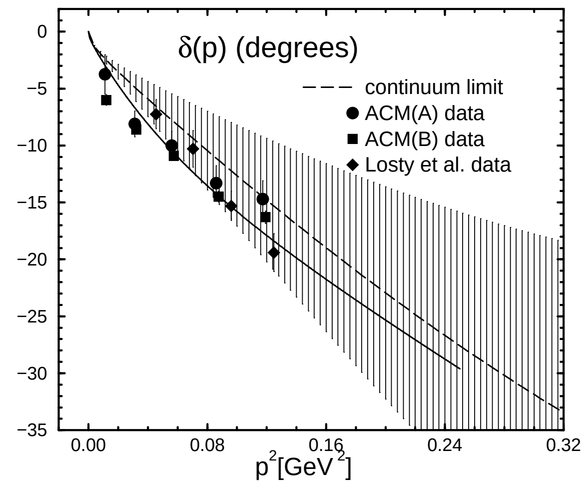

Figure 4. Phase shifts in the continuum limit (indicated by the dashed curve). The band of bars shows our estimate of errors. The solid curve is parametrized by experimental inputs [7] and the symbols are the experimental data.

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