$I = 2$ Pion Scattering Length with Wilson Fermions *

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We present results for $I = 2$ pion scattering length with the Wilson fermions in the quenched approximation. The finite size method presented by Lüscher is employed, and calculations are carried out at $\beta = 5.9$, 6.1, and 6.3. In the continuum limit, we obtain a result in reasonable agreement with the experimental value.

1. Introduction

Lattice calculations of scattering lengths of the two-pion system is an important step for understanding of strong interactions beyond the hadron mass spectrum. For the $I = 0$ process, which is difficult due to the presence of disconnected contributions, only one group carried out the calculation [1]. For the $I = 2$ process, on the other hand, a number of calculations has been carried out with the Staggered [1,2] and the Wilson fermions [1,3]. These calculations reported results in agreement with the prediction of current algebra or lowest-order chiral perturbation theory(CHPT) [4]. It is known, however, that this prediction differs from the experimental value over 1.4σ, and that the higher order effects of CHPT are small [5].

The past lattice calculations were made on coarse lattices with small sizes, and the continuum extrapolation was not taken. In this article we report on our high statistics calculation of the $I = 2$ pion scattering length aiming to improve on these points. This work is carried out in quenched lattice QCD employing the standard plaquette action for gluons with the Wilson fermions. The number of configurations (and lattice size) are $187(16^3 \times 64)$, $120(24^3 \times 64)$, and $100(32^3 \times 80)$ for $\beta = 5.9$, 6.1, and 6.3. The pion mass covers the range $450 - 900$MeV.

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2. Method

The energy eigenvalue of a two-pion system in a finite periodic box L^3 is shifted by finite-size effect. Lüscher presented a relation between the energy shift ΔE and the S-wave scattering length a_0 given by [6]

$$-\Delta E \cdot \frac{m_\pi L^2}{4\pi^2} = T + A \cdot T^2 + B \cdot T^3 + O(T^4) \ , \quad (1)$$

where $T = a_0/(\pi L)$. Since T takes a small value, typically $\sim -10^{-2}$, in our simulation we can neglect the higher order terms $O(T^4)$. The constants A and B are geometrical values $A = -8.9136\cdots$ and $B = 62.9205\cdots$.

The energy shift ΔE can be obtained from the ratio $R(t) = G(t)/D(t)$, where

$$G(t) = \langle \pi^+(t)\pi^+(t)W^-(t_1)W^-(t_2)\rangle$$
$$D(t) = \langle \pi^+(t)W^-(t_1)\rangle \ \langle \pi^+(t)W^-(t_2)\rangle \ . \quad (2)$$

In order to enhance signals we use wall sources (denoted by W^-) and fix gauge configurations to the Coulomb gauge. The two wall sources are

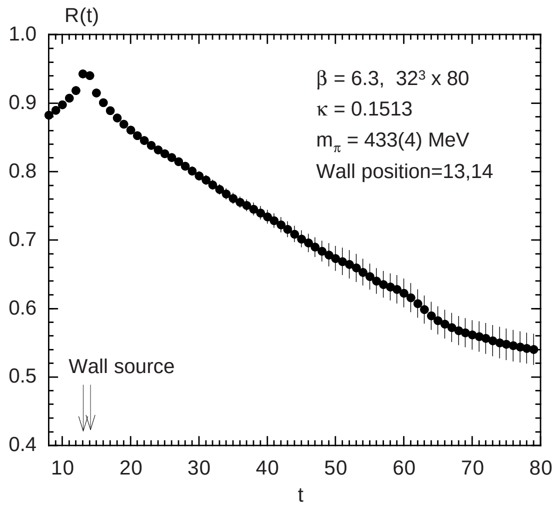

Figure 1. $R(t) = G(t)/D(t)$.

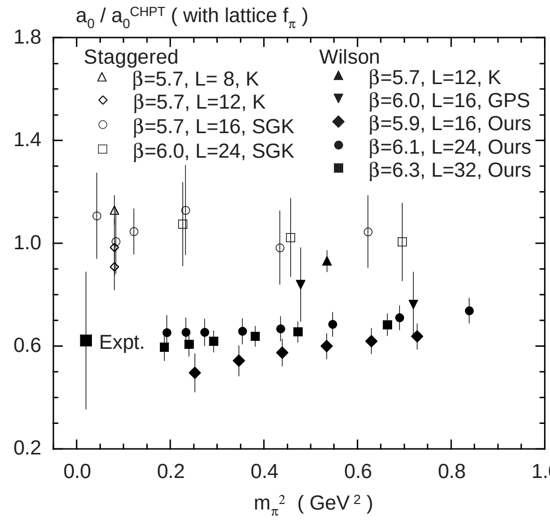

Figure 2. Comparison of our results with the old calculations by a_0/a_0^{CHPT}.

placed at different time slices t_1 and t_2 to avoid contaminations from Fierz-rearranged terms in the two-pion state that would occur for the choice $t_1 = t_2$. In this work we set $t_2 = t_1 + 1$ and $t_1 = 8$, 10, 13 for $\beta = 5.9$, 6.1, 6.3. Quark propagators are solved with the Dirichlet boundary condition imposed in the time direction and the periodic boundary condition in the space directions. In region $t >> t_1, t_2$ the ratio behaves as $R(t) \sim Z(1 - \Delta E \cdot (t - t_1) + O(t^2))$.

As an example, the ratio $R(t)$ at $\beta = 6.3$ and $\kappa = 0.1513$ corresponding to $m_\pi = 433(4)$MeV is plotted in Fig. 1. The signal is very clear and $Z \sim 1$. This means the overlap of our wall sources with the two-pion state is sufficiently large. In general there are higher order terms $O(t^2)$ in $R(t)$, but we cannot resolve them in Fig. 1. Making a linear fitting in the range $t = 27 - 62$, we obtain $(a\Delta E) = 5.97(60) \times 10^{-3}$. Solving equation (1) with this value we obtain $T = -1.73(15) \times 10^{-2}$, which corresponds to $a_0 = -0.525(45)(1/\mathrm{GeV})$.

3. Result

In Fig. 2 we compare our new results for a_0/a_0^{CHPT} with those of old calculations, where a_0^{CHPT} is the prediction of current algebra : $a_0^{\mathrm{CHPT}} = -m_\pi/(16\pi f_\pi^2)$. For the decay constant f_π we use the value at finite m_π at finite lattice spacing referred in each study. This ratio has been commonly employed to make a comparison of current algebra and lattice calculations with different quark actions and parameters. The open symbols refer to results with the Staggered fermions and filled ones are those of the Wilson fermions. The legends give β, the spatial lattice size L, and collaborations of the studies (K : Kuramashi *et.al.* [1], SGK : Sharpe *et.al.* [2], GPS : Gupta *et.al.* [3], Ours : our calculation). We also plot the experimental value at $m_\pi = 140$MeV. We find that our results are inconsistent with old results, especially with those of the Staggered fermions.

A possible cause of the discrepancy is the systematic error of determination of f_π needed to calculate a_0^{CHPT}. In Fig. 3 we compare our results with old calculations in terms of a_0/m_π. The same symbols as those in Fig. 2 are used. The lattice results including ours are almost consistent with each other. Also they appear to be in more agreement with the experiment than with the prediction of current algebra.

We note that the calculation of f_π, being determined by the amplitude of correlation function of pion and axial vector current, is quite difficult. Various systematic errors may well enter in their determinations. Further the mass dependence of f_π is not small and a_0/a_0^{CHPT} is very sensitive to it. For these reasons we analyze a_0/m_π below.

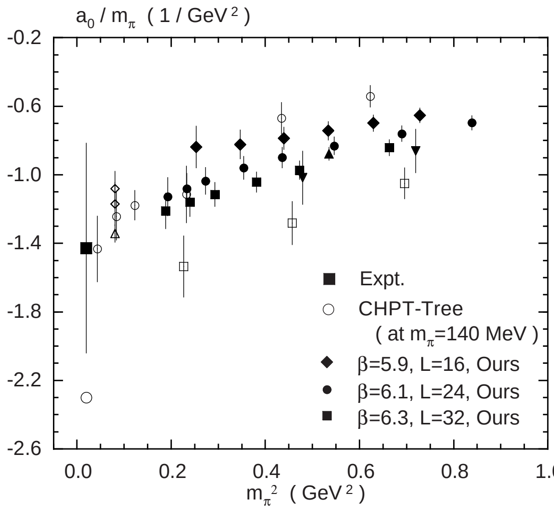

Figure 3. Comparison of our results with the old calculations by a_0/m_π.

From chiral symmetry a_0/m_π behaves as

$$a_0/m_\pi = A + B\cdot m_\pi^2 + C\cdot m_\pi^2 \log(m_\pi^2/\Lambda^2) + O(m_\pi^4) \ . \tag{3}$$

For the Wilson fermions we should consider another term $\propto 1/m_\pi^2$ that arises from breaking of chiral symmetry. Golterman and Bernard also proposed the same term pointing out that it can appear from quenching effect [7]. However, these effects are very small in our simulation as we do not observe a rapid variation of a_0/m_π expected from such a term in Fig. 3. Further the chiral logarithm term, $m_\pi^2 \log(m_\pi^2/\Lambda^2)$, is also small.

In Fig. 4 our results for a_0/m_π in the chiral limit obtained by a linear fitting in m_π^2 are plotted, together with the experimental value and the prediction of current algebra at $m_\pi = 140$MeV. We observe a very clear linear dependence in the lattice spacing a. By a linear extrapolation, we then obtain

$$\begin{aligned} a_0/m_\pi &= -1.91(25)\ (1/\mathrm{GeV}^2) \\ a_0 m_\pi &= -0.0374(49)\ , \end{aligned} \tag{4}$$

in the continuum limit.

This result is consistent with the experimental value: $a_0/m_\pi = -1.43(61)(1/\mathrm{GeV}^2)$ ($a_0 m_\pi = -0.028(12)$). The difference from the prediction of the current algebra given by $a_0/m_\pi = -2.3(1/\mathrm{GeV}^2)$ ($a_0 m_\pi = -0.045$) is about 1.5σ. Since scaling violation is not small, and our data points are far from the continuum limit, as seen in

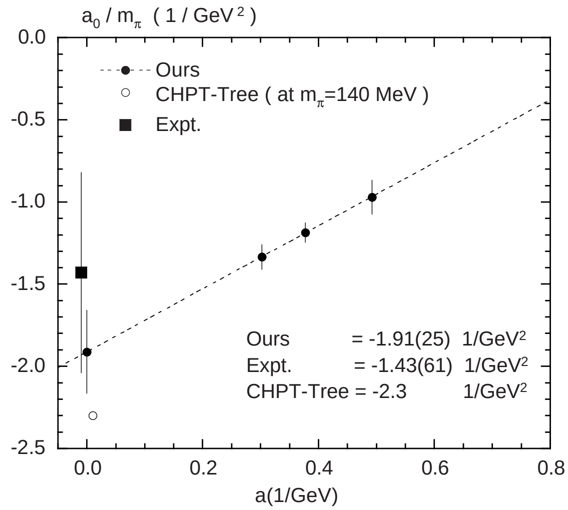

Figure 4. a_0/m_π at the chiral limit at each β.

Fig. 4, further calculations nearer to the continuum limit is desirable. In addition studies with the Staggered fermions should be repeated in a systematic manner for comparison with present results.

This work is supported by the Supercomputer Project No.45 (FY1999) of High Energy Accelerator Research Organization (KEK), and also in part by the Grants-in-Aid of the Ministry of Education (Nos. 09304029, 10640246, 10640248, 10740107, 10740125, 11640294, 11740162). K-I.I is supported by the JSPS Research Fellowship.

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