Hybrid Quarkonia on Asymmetric Lattices

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We report on a study of heavy quark bound states containing an additional excitation of the gluonic degrees of freedom. To this end we employ the nonrelativistic QCD approach on coarse and asymmetric lattices, where we discard vacuum polarization effects and neglect all spin-correction terms. We find a clear hybrid signal on all our lattices ($a_s = 0.15, \ldots, 0.47$ fm). We have studied in detail the lattice spacing artifacts, finite volume effects, and mass dependence. Within the above approximations we predict the lowest lying hybrid excitation in charmonium to be 1.323(13) GeV above the ground state, where we use the $1P$-$1S$ splitting to set the scale. The bottomonium hybrid was found to be 1.542(8) GeV above its ground state. [S0031-9007(99)09172-3]

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Gluonic excitations are ideal objects for investigating the nonperturbative nature of the gluonic degrees of freedom in QCD. Hybrid mesons can be thought of as hadronic bound states with an additional excitation of the gluon flux. This can give rise to states with nonconventional quantum numbers and has triggered an intense experimental and theoretical search for such particles. Previous predictions for the energies of hybrid states come from phenomenological models [1], static potential models [2,3], and lattice simulations with propagating quarks [4–6]. So far, lattice QCD is the only approach in which hybrids can be treated from first principles. However, the errors from such calculations on isotropic lattices are still much larger than for conventional states. This is because the correlation functions decay too rapidly when the excitation energy is very large with respect to the inverse lattice spacing. To obtain a similar signal-to-noise ratio one needs a much finer resolution in the temporal direction.

While in recent years there has been much progress in obtaining more reliable results from improved actions on spatially coarse lattices, it has also been demonstrated that anisotropic lattices can be employed to accommodate different physical scales on the same lattice. In particular, the study of glueball states on coarse and anisotropic lattices [7] has prompted us to study heavy hybrid states on such lattices in order to increase both the scope and the precision of a previous calculation [8] significantly. There a nonrelativistic approach (NRQCD) was used for the heavy b quarks on an isotropic lattice with $a \approx 0.08$ fm. NRQCD has frequently been employed to allow high precision measurements for the $b\bar{b}$ system [9–12]. Also the combination of improved gluon actions and the NRQCD approach for heavy quarks has already been used to determine the spectrum of conventional quarkonia [13–15]. Previous attempts to measure heavy hybrid states on the lattice were reviewed in [16]. An initial preliminary study using asymmetric lattices for bottomonium hybrids was presented in [3,17].

In this paper we implemented such an efficient approach to study in detail the lattice artifacts and finite size effects for both the $b\bar{b}g$ and $c\bar{c}g$ hybrid. For the latter state we obtain excellent agreement with a relativistic simulation on isotropic lattices [4]. As with this other study we have neglected dynamical sea quark effects, but we have succeeded in lowering the statistical error to about 1%.

In our study we generated the gauge field configurations using a tadpole-improved action which has been employed by different groups [7,15]:

$$S = -\beta\xi^{-1} \sum_{x,i>j} \left\{ \frac{5}{3} P_{ij} - \frac{1}{12} (R_{ij} + R_{ij}) \right\} - \beta\xi \sum_{x,i} \left\{ \frac{4}{3} P_{it} - \frac{1}{12} R_{it} \right\}, \qquad U_i \to U_i/u_s . \quad (1)$$

Here P_{ij} and R_{ij} denote the trace of the standard spatial plaquette and rectangle, respectively. Where the index t appears the plaquette or rectangle extends only one link into the temporal direction. This theory has two parameters, β and ξ, the second of which determines the asymmetry of our lattices. At tree level the "aspect ratio" is $\xi = a_s/a_t$, where a_s and a_t are the spatial and temporal lattice spacings, respectively. From [7] we note that the radiative corrections to this relation are small when tadpole improvement is implemented, as described below.

The action in Eq. (1) is designed to be accurate up to $\mathcal{O}(a_s^4, a_t^2)$ classically. To account for radiative corrections all spatial gauge links, U_i, are self-consistently tadpole improved with $u_s = \langle 0|(1/3)P_{ij}|0\rangle^{1/4}$, as suggested in

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[18]. Such a mean-field treatment was demonstrated to reduce significantly the leading corrections, which are due to unphysical tadpoles in lattice perturbation theory. Our scaling analysis shows that errors $\mathcal{O}(\alpha a_s^2)$ are indeed negligible if the lattice spacing is sufficiently small. Since we will use this action only for small temporal lattice spacings we expect $\mathcal{O}(\alpha a_t^2)$ errors to be very small. Therefore we have not employed the tadpole improvement description for the temporal gauge links. Our results for two different aspect ratios ξ justify such an assumption.

To propagate the heavy quarks through the lattice we expand the NRQCD Hamiltonian correct to $\mathcal{O}(mv^2)$, where all spin-dependent terms are absent. This accuracy was already employed in [8]. The only additional improvement is to correct for temporal and spatial lattice spacing errors by adding two extra terms (c_7, c_8) to the evolution equation in [10]:

$$H_0 = -\frac{\Delta^2}{2m_b}, \qquad \delta H = -c_7 \frac{a_t \Delta^4}{16 n m_b^2} + c_8 \frac{a_s^2 \Delta^{(4)}}{24 m_b}. \tag{2}$$

In those operators all spatial links are also tadpole improved using the same u_s as for generating the configurations. After this modification we take the tree-level values for all the coefficients in the Hamiltonian. In this case, $c_7 = c_8 = 1$.

For the nonrelativistic meson operators we used only the simplest possible choices and have not tuned the overlap to optimize the signal. The gauge-invariant construction of S-state and P-state operators is described in [10]. For the *magnetic* hybrid signal studied here, we have inserted the lattice version of the magnetic field into the $Q\bar{Q}$ state ($B_i = \epsilon_{ijk}\Delta_j\Delta_k$):

$$\begin{aligned} {}^1H_i(x) &= \psi^\dagger(x) B_i \chi(x), \\ {}^3H_{jk}(x) &= \psi^\dagger(x) \sigma_j B_k \chi(x). \end{aligned} \tag{3}$$

For the leading order in the NRQCD Hamiltonian, the operators in Eq. (3) create a whole set of degenerate states: $1^{--}, 0^{-+}, 1^{-+}, 2^{-+}$. These are the spin-singlet and spin-triplet states with zero orbital angular momentum, including the exotic combination 1^{-+}. This degeneracy will be lifted when higher order relativistic corrections are reintroduced into the NRQCD Hamiltonian. We expect this to be a small effect, in as much as the heavy quarks are very slow in the shallow hybrid potential [3]. By the same argument, we expect hybrid states with additional orbital angular momentum to be almost degenerate as it was observed in [8]. The definition of the operators in Eq. (3) has been augmented by a combination of fuzzing [19] for the links and Jacobi smearing for the quark fields [20]. No effort has been made to optimize the signal further, but one could do so if even higher precision is needed, or if higher excited states are to be determined. To extract the hadron masses we simply fit the meson correlators to a single exponential;

$$C_\alpha(t) = \langle H_\alpha^\dagger(t) H_\alpha(0) \rangle = A_\alpha e^{-m_\alpha t}. \tag{4}$$

We measured correlators every 10 trajectories in the Monte Carlo update, and for the error analysis we binned 50 such measurements into one. After this binning we still have an ensemble of 100–1000 configurations depending on the lattice and the state of interest. As it can be seen from the representative example of an effective mass plot in Fig. 1, the data are very good and the goodness of the single exponential fits is always bigger than $Q = 0.1$, which we called acceptable. In Tables I and II we present our results and the simulation parameters.

To determine the lattice spacing a_t^{-1}, we used the $1P$-$1S$ splitting in charmonium and bottomonium. As expected, the values from charmonium are smaller than those from bottomonium, because in the quenched approximation the coupling does not run as in full QCD. At $(\beta, \xi) = (2.4, 5)$ we observe a 16% effect. In order to give another estimate of quenching errors we also determined the radial excitations, nS, and calculated the ratio $R_{SP} = 2S\text{-}1S/1P\text{-}1S$, which can be compared with the experimental value of 1.28 for bottomonium. At $(\beta, \xi) = (2.7, 5)$ we find $R_{SP} = 1.424(89)$. From these findings we quote quenching errors of (10–20)%. Several suggestions have been made how to measure the spatial lattice spacing [15,21], but to convert our results into dimensionful numbers we can use a_t^{-1} throughout.

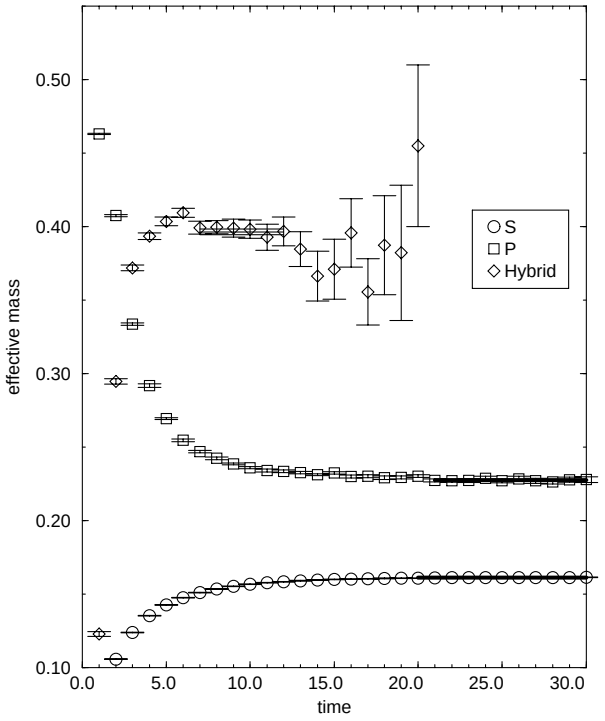

FIG. 1. A representative effective mass plot for the S, P, and hybrid states in bottomonium at $(\beta, \xi) = (2.7, 5), a_s m_b = 3.15$ on a $9^3 \times 40$ lattice.

TABLE I. Results for charmonium. The dimensionful numbers in the scaling region are given in boldfaced characters. From their average we obtain **1.323(13)** GeV for the lowest lying hybrid excitation from our simulation with accuracy $\mathcal{O}(mv^2, a_s^4, a_t^2)$. In the last column we give the spin averaged results from a higher order accuracy $\mathcal{O}(mv^6, a_s^4, a_t^2)$.

(β, ξ)	(1.7, 5)	(1.9, 5)	(2.2, 5)	(2.4, 5)	(2.4, 5)
Volume	$4^3 \times 40$	$4^3 \times 40$	$8^3 \times 40$	$8^3 \times 40$	$8^3 \times 40$
u_s	0.7370	0.7568	0.7841	0.7997	0.7997
$a_s m_c$	3.0	2.66	2.0	1.62	1.62, $\mathcal{O}(mv^6)$
P-S	0.2196(18)	0.1689(26)	0.1299(13)	0.1068(21)	0.1047(42)
a_t^{-1}/GeV	2.084(18)	2.709(42)	3.522(37)	4.286(86)	4.37(18)
H-S	0.5713(98)	0.4860(48)	0.3821(33)	0.3048(17)	0.3011(24)
H-S/P-S	2.602(49)	2.877(53)	2.928(53)	2.855(59)	2.87(12)
H-S/GeV	1.191(23)	**1.317(24)**	**1.346(18)**	**1.306(27)**	1.316(54)

We have also tested the velocity expansion and included relativistic corrections up to $\mathcal{O}(mv^6)$ for some of our lattices. At this level of accuracy we could not resolve any significant change in our results. A more detailed analysis of the spin structure is the subject of a future project.

In this study we were mainly interested in the gluonic excitations of heavy quarkonia. For this purpose it was irrelevant to adjust the quark masses to their exact values. For some lattices we have changed the quark masses by 25% and did not find any noticeable change in the ratio $R_H = (1H\text{-}1S)/(1P\text{-}1S)$.

Finite volume effects were a source of immediate concern for us. This is because hybrid states are expected to reside in a very flat potential [3]. The bag model also suggests a very large bound state as the result of the gluonic excitation [22]. As shown in Fig. 2, we have found that for spatial extents of 1.2 fm or larger the masses of all bottomonium and charmonium states remain constant within small statistical errors. For even smaller volumes we can resolve a slight increase in the mass of the $b\bar{b}g$ hybrid. In other words, the spatial extent of the hybrid excitation seems to be almost independent of the heavy quark mass. In this sense the $b\bar{b}g$ hybrid is more difficult to calculate, as we need similar volumes but finer lattices than for charmonium.

Finally, we carried out a scaling analysis to demonstrate that discretization errors are under control. This is of utmost importance for the NRQCD approach since one cannot extrapolate to zero lattice spacing in this effective field theory and one has to model continuum behavior already for finite lattice spacings. In Fig. 3 we show the scaling of the hybrid excitation above the ground state. From this, one can see that we have found convincing scaling windows for both bottomonium and charmonium. Scaling violations can be seen only on the coarsest lattices ($a_s > 0.36$ fm for $c\bar{c}g$ and $a_s > 0.19$ fm for $b\bar{b}g$), but this is not totally unexpected—it is questionable how well our simple-minded implementation of the tadpole prescription works to remove the $\mathcal{O}(\alpha p^2 a^2)$ errors for heavy quark systems on such coarse lattices. In the case of $b\bar{b}g$ we also plot the results for two different aspect ratios. Both results are consistent and confirm the initial assumption of small temporal lattice spacing errors. To quote our final result for the lowest lying hybrid excitations we take the averaged value of all the results in the scaling region and employ the experimental values for the $1P$-$1S$ splitting to set the scale. We find 1.323(13) GeV for the case of charmonium and 1.542(8) GeV for the first gluonic excitation in bottomonium, in good agreement with a previous estimate of 1.68(10) GeV [8].

In conclusion, we have demonstrated the usefulness of coarse and anisotropic lattices for the nonperturbative study of gluonic excitations in heavy quark systems. Furthermore, this should also be considered a success of the NRQCD approach, which allowed us to predict the lowest lying charmonium hybrid state at the same

TABLE II. Results for bottomonium. From the average over the scaling region we obtain **1.542(8)** GeV for the lowest lying $b\bar{b}g$ hybrid, when the P-S splitting is used to set the scale. In column 5 we show results with accuracy $\mathcal{O}(mv^6, a_s^4, a_t^2)$.

(β, ξ)	(2.4, 5)	(2.6, 5)	(2.7, 5)	(2.7, 5)	(2.5, 3)	(2.6, 3)	(2.8, 3)
Volume	$8^3 \times 40$	$6^3 \times 40$	$9^3 \times 40$	$9^3 \times 40$	$6^3 \times 40$	$8^3 \times 40$	$10^3 \times 40$
u_s	0.7997	0.8139	0.8193	0.8193	0.8100	0.8193	0.8314
$a_s m_b$	4.73	3.50	3.15	3.15, $\mathcal{O}(mv^6)$	4.10	4.00	3.33
P-S	0.086 65(68)	0.074 44(65)	0.065 73(62)	0.0636(23)	0.1388(10)	0.130 68(82)	0.109 88(77)
a_t^{-1}/GeV	5.075(43)	5.907(54)	6.690(67)	6.91(25)	3.169(25)	3.365(23)	4.002(30)
H-S	0.309 88(84)	0.2591(11)	0.2275(36)	0.2305(46)	0.4882(41)	0.4568(21)	0.3852(32)
H-S/P-S	3.576(30)	3.481(34)	3.462(65)	3.62(15)	3.517(39)	3.496(27)	3.506(38)
H-S/GeV	**1.573(14)**	**1.531(15)**	**1.522(29)**	1.594(67)	**1.547(18)**	**1.537(13)**	**1.542(17)**

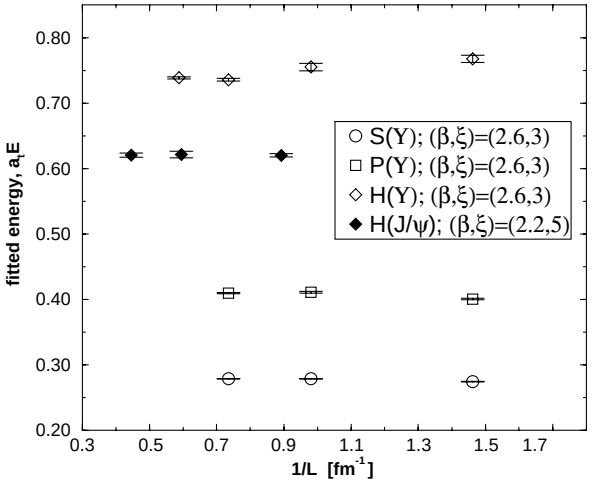

FIG. 2. Finite volume analysis for charmonium and bottomonium. We plot the dimensionless energies for different states against the inverse spatial extent, $1/L$, of the lattice.

mass as from a relativistic calculation [4]. Apart from the very accurate predictions for hybrid quarkonia, it is also interesting to notice that all of the above results could be obtained in a comparatively short period of time. Whereas our present calculation confirms that the $b\bar{b}g$ hybrid will lie above the $S + S$ threshold for decay into B mesons, the issue whether it may be found below the $S + P$ threshold has to be decided in a simulation where dynamical sea quarks are included in order to control this last remaining systematic error.

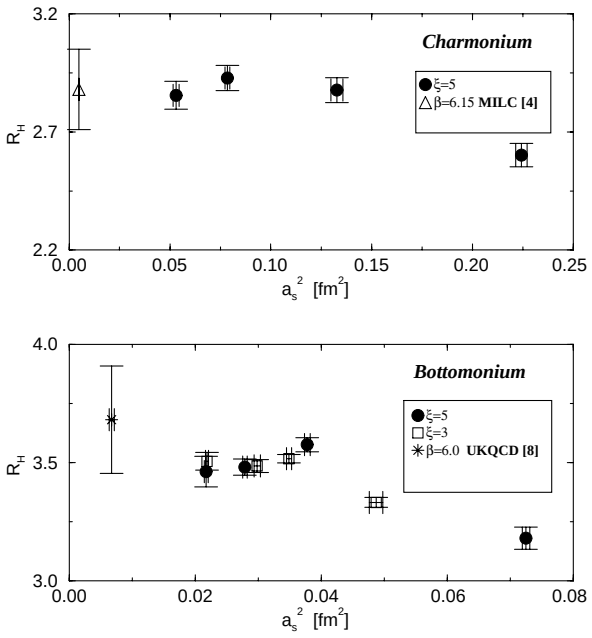

FIG. 3. Scaling of the hybrid excitation, H-S. We plot the ratio $R_H = (H\text{-}S)/(P\text{-}S)$ against the squared spatial lattice spacing; $a_s = \xi a_t$ at tree level. We also show the results from a previous calculation [8] on a symmetric lattice at $\beta = 6.0$ (burst). To display the result for the lowest $c\bar{c}g$ hybrid at $\beta = 6.15$ [4], we use their value 1.32(8) GeV and the experimental P-S splitting in charmonium (triangle).

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