Heavy quark mass dependence of semileptonic form factors for B decays*

JLQCD Collaboration
S. Aoki[a], M. Fukugita[b], S. Hashimoto[c], K-I. Ishikawa[d], N. Ishizuka[a,e], Y. Iwasaki[a,e], K. Kanaya[a,e], Y. Kuramashi[f], H. Matsufuru[d], M. Okawa[f], T. Onogi[d], S. Tominaga[f], A. Ukawa[a], N. Yamada[d], T. Yoshié[a,e]

[a]Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan

[b]Institute for Cosmic Ray Research, University of Tokyo, Tanashi, Tokyo 188, Japan

[c]Computing Research Center, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305, Japan

[d]Department of Physics, Hiroshima University, Higashi-Hiroshima 739, Japan

[e]Center for Computational Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan

[f]Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305, Japan

We present our study of the dependence of the heavy-to-light semileptonic B decay form factors on the heavy-light meson mass M_{PS}. Simulations are made over a range of the heavy quark mass covering both the charm and bottom quarks using the $O(a)$-improved clover action at $\beta = 5.9$ on a $16^3 \times 40$ and $24^3 \times 64$ lattice. We find that a weak dependence of form factors on M_{PS} observed in previous studies in the region of charm quark persists up to the region of b quark. The soft pion relation $f^0(q^2_{max}) = f_B/f_\pi$ is examined and found to be largely violated.

1. Introduction

In spite of their small branching fraction, the exclusive semileptonic decays $B \to \pi l \nu$ and $B \to \rho l \nu$ are expected to become important processes to determine V_{ub}. A model independent calculation of the form factors relevant for these decay processes is possible using lattice QCD, and several attempts have already been made[1–4]. To avoid problems associated with large heavy quark mass m_Q, however, these studies made simulations in the charm quark mass region and extrapolated results to the b quark.

In this report we describe our examination of the heavy quark mass dependence of the form factors. For this purpose we employ the $O(a)$-improved clover action in the formalism of the FNAL group[5], and carry out simulations over a wide range of heavy quark mass including the bottom as well as charm quark masses[6]. Our results for f_B with this formalism show that systematic errors due heavy quark are small enough for the b quark[7], and we expect the systematic error to be also well under control for the form factors.

2. Simulation

We use the plaquette action and the clover action with the clover coefficient determined at the one-loop level[7]. Measurements are made at $\beta = 5.9$ with 100 configurations on a $16^3 \times 40$ lattice and 110 configurations on a $24^3 \times 64$ lattice. The lattice scale is set by the ρ meson mass which gives $a^{-1} = 1.64(2)$ GeV. Six values of the heavy quark hopping parameter in the range $\kappa_h = 0.0718 - 0.1245$ are employed. The chiral limit is taken for the light quark with results obtained for the light quark hopping parameter in the range $\kappa_l = 0.13630 - 0.13816$ to the critical value $\kappa_c = 0.13901(1)$.

*presented by S. Tominaga

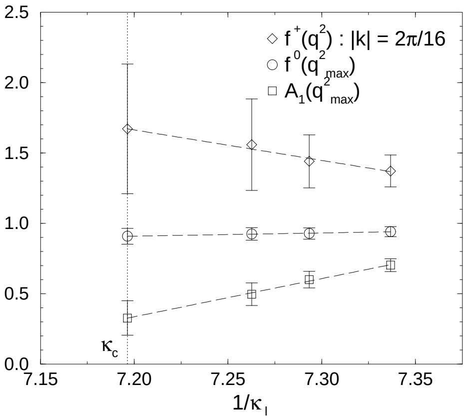

Figure 1. Chiral extrapolation of $f^+(q^2)$ for minimum pion momentum, $f^0(q^2_{max})$ and $A_1(q^2_{max})$ obtained near B mass region ($\kappa_h = 0.0973$) for $16^3 \times 40$ lattice.

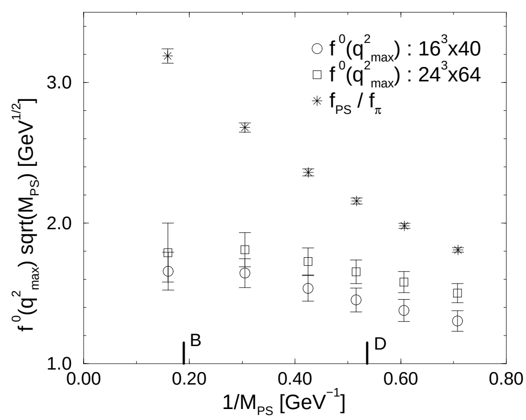

Figure 2. $f^0(q^2_{max})\sqrt{M_{PS}}$ as a function of $1/M_{PS}$. Circles correspond to data on a $16^3 \times 40$ lattice and squares on a $24^3 \times 64$ lattice. Crosses represent $f_{PS}\sqrt{M_{PS}}/f_\pi$.

The form factors for the B to π decay are obtained from the heavy-to-light three-point function $\langle P_{HL}(T/2)V_\mu(t)P^\dagger_{LL}(0)\rangle$ for the vector current $V_\mu(t)$ by dividing out the appropriate two-point functions. The pseudo scalar fields $P_{HL}(T/2)$ and $P_{LL}(0)$ are smeared with the measured wave function obtained in Ref. [7]. The B meson is taken at rest ($|\vec{p}|$=0) and the pion is given up to a unit of momentum ($|\vec{k}|$=0, $2\pi/L$), above which the signal becomes unacceptably noisy. The form factors for the B to ρ decay is obtained in a similar manner, for which we consider only the zero-recoil decay $|\vec{p}| = |\vec{k}| = 0$.

We do not include the perturbative Z factor for the currents in this study, since results for finite heavy quark masses are not fully available.

3. Results

In Fig. 1 we plot $f^+(q^2)$, $f^0(q^2_{max})$ and $A_1(q^2_{max})$ as a function of $1/\kappa_l$ for $\kappa_h = 0.0973$ which is near the b quark mass. Results for f^+ are for the minimal non-zero pion momentum. We observe that the dependence in $1/\kappa_l$ is quite small for $f^0(q^2_{max})$. In contrast there is a clear slope for $A_1(q^2_{max})$, which differs from previous results[2,3]. For the chiral extrapolation we adopt a linear form in $1/\kappa_\ell$ as shown in Fig. 1, with which our results are consistent.

Heavy quark symmetry predicts that $f^0\sqrt{M_{PS}}$, $f^+/\sqrt{M_{PS}}$ and $A_1\sqrt{M_{PS}}$ scales toward the heavy quark mass limit. In Fig. 2 we plot $f^0(q^2_{max})\sqrt{M_{PS}}$ in the chiral limit as a function of $1/M_{PS}$. We observe that a small $1/M_{PS}$ correction, as suggested by results of previous studies performed around the charm quark mass[1,3,4], also holds in our data. Furthermore the weak dependence persists to the region of b quark mass. We also note a volume effect of about 10% between $16^3 \times 40$ and $24^3 \times 64$ lattices, which is statistically significant toward light meson masses.

In the chiral limit, an application of the soft pion technique predicts the relation $f^0(q^2_{max}) = f_{PS}/f_\pi$[8]. In Fig. 2 we also plot $f_P\sqrt{M_{PS}}/f_\pi$ (crosses) obtained in Ref. [7]. There is a significant discrepancy with $f^0(q^2_{max})\sqrt{M_{PS}}$, particularly toward heavy quark masses. Possible origins of the discrepancy are (i) subtleties in the chiral extrapolation of $f^0(q^2_{max})$ since q^2_{max} changes with $1/\kappa_\ell$, (ii) systematic errors due to heavy quark including corrections from the Z factor, (iii) scaling violation and breaking of chiral symmetry for light quark. It is not clear at present if these could account for the large difference seen in Fig. 2.

The $1/M_{PS}$ dependence of $f^+(q^2)/\sqrt{M_{PS}}$ is shown in Fig. 3. It is negligible for this quantity.

For f^0 and f^+ the slope in the chiral extrapolation is almost independent of the heavy quark

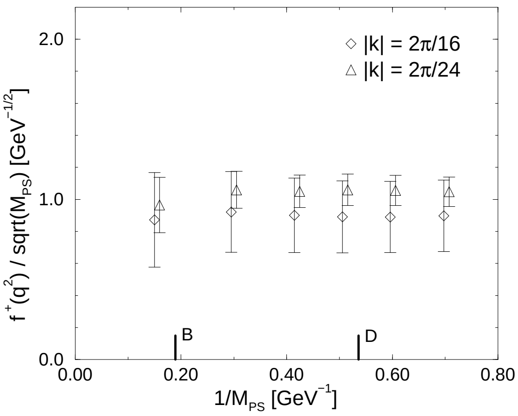

Figure 3. $f^+(q^2)/\sqrt{M_{PS}}$ in the chiral limit as a function of $1/M_{PS}$. Triangles correspond to the results on a $16^3 \times 40$ lattice and diamonds on a $24^3 \times 64$ lattice. B meson is at rest and pion carries one unit of momentum.

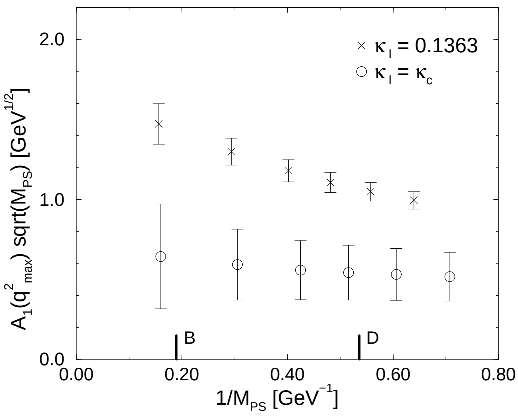

Figure 4. $A_1(q^2_{max})\sqrt{M_{PS}}$ on a $16^3 \times 40$ lattice at $\kappa_l = 0.1363$ and κ_c as a function of $1/M_{PS}$.

mass. Therefore the $1/M_{PS}$ dependence of these form factors is almost unchanged by the chiral extrapolation, as was seen in Ref. [3]. On the other hand, the large slope for A_1 shown in Fig. 1 becomes smaller as the heavy quark becomes lighter. As a result the $1/M_{PS}$ behavior of A_1 depends significantly on the light quark mass as shown in Fig. 4, where we plot results at a finite light quark mass (crosses) together with that in the chiral limit(circles).

4. Conclusions

Previous studies[1,3,4] performed on the charm quark mass region suggested that the $1/M_{PS}$ dependence of the form factors is small contrary to the case of the heavy-light decay constant which varies significantly between the static limit and the region of charm quark. Our calculation confirms this trend and extends it to the region of b quark. This is particularly puzzling for f^0 for which we find a large violation of the soft pion relation $f^0(q^2_{max}) = f_B/f_\pi$. A weak $1/M_{PS}$ dependence and a discrepancy from the soft pion relation are also observed if non-relativistic QCD is employed for heavy quark[9]. We feel that further understanding of the heavy quark mass dependence is required for a reliable calculation of heavy-to-light form factors from lattice QCD.

This work is supported by the Supercomputer Project (No. 97-15) of High Energy Accelerator Research Organization (KEK), and also in part by the Grants-in-Aid of the Ministry of Education (Nos. 08640349, 08640350, 08640404, 09246206, 09304029, 09740226). Two of us (H.M. and S.T.) are supported by the JSPS Research Fellowships.

REFERENCES

1. T. Onogi, these proceedings; J. Flynn, Nucl. Phys. B (Proc. Suppl.) **53** (1997) 168.
2. C. W. Bernard *et al.*, Phys. Rev. **D43** (1991) 2140; Phys. Rev. **D45** (1992) 869.
3. C.R. Allton *et al.*, Phys. Lett. **B345** (1995) 513; As. Abada *et al.*, Nucl. Phys. **B416** (1994) 675.
4. D. R. Burford *et al.*, Nucl. Phys. **B447** (1995) 425; J. M. Flynn *et al.*, Nucl. Phys. **B461** (1996) 327.
5. A.X. El-Khadra, A.S. Kronfeld, P.B. Mackenzie, Phys. Rev. **D55** (1997) 3933.
6. For a similar work, see J. Simone, Nucl. Phys. B (Proc. Suppl.) **47** (1996) 17.
7. JLQCD Collaboration (presented by S. Hashimoto), these proceedings.
8. See, *e.g.*, G. Burdman *et al.,* Phys. Rev. **D49** (1994) 2331.
9. H. Matsufuru *et al.*, these proceedings.