# Heavy quark expansion parameters from lattice NRQCD *

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Using the lattice NRQCD action for heavy quark, we calculate the heavy quark expansion parameters $\mu_\pi^2$ and $\mu_G^2$ for heavy-light mesons and heavy-light-light baryons. The results are compared with the mass differences among heavy hadrons to test the validity of HQET relations on the lattice.

## 1. Introduction

In the calculation of inclusive decay rates of the heavy hadron, the heavy quark expansion (HQE) technique is widely used. At the order $1/m_Q^2$ of HQE two nonperturbative parameters

$$\mu_\pi^2(H_Q) \equiv \frac{1}{2M_{H_Q}} \left\langle H_Q \left| \bar{Q}(i\vec{D})^2 Q \right| H_Q \right\rangle, \quad (1)$$

$$\mu_G^2(H_Q) \equiv \frac{1}{2M_{H_Q}} \left\langle H_Q \left| \bar{Q}\vec{\sigma}\cdot\vec{B} Q \right| H_Q \right\rangle, \quad (2)$$

appear in the calculation. $H_Q$ represents a heavy-light meson or heavy-light-light baryon (for $b$ hadrons, $H_b = B$, $B^*$, $\Lambda_b$, $\Sigma_b$, $\Sigma_b^*$). For instance, the lifetime ratio of $b$ hadrons is given as [1]

$$\frac{\tau(H_b^{(1)})}{\tau(H_b^{(2)})} = 1 + \frac{\mu_\pi^2(H_b^{(1)}) - \mu_\pi^2(H_b^{(2)})}{2m_b^2} + c_G \frac{\mu_G^2(H_b^{(1)}) - \mu_G^2(H_b^{(2)})}{m_b^2} + O(1/m_b^3), \quad (3)$$

with $c_G \simeq 1.2$. While $\mu_G^2$ may be evaluated from experimental values of hyperfine splitting, the determination of $\mu_\pi^2$ requires some theoretical inputs. It should be noted that the parameters are defined in the static limit: $m_Q \to \infty$. For heavy-light meson, $\mu_{\pi,G}^2$ has been calculated using the lattice version of the Heavy Quark Effective Theory [2].

In this work we calculate $\mu_\pi^2$ and $\mu_G^2$ on the lattice using the NRQCD action for heavy quark. Although the individual matrix element suffers from large perturbative uncertainty due to power divergence in the matching calculation, their differences like $\mu_\pi^2(H_b^{(1)}) - \mu_\pi^2(H_b^{(2)})$ are free from the uncertainty of the operator. We calculate both $\mu_\pi^2(H_b^{(1)}) - \mu_\pi^2(H_b^{(2)})$ and $\mu_G^2(H_b^{(1)}) - \mu_G^2(H_b^{(2)})$, and compare them with the corresponding predictions for mass splittings.

## 2. HQET mass formula

The parameters $\mu_\pi^2$ and $\mu_G^2$ can be indirectly obtained from hadron masses, using

$$M_{H_Q} - m_Q = \overline{\Lambda} + \frac{-\mu_\pi^2 - \mu_G^2}{2m_Q} + O\left(\frac{1}{m_Q^2}\right), \quad (4)$$

where $\overline{\Lambda}$ is the residual energy difference between $M_{H_Q}$ and $m_Q$ surviving in the infinite heavy quark limit. $\mu_\pi^2$ and $\mu_G^2$ appear in the correction terms of $O(1/m_Q)$. Therefore, by consider-

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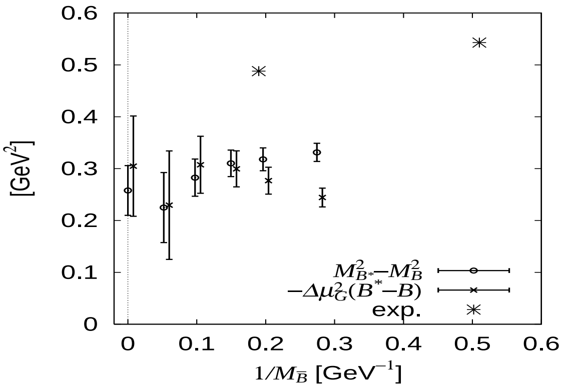


Figure 1. Hyperfine splitting of ground state mesons. Circles is obtained from the energy differences, while crosses are from the matrix elements.

ing proper mass differences, certain combinations of $\mu_\pi^2$ and $\mu_G^2$ can be extracted.

For example, a difference of $\mu_G^2$ can be obtained from the mass splitting in a spin multiplet, because $\overline{\Lambda}$ and $\mu_\pi^2$ have the same value. Also, the spin averaged mass $M_{\bar{B}} = (M_B + 3M_B^*)/4$ does not depend on $\mu_G^2$, because $\mu_G^2$ is proportional to the spin of the light degrees of the freedom and the sum of $\mu_G^2$ in the spin multiplet vanishes.

## 3. Lattice calculations

We carry out quenched QCD simulations at $\beta$=6.0 on a $20^3 \times 48$ lattice. The NRQCD action including all $O(1/m_Q)$ terms and the non-perturbatively improved clover action ($c_{sw}$=1.769) is adapted for heavy quark and light quark, respectively. Five heavy quark masses $am_Q$=1.3, 2.1, 3.0, 5.0, and 10.0 are used to study the $1/m_Q$ dependence of hadron masses and matrix elements, while three hopping parameters $K$=0.13331, 0.13384, and 0.13432 are simulated to extrapolate to the chiral limit $K_c$=0.135284(8). The inverse lattice spacing $a^{-1}$=1.85(5) GeV is determined with the $\rho$ meson mass $m_\rho$=770 MeV.

We measure the three-point functions $\langle \mathcal{O}_{H_Q}(t)\mathcal{O}_{\pi,G}(t_\mathcal{O})\mathcal{O}^\dagger_{H_Q}(0)\rangle$, where $\mathcal{O}_{H_Q}$ is an interpolating field to create or annihilate the hadron $H_Q$, and $\mathcal{O}_{\pi,G}$ is the operator to be measured, $\bar{Q}(i\vec{D})^2Q$ or $\bar{Q}\vec{\sigma}\cdot\vec{B}Q$. We divide them by

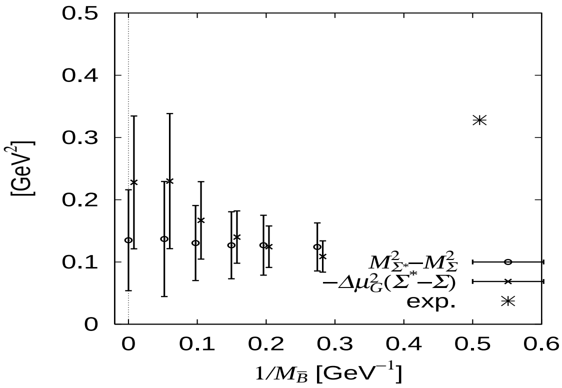


Figure 2. Hyperfine splitting of heavy-light-light baryons.

$\langle \mathcal{O}_{H_Q}(t_1)\mathcal{O}^\dagger_{H_Q}(0)\rangle$ to obtain the desired matrix elements $\mu_\pi^2$ and $\mu_G^2$.

## 4. Hyperfine splittings

From (4) the hyperfine splitting $M_{B^*} - M_B$ is given by $-\Delta\mu_G^2/2m_Q$, or equivalently

$$M^2_{B^*} - M^2_B = -\Delta\mu^2_G \equiv -(\mu^2_G(B^*) - \mu^2_G(B)), \quad (5)$$

at the leading order. In Figure 1, we plot our results for $-\Delta\mu_G^2$ together with the measurement of $M^2_{B^*} - M^2_B$. We observe that the relation (5) is satisfied very well, while both are significantly lower than the experimental values for $B$ and $D$ mesons.

In deriving (5) we used a relation

$$\Delta\mu^2_\pi = \mu^2_\pi(B^*) - \mu^2_\pi(B) = 0, \quad (6)$$

which holds in the static limit. However, for the NRQCD action including the spin-magnetic interaction term at $O(1/m_Q)$, the operator $\mathcal{O}_\pi$ mixes with $\mathcal{O}_\mathcal{G}$ at order $\alpha_s/m_Q$. This is the reason why our result for $-\Delta\mu_G^2$ deviates from that of the mass difference in the lighter heavy quark mass region. In other words, the relation (6) may be considered as a renormalization condition for the operator $\mathcal{O}_\pi$.

Similar analysis can be made for the hyperfine splitting of heavy-light-light baryon, *i.e.* $\Sigma^* - \Sigma$ splitting. Figure 2 shows the mass difference and the matrix element $-\Delta\mu_G^2$. Both are in good agreement.

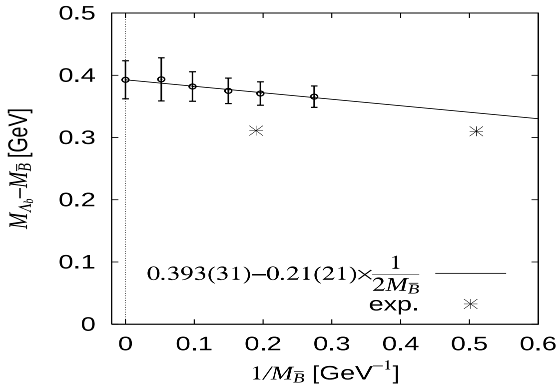


Figure 3. $1/M_{\bar{B}}$ dependence of $M_{\Lambda_b} - M_{\bar{B}}$.

## 5. $M_{\Lambda_b} - M_{\bar{B}}$

The heavy-light meson-baryon mass difference $M_{\Lambda_b} - M_{\bar{B}}$ is given as

$$M_{\Lambda_b} - M_{\bar{B}} = \overline{\Lambda}(\Lambda_b) - \overline{\Lambda}(B) + \frac{1}{2m_Q}\left[-\mu_\pi^2(\Lambda_b) + \mu_\pi^2(B)\right]. \quad (7)$$

The intercept at $1/M_{\bar{B}}$=0 yields $\overline{\Lambda}(\Lambda_b) - \overline{\Lambda}(B)$ while the slope is described by $-\Delta\mu_\pi^2 = -(\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B))$.

In Figure 3 we plot $M_{\Lambda_b} - M_{\bar{B}}$ as a function of $1/M_{\bar{B}}$. For the intercept we obtain $\overline{\Lambda}(\Lambda_b) - \overline{\Lambda}(B)$=393(31) MeV. in agreement with a previous work by Ali Khan *et al.*, $\overline{\Lambda}(\Lambda_b) - \overline{\Lambda}(B)$=415(156) MeV. Our result is slightly larger than the experimental values for $b$ and $c$ hadrons. However, to draw a definite conclusion we have to consider several systematic errors, especially the finite volume effect, because our lattice may not be large enough for baryons.

The slope obtained from the fit of the mass difference is consistent with zero: $-0.21(21)$ GeV$^2$. Our results of direct measurement of $-\Delta\mu_\pi^2$ is plotted in Figure 4, which is consistent with the result from mass difference, but have much better accuracy. Our result is also compatible with the phenomenological estimate $-0.01(3)$ GeV$^2$ [1] obtained from a combination $(M_{\Lambda_b} - M_{\bar{B}}) - (M_{\Lambda_c} - M_{\bar{D}})$.

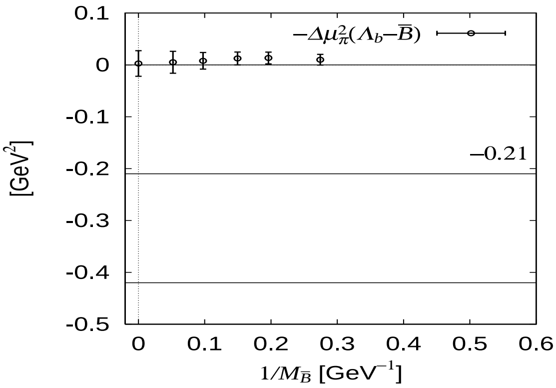


Figure 4. $-\Delta\mu_\pi^2$ measured from the matrix elements is compared with the slope of mass difference $M_{\Lambda_b} - M_{\bar{B}}$, $-0.21(21)$ GeV$^2$.

## 6. Conclusions

We confirm that the lattice measurements of the matrix elements $\mu_\pi^2$ and $\mu_G^2$ are consistent with the HQET mass relations. The well-known problem of quenched lattice calculation that the hyperfine splitting is much smaller than the experiments is also reproduced.

An important extension of our work is to measure the matrix elements of four-quark operators, which are relevant to the $1/m_Q^3$ corrections to the lifetime ratios [1].

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