Hadron matrix elements for nucleon decay with the Wilson quark action *

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We report preliminary results of our study of matrix elements of baryon number violating operators which appear in the low-energy effective Lagrangian of (SUSY-)Grand Unified Theories. The calculation is performed on a $32^3 \times 80$ lattice at $\beta = 6.1$ using Wilson fermions in the quenched approximation. Our calculation is independent of details of (SUSY-)GUT models and covers all interesting decay modes.

1. Introduction

Proton decay is one of the most exciting predictions of Grand Unified Theories (GUTs). Experimental effort over the years has pushed the lower limit on the partial lifetimes to $\tau/B_{p\to\pi^0 e^+} > 5.5 \times 10^{32}$ years and $\tau/B_{p\to K^+\overline{\nu}} > 1.0 \times 10^{32}$ years at the 90% confidence level[1], and further improvement is expected from the SuperKamiokande experiment.

A crucial link to relate these numbers to constraints on (SUSY-)GUT models is the values of hadron matrix elements relevant for proton decay. Model calculations suffer from high degree of uncertainty, various estimations easily differing by a factor ten.

Pioneering lattice QCD studies to remove this source of uncertainty were made about ten years ago, first combining calculations of the matrix element $\langle 0|O^{\not B}|p\rangle$ and soft-pion theorems to estimate $\langle \pi^0|O^{\not B}|p\rangle$[2,3], and subsequently directly evaluating the matrix element $\langle \pi^0|O^{\not B}|p\rangle$ itself[4].

In this article we report preliminary results of our renewed effort to determine the matrix elements from first principles of QCD. In addition to the use of larger lattice sizes and higher statistics to achieve a much better precision, which is made possible through the increase of computing power, we aim to advance the calculation on two fronts: (i) calculation of $p \to K$ matrix elements relevant for SUSY-GUT as well as those for the $p \to \pi$ mode, for physical values of u-d and s quark masses and for physical momenta, and (ii) evaluation of matrix elements of all dimension-6 baryon number violating operators classified according to $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariance[5,6], so as to cover various GUT models and decay processes.

2. Calculational procedure

Our calculation is carried out in quenched QCD at β=6.1 with the Wilson quark action on a $32^3 \times 80$ lattice. We analyze 100 configurations for the hopping parameter K=0.15428, 0.15381, 0.15333, 0.15287. The lattice scale fixed by $m_\rho =$ 769MeV in the chiral limit ($K_c = 0.15499(2)$) equals $a^{-1} = 2.56(4)$GeV, and the point for strange quark estimated from $m_K/m_\rho = 0.648$ is given by $K = 0.15304(5)$. All errors are estimated by the single elimination jackknife procedure.

*presented by N. Tsutsui

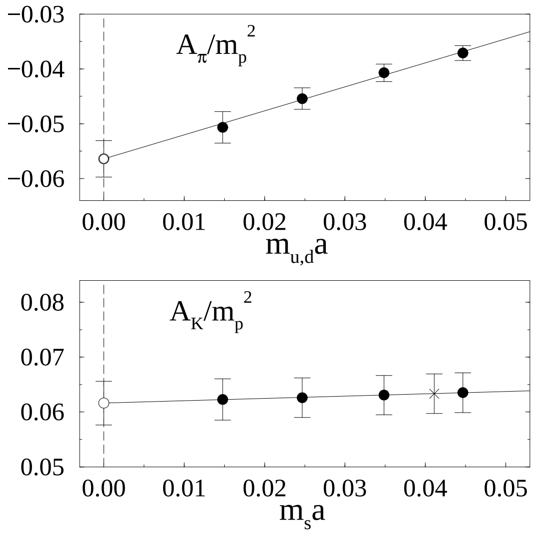

Figure 1. Matrix element relevant for $p \to \pi^0$ and $p \to K^0$ decay normalized by measured proton mass at $\vec{p} = 0$ of pseudo scalar meson as a function of u-d and s quark mass in lattice units. u-d quark mass is taken to the chiral limit for A_K.

To calculate the nucleon (N) to pseudo scalar (PS) meson matrix element of a baryon number violating operator $O^{\not B}$, we form the ratio,

$$\langle PS|O^{\not B}|N\rangle = \frac{\langle 0|J_{PS}O^{\not B}\bar{J}_N|0\rangle}{\langle 0|J_N\bar{J}_N|0\rangle\langle 0|J_{PS}J^\dagger_{PS}|0\rangle} \times \langle 0|J_{PS}|PS\rangle\langle N|\bar{J}_N|0\rangle, \quad (1)$$

where $\langle 0|J_{PS}|PS\rangle$ and $\langle N|\bar{J}_N|0\rangle$ are extracted from local-local hadron propagators. We fix the nucleon source at t=0, PS sink at $t = 32$ and move the operator between them. Matrix elements are evaluated for four spatial momenta $\vec{p}a$=(0,0,0), (1,0,0), (0,1,0), (0,0,1) in units of minimum momentum $ap_{min} = \pi/16$ injected in the PS meson.

We distinguish u-d and s quark masses; the former is taken to the chiral limit, and the latter interpolated to the physical s quark mass in our calculations. After this procedure, we interpolate the spatial momentum to the physical value.

Matrix elements are renormalized, with mixing included, by tadpole-improved one-loop renormalization factors to the $\overline{\text{MS}}$ scheme[7] calculated at the scale $\mu = 1/a$.

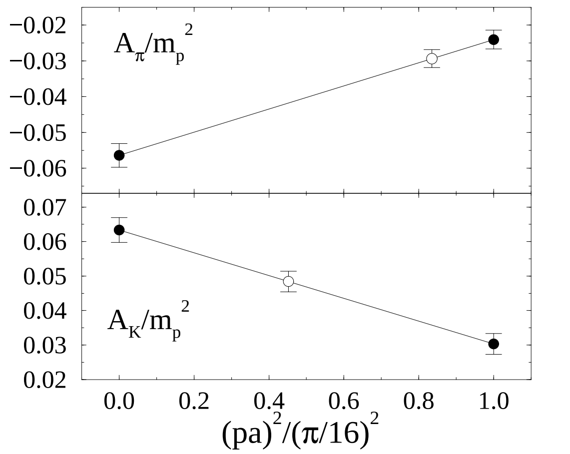

Figure 2. Momentum dependence of $p \to \pi^0$ and $p \to K^0$ matrix elements. Quark masses are taken to the physical point.

3. Results

Let us define the $p \to \pi^0$ and $p \to K^0$ matrix elements A_π and A_K by

$$\langle \pi^0(K^0)|\epsilon_{ijk}(u^iCd^j_L(s^j_L))u^k_R|p\rangle \equiv A_{\pi(K)}N_R. \quad (2)$$

In Fig. 1 we show our results for these matrix elements at zero momentum $\vec{p} = 0$, normalized by proton mass measured for relevant values of u-d quark mass. For A_π, abscissa represents the u-d quark mass, while it represents the s quark mass for A_K, the u-d quark mass having been taken to the chiral limit.

Compared to a first calculation of the A_π matrix element[4], whose results are consistent with ours, our statistical errors of 4–8% are improved by about a factor five. This allows us to observe that the amplitude exhibits a clear decrease of about 40% from the region of s quark ($m_{u,d}a \approx 0.04$) to the chiral limit, which was not apparent in results of Ref. [4].

For the $p \to K^0$ matrix elements A_K, after chiral extrapolation of u-d quark mass, the dependence on the s quark mass is small. The point plotted with a cross shows the value interpolated to the physical s quark mass.

In Fig. 2 we plot A_π and A_K as a function of squared momentum after quark masses are taken to the physical values. We observe a quite signifi-

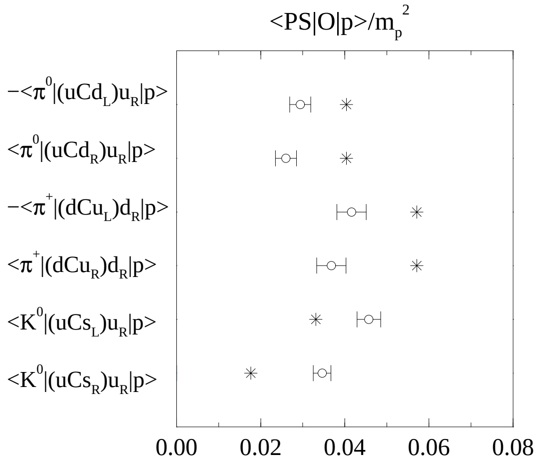

Figure 3. Matrix elements for $p \to \pi^0, \pi^+, K^0$ decay modes from present work (open circles) compared with predictions of tree-level chiral Lagrangian (asterisks).

cant momentum dependence for both amplitudes, necessitating an interpolation for a precise estimate of their physical values. The open circles in Fig. 2 show results of a linear interpolation in $\vec{p}^2$ to physical momentum.

Phenomenological analyses of proton decay often employ the soft-pion relation[8,9]

$$\langle \pi^0 | O^{\not B} | p \rangle = \langle 0 | O^{\not B} | p \rangle \frac{1}{\sqrt{2} f_\pi} (1 + F + D), \qquad (3)$$

to estimate the $p \to \pi^0$ matrix element, where F and D are the axial vector matrix elements of proton. With our results for physical quark masses and momentum, the right hand-side is about four times larger than the left hand-side of this relation. A similar discrepancy was observed in Ref. [4]. Examining the above relation for zero pion momentum in the chiral limit, *i.e.,* the real soft-pion limit, we find that the two sides are still discrepant by about a factor two. The origin of the discrepancy is not clear to us at present.

4. Phenomenology

We plot our preliminary results for matrix elements normalized by proton mass squared relevant for $p \to \pi^0, \pi^+$ and K^0 decay in Fig.3. For comparison, predictions from tree-level chiral Lagrangian with a choice of the parameters $-\alpha = \beta = 0.003$ GeV^3 are shown, which represent the smallest values among various model estimations. Here α and β are defined as $\langle 0 | \epsilon_{ijk} (u^i C d^j_{L(R)}) u^k_R | p \rangle = \alpha(\beta) N_R$. Our lattice results are even smaller for some of the channels, and predictions for the ratio of $p \to \pi^0$ and $p \to K^0$ amplitudes are also different.

For non-SUSY minimal SU(5) GUT, the decay width for the $p \to \pi^0$ mode is given by[4]

$$\Gamma(p \to \pi^0 e^+) = \frac{5\pi}{2} |F_s|^2 \alpha_5^2(M_X) A_\pi^2 \frac{m_p}{M_X^4}. \qquad (4)$$

Employing the GUT gauge coupling $\alpha_5(M_X) = 0.024$ and the short distance renormalization factor $|F_s|^2 = 10$ at $\mu = 2$GeV as in Ref. [4], substituting our preliminary result for A_π yields $\tau / B_{p \to \pi^0 e^+} = (1.1 \pm 0.1) \times 10^{30} (\frac{M_X}{2.0 \times 10^{14} \mathrm{GeV}})^4$ years where the error is only statistical.

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