Further study of the finite-temperature chiral phase transition of two-flavor lattice QCD at a small quark mass

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(Received 10 July 1990)

A previous finite-size study for the chiral phase transition of two-flavor QCD is extended to a smaller quark mass of $m_q=0.0125$ in lattice units. The characteristics of the system for lattice sizes $(6^3-12^3)\times 4$ are found to be quite similar to those for $m_q=0.025$. The increase of susceptibilities over this range of the spatial size is still too mild to discriminate among the order of the transition also at this small quark mass.

In a previous publication[1] we reported a finite-size study of the finite-temperature chiral phase transition of QCD for a light-quark mass employing the Kogut-Susskind quark action on lattices of a size ranging from $4^3\times 4$ to $12^3\times 4$. For four flavors $N_f=4$, strong evidence was found for a first-order transition at the quark mass of $m_q=0.025$ in lattice units. For $N_f=2$, on the other hand, we could not determine the order of the transition at the same quark mass. One of the reasons for this unsuccessful attempt may be ascribed to the possibility that the quark mass used in the analysis was not small enough to observe clear phase transition signatures. Since then we have, therefore, carried out a simulation decreasing the quark mass down to $m_q=0.0125$. This value corresponds to 7 MeV in physical units, if the lattice scale is determined by the ρ-meson mass.[2]

In this Brief Report we report on these additional runs. We made a hybrid R simulation for the time interval $\tau=4000-8000$ on $6^3\times 4$, $8^3\times 4$ and $12^3\times 4$ lattices with the molecular-dynamics time step chosen to be $\delta\tau=0.02$ and one trajectory corresponding to $\tau=1$. The other details of the runs are the same as in Ref. 1. In Table I we list the parameters of all of our runs including those reported in Ref. 1.

We present in Fig. 1 the time history of the Polyakov line and the corresponding histogram of the new runs on the three sizes of the lattice. Similar data for the chiral order parameter are shown in Fig. 2. The run on a $6^3\times 4$ lattice [(A) and (B) in Figs. 1 and 2] exhibits the behavior suggestive of a first-order transition; the time history shows reasonably clear flip-flops and the histogram appears to have a double-peak shape. This behavior seems to persist for an $8^3\times 4$ lattice [(C) and (D)].

With a further increase of the lattice size to $12^3\times 4$, however, the two-state signal in the time history becomes less apparent, and the histogram shows only a single peak [(E) and (F) in Figs. 1 and 2]. We also note that the separation between the two states decreases as the spatial lattice size increases. These observations contrast sharply to the case for $N_f=4$ (Ref. 1), for which, already at $m_q=0.025$, the two-state signals become increasingly clearer for a larger spatial size and the gap stays almost at a constant value independent of the lattice size.

We made a finite-size scaling analysis for the susceptibilities of the Polyakov line (χ_Ω), the chiral order parameter (χ_c) and the average plaquette (χ_P) which are defined by

TABLE I. The value of the coupling constant $\beta=6/g^2$ where our runs were made. The numbers in parentheses denote the length of the run in time units. The runs with $m_q=0.025$ are those reported in Ref. 1.

Size	m_q	$N_f=2$	$N_f=4$
$4^3\times 4$	0.025	5.28 (10 000)	4.94 (10 000)
$6^3\times 4$	0.025	5.285 (10 000)	4.98 (10 000)
$8^3\times 4$	0.025	5.28 (10 000)	4.98 (8500)
$10^3\times 4$	0.025		4.9825 (4000 hot)
			4.9825 (4000 cold)
$12^3\times 4$	0.025	5.29 (8000)	4.9833 (2500)
$6^3\times 4$	0.0125	5.27 (8000)	
$8^3\times 4$	0.0125	5.27 (5000	
$12^3\times 4$	0.0125	5.27 (4000)	

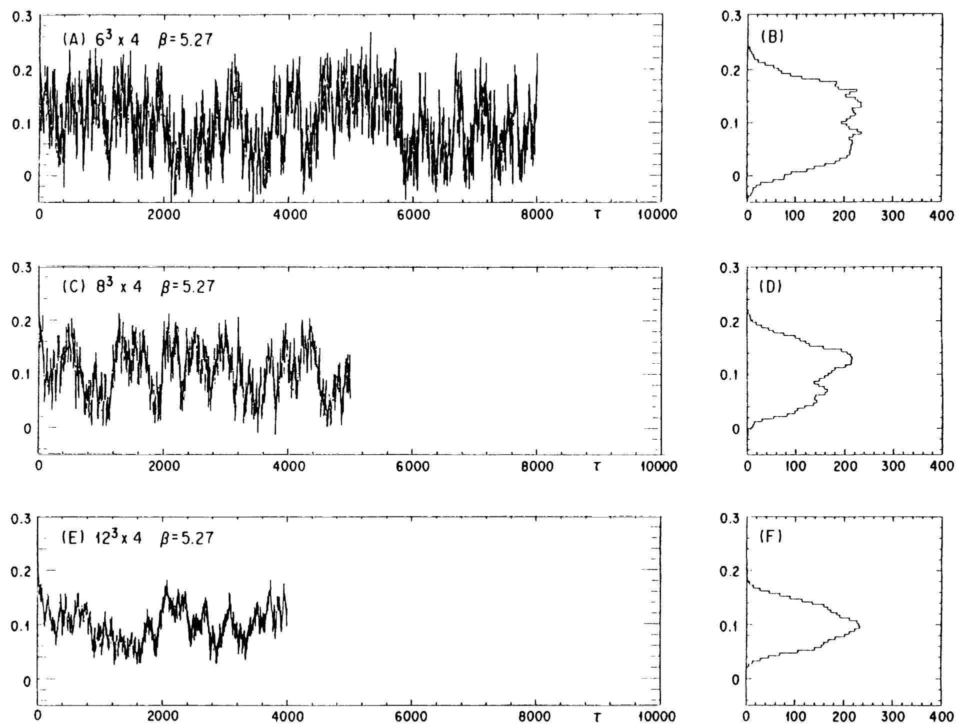

FIG. 1. Time history of the Polyakov line ReΩ for $N_f=2$ at $m_q=0.0125$ on an $N_s^3\times 4$ lattice with $N_s=6$, 8, and 12, and the corresponding histograms.

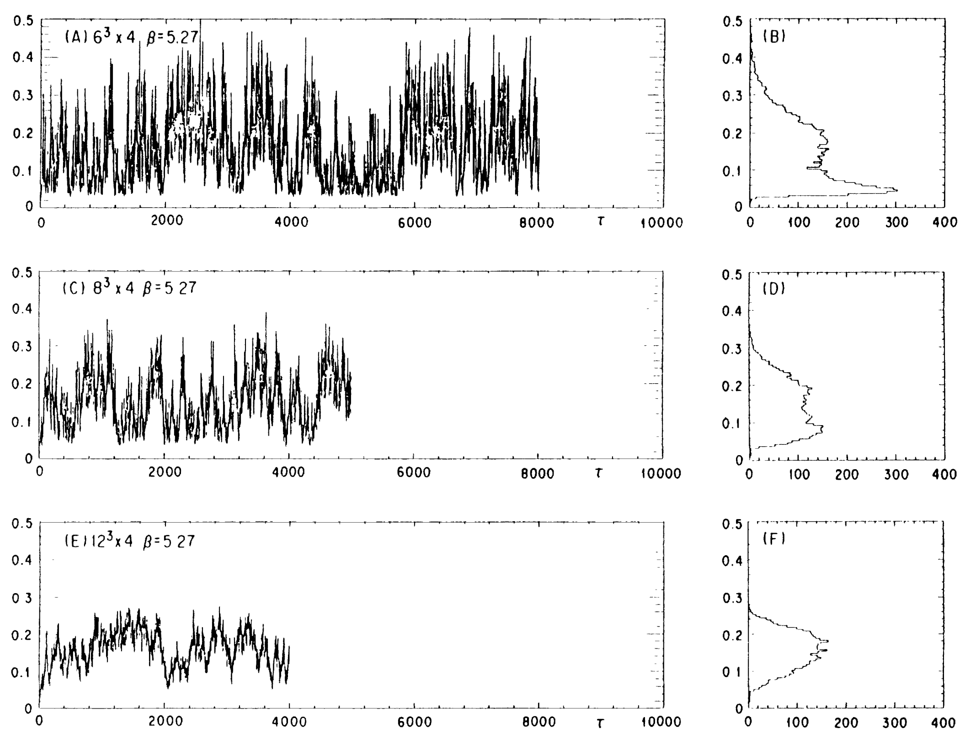

FIG. 2. Time history of the chiral order parameter $\bar{\chi}\chi$ computed as $\mathrm{tr}(D^{-1})/3$ and the histograms corresponding to the runs in Fig. 1.

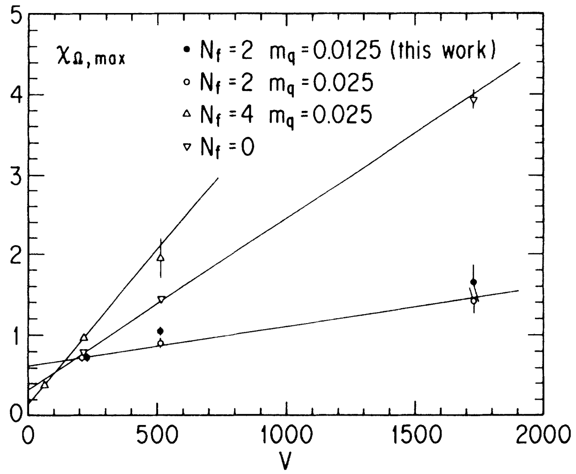

FIG. 3. The peak height of the Polyakov line susceptibility as a function of the spatial volume $V=N_s^3$. Filled circles represent the new data for $N_f=2$ at $m_q=0.0125$. Open symbols are the previous data for $m_q=0.025$ (Ref. 1) and for the pure gauge theory (Ref. 4), with lines showing a fit of form $\chi_{\Omega,\max}=c+aV$.

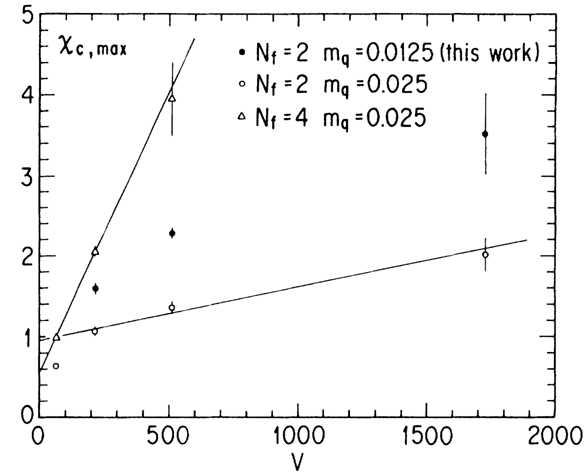

FIG. 4. The peak height of the chiral order parameter susceptibility. The meaning of the symbols is the same as in Fig. 3.

$$\chi_\Omega = V[\langle(\mathrm{Re}\Omega)^2\rangle - \langle \mathrm{Re}\Omega\rangle^2] ,$$

$$\chi_c = V\left[\left\langle \left[\frac{1}{3}\mathrm{tr}\frac{1}{D}\right]^2\right\rangle - \left\langle \frac{1}{3}\mathrm{tr}\frac{1}{D}\right\rangle^2\right] ,$$

$$\chi_P = V(\langle P^2\rangle - \langle P\rangle^2) ,$$

where $V=N_s^3$ is the spatial volume, D is the Kogut-Susskind quark operator, and the Polyakov line Ω and the average plaquette P are normalized to unity for the completely ordered gauge configuration. Figure 3 shows the peak height of the Polyakov line susceptibility $\chi_{\Omega,\max}$ as a function of the spatial volume V, obtained with the aid of the extrapolation technique.[3] (The peak position that may be taken as an estimate of the critical coupling is tabulated in Table II for all the runs we made, together with the full width at half maximum.) In the same figure we also plot data for $m_q=0.025$ (Ref. 1) for $N_f=2$ as well as those for $N_f=4$ (Ref. 1) and 0 (Ref. 4) for comparison. The curves for $m_q=0.0125$ and $m_q=0.025$ are very similar, whereas a sharper increase with V is generally expected if the chiral transition is substantially stronger for $m_q=0.0125$. The slow increase of $\chi_{\Omega,\max}$ as a function of V makes it difficult to conclude the order of the transition with the aid of finite-size scaling; while the data for $N_s \geq 8$ seem to lie on a line parallel to the linear fit $(c+aV)$ for $m_q=0.025$ as consistent with a first-order transition, they may also be fitted by a power $c+aV^p$ with p smaller than unity.

A similar figure is shown in Fig. 4 for $\chi_{c,\max}$. Here we observe that the intercept at $V=0$ increases from

TABLE II. The peak position of the susceptibility of the Polyakov line χ_Ω taken as an estimate of the critical coupling β_c. The numbers written below β_c are the full widths at half maximum of the peak.

Size	m_q	$N_f=2$	$N_f=4$
$4^3\times 4$	0.025		4.952(5)
			0.061(2)
$6^3\times 4$	0.025	5.290(2)	4.982(2)
		0.059(6)	0.023(1)
$8^3\times 4$	0.025	5.286(2)	4.983(3)
		0.031(5)	0.010(1)
$12^3\times 4$	0.025	5.288(2)	
		0.016(4)	
$6^3\times 4$	0.0125	5.271(3)	
		0.051(5)	
$8^3\times 4$	0.0125	5.269(2)	
		0.027(2)	
$12^3\times 4$	0.0125	5.271(2)	
		0.013(3)	

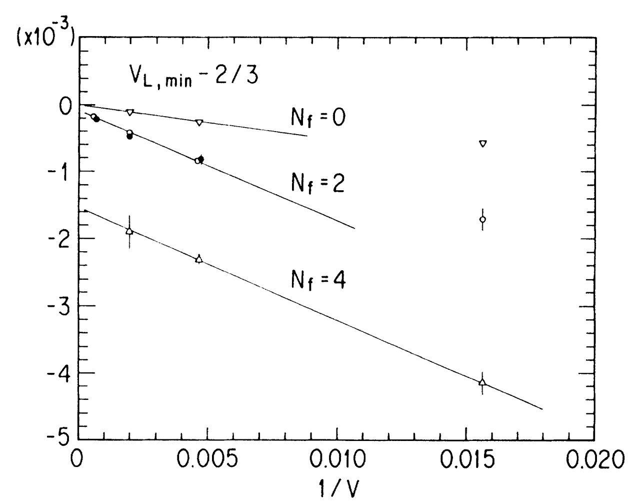

FIG. 5. The minimum value of the Challa-Landau-Binder cumulant for the average plaquette as a function of the inverse volume. The meaning of the symbols is the same as in Fig. 3.

$m_q = 0.025$ to $m_q = 0.0125$. The slope, however, changes little when the quark mass is decreased.

The minimum values of the Challa-Landau-Binder reduced cumulant[5] for the average plaquette are shown in Fig. 5 as a function of the inverse volume. The values for $m_q = 0.0125$ are located very close to those for $m_q = 0.025$. Here again we cannot draw a conclusion about the order of the transition.

To summarize, decreasing the quark mass down to $m_q = 0.0125$ served little to obtain improved signals for the order of the chiral phase transition for two flavors. We feel that the only alternative to determine the nature of the transition is to carry out a simulation on a lattice much larger ($\geq 24^3 \times 4$) than those used in the present study.

The numerical computation was made on HITAC S820/80 at KEK. Generous allowance of the computer time is gratefully acknowledged. This work was supported in part by the Grants-in-Aid of the Ministry of Education (Nos. 01460017 and 02740140).

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