

# Equation of state for pure SU(3) gauge theory on anisotropic lattices\*

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We present results for the equation of state for pure SU(3) gauge theory obtained on anisotropic lattices with the anisotropy  $\xi \equiv a_s/a_t = 2$ . The pressure and energy density are calculated on  $N_t/\xi = 4, 5$  and 6 lattices with the integral method. They are found to satisfy the leading  $1/N_t^2$  scaling from our coarsest lattice  $N_t/\xi = 4$ . This enables us to carry out well controlled continuum extrapolations. We find that the pressure and energy density agree with those obtained using the isotropic plaquette action, but have smaller and more reliable errors.

## 1. Introduction

The study of lattice QCD at finite temperatures is important to clarify the dynamics of the quark gluon plasma [1]. In order to extract predictions for the real world from lattice data, we have to extrapolate them to the continuum limit. However the continuum limit of the equation of state (EOS) has not been obtained in full QCD because of large lattice spacing dependence [2–4].

We have proposed using anisotropic lattices to solve this problem. As a first test, we have applied them for calculations of the EOS in pure SU(3) gauge theory and studied their efficiency [5]. We report a summary of our results in this article.

## 2. Simulation

We employ the plaquette action given by

$$S = \beta \left( \frac{1}{\xi_0} \sum_{n,ij} (1 - P_{ij}(n)) + \xi_0 \sum_{n,i} (1 - P_{i4}(n)) \right) \quad (1)$$

where  $\xi_0$  is the bare anisotropy and  $\beta = 6/g_0^2$ . We use the values of the renormalization factor  $\eta(\beta, \xi) \equiv \xi/\xi_0(\beta, \xi)$  obtained by Klassen [6], where  $\xi \equiv a_s/a_t$ .

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Our study in the Stephan-Boltzmann limit shows that  $\xi = 2$  is the optimal choice to reduce discretization errors in the EOS [5]. Therefore we use  $\xi = 2$  anisotropic lattices with sizes  $N_s^3 \times N_t = 16^3 \times 8, 20^3 \times 10$  and  $24^3 \times 12$ . For  $N_t = 8$ , we make additional runs on  $12^3 \times 8$  and  $24^3 \times 8$  lattices to examine finite size effects. The zero-temperature runs are made on  $N_s^3 \times \xi N_s$  lattices with  $\xi = 2$ . After thermalization, we perform 20,000 to 100,000 iterations on finite-temperature lattices and 5,000 to 25,000 iterations on zero-temperature lattices.

## 3. Scale and critical temperature

We determine the physical scale of our lattices from the string tension. The string tension  $\sigma$  is extracted from the static potential  $V(R)$  at zero temperature assuming a form [7],

$$V(R) = V_0 + \sigma R - e \frac{1}{R} + l \left( \frac{1}{R} - \left[ \frac{1}{R} \right] \right), \quad (2)$$

where  $[1/R]$  is the lattice Coulomb term from one gluon exchange. Then we fit the string tension in  $\beta$  using an ansatz proposed by Allton [8],

$$a_s \sqrt{\sigma} = f(\beta) (1 + c_2 \hat{a}(\beta)^2 + c_4 \hat{a}(\beta)^4) / c_0, \quad (3)$$

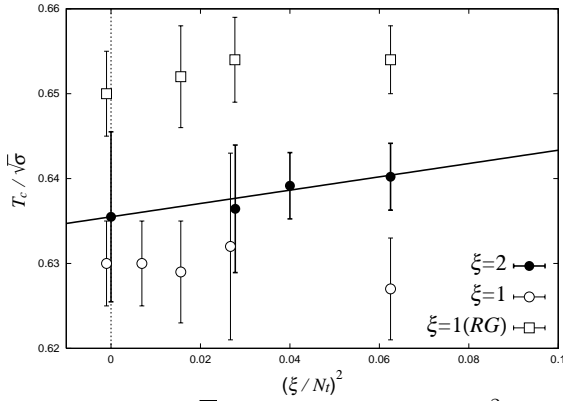


Figure 1.  $T_c/\sqrt{\sigma}$  as a function of  $(\xi/N_t)^2$ .

where  $f(\beta)$  is the two-loop scaling function of SU(3) gauge theory and  $\hat{a}(\beta) \equiv f(\beta)/f(\beta = 6.0)$ .

We define the critical coupling  $\beta_c(N_t, N_s)$  from the peak location of the susceptibility for a Z(3)-rotated Polyakov loop. We extrapolate  $\beta_c(N_t, N_s)$  to the thermodynamic limit assuming a finite-size scaling law,

$$\beta_c(N_t, N_s) = \beta_c(N_t, \infty) - h(N_t/(\xi N_s))^3. \quad (4)$$

From  $\beta_c$  on anisotropic  $12^3 \times 8$ ,  $16^3 \times 8$  and  $24^3 \times 8$  lattices with  $\xi = 2$ , we find  $h = 0.031(16)$  for  $N_t/\xi = 4$ . We adopt this value for all  $N_t$ . The critical temperature in units of the string tension is given by  $T_c/\sqrt{\sigma} = \xi/N_t a_s \sqrt{\sigma}(\beta_c(N_t, \infty))$ .

We extrapolate results for  $F = T_c/\sqrt{\sigma}$  to the continuum limit with the leading scaling form,

$$F|_{N_t} = F|_{\text{continuum}} + c_F/N_t^2. \quad (5)$$

In the continuum limit, we obtain  $T_c/\sqrt{\sigma} = 0.635(10)$  from the  $\xi = 2$  plaquette action.

Fig. 1 shows our results of  $T_c/\sqrt{\sigma}$ . In Fig. 1, we also plot the results obtained on isotropic lattices using the plaquette action [9] and the RG-improved action [10,11]. Our value of  $T_c/\sqrt{\sigma}$  in the continuum limit is consistent with that from the  $\xi = 1$  plaquette action within the error of about 2%, but slightly smaller than that from the  $\xi = 1$  RG-improved action. This may be caused by the difference in the details of the potential fit.

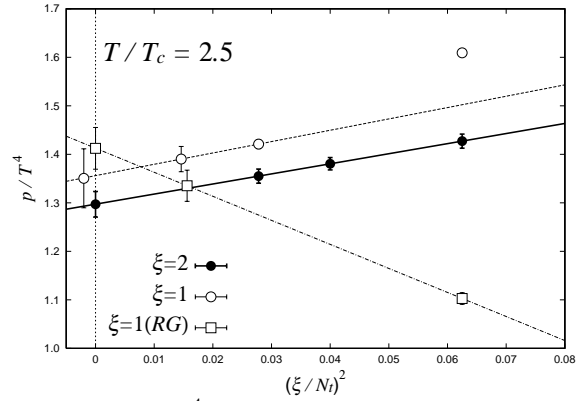


Figure 2.  $p/T^4$  at  $T = 2.5T_c$ .

#### 4. Equation of state

We calculate the pressure  $p$  and energy density  $\epsilon$  using the integral method[12],

$$\frac{p}{T^4} \Big|_{\beta_0}^{\beta} = \int_{\beta_0}^{\beta} d\beta' \Delta S, \quad (6)$$

$$\frac{\epsilon - 3p}{T^4} = -a_s \frac{\partial \beta}{\partial a_s} \Big|_{\xi=2} \Delta S, \quad (7)$$

where

$$\Delta S \equiv \left( \frac{N_t^4}{\xi^3} \right) \frac{1}{N_s^3 N_t} \frac{\partial \log Z}{\partial \beta} \Big|_{\xi=2} - (T = 0). \quad (8)$$

The beta function  $\partial \beta / \partial a_s|_{\xi=2}$  is determined through the string tension parametrized by Eq. (3). We extrapolate the EOS to the continuum limit with Eq. (5).

Figs. 2 and 3 show the pressure and energy density at  $T/T_c = 2.5$ , as a function of  $(\xi/N_t)^2$  (filled circles). For comparison, results from isotropic lattices using the plaquette action [13] (open circles) and the RG-improved action [11] (open squares) are also plotted.

The advantage of anisotropic lattices is clearly seen in Figs. 2 and 3. On the coarsest lattice  $N_t/\xi = 4$ , the cutoff errors at  $\xi = 2$  are much smaller than those at  $\xi = 1$  with the same plaquette action. Comparing the computational cost to achieve comparable systematic and statistical errors on isotropic and  $\xi = 2$  anisotropic lattices, we find that the anisotropic calculation has a factor of approximately 5 gain. Furthermore, since the  $\xi = 2$  data satisfy the  $1/N_t^2$  scaling from  $N_t/\xi = 4$

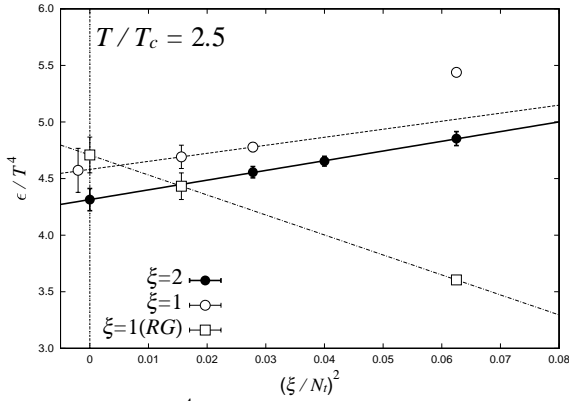


Figure 3.  $\epsilon/T^4$  at  $T = 2.5T_c$ .

(the right-most point), we can reliably perform an extrapolation to the continuum limit using three data points. Results at other temperatures are similar.

Our results for the EOS in the continuum limit are shown in Fig. 4. The EOS from  $\xi = 2$  is consistent with that from the  $\xi = 1$  plaquette action within the error of about 5%. Notice that our  $\xi = 2$  results have smaller and more reliable errors of 2–3%. On the other hand, the EOS from the  $\xi = 1$  RG-improved action is larger by 7–10% (about  $2\sigma$ ) than that from our  $\xi = 2$  results in the continuum limit. A possible origin of this discrepancy is the use of the  $N_t/\xi = 4$  data of the RG-improved action, which show a large (about 20%) deviation from the continuum limit.

## 5. Conclusion

We have studied the continuum limit of the EOS in pure SU(3) gauge theory on  $\xi = 2$  anisotropic lattices, using the plaquette action. Anisotropic lattices are shown to be efficient in reducing the cutoff dependence of the EOS. As a result, we can perform continuum extrapolations from our coarsest lattice  $N_t/\xi = 4$ . The EOS in the continuum limit agrees with that obtained on isotropic lattices using the same action, but has much smaller and better controlled errors.

Anisotropic lattices may help us extract the continuum limit of the EOS in full QCD.

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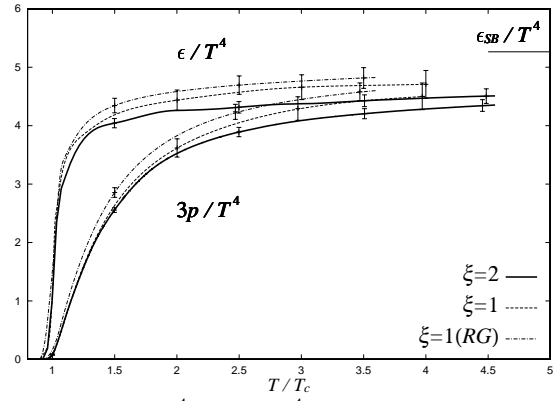


Figure 4.  $p/T^4$  and  $\epsilon/T^4$  as functions of  $T/T_c$ .

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