Differential decay rate for $B \to \pi l \nu$ semileptonic decays*

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We present our study on $B \to \pi l \nu$ semileptonic decay form factors using NRQCD action for heavy quark. In the analysis, we use the form factors $f_1(v \cdot k_\pi)$ and $f_2(v \cdot k_\pi)$ defined in the context of heavy quark effective theory by Burdman *et al.*. Since the NRQCD action for heavy quark respects the heavy quark symmetry, our results are described by the HQET form factors most naturally. From a quenched lattice QCD simulation at β=5.9 on a $16^3 \times 48$ lattice, we obtain the form factors $f_1(v \cdot k_\pi)$ and $f_2(v \cdot k_\pi)$, and find that their $1/m_B$ correction is small. The limit of physical heavy and light quark masses can be reached without introducing any model function, and we obtain a prediction for the differential decay rate $d\Gamma/dq^2$. We also discuss the soft pion limit of the form factors.

1. Introduction

Model-independent calculation of the B meson semileptonic decay form factors is essential for extracting $|V_{ub}|$, which is one of the most poorly determined elements of the CKM matrix. While the experimental results are being improved by the high statistics data from the B factories, there are also theoretical progresses in lattice QCD[1–3], which has a potential of making a precise prediction of the form factors near zero recoil.

The success of lattice QCD for heavy-light physics especially in the decay constant may be attributed to the use of the heavy quark (or non-relativistic) effective theory (HQET). The effective lattice actions constructed as a systematic expansion in $1/m_Q$ provide a control over the systematic errors from the large heavy quark mass. Furthermore, by identifying a quantity which has a well-defined heavy quark mass limit using the HQET, one may consider an expansion in $1/m_B$ and study the effect of finite heavy quark mass in a systematic way. Such a quantity for the decay constant is $f_B\sqrt{m_B}$, for which many lattice calculations found that the $1/m_B$ correction is significant.

In this study we take the same strategy for $B \to \pi l \nu$ semileptonic decay and use HQET motivated form factors $f_1(v \cdot k_\pi)$ and $f_2(v \cdot k_\pi)$ proposed by Burdman *et al.* as [4]

$$\frac{2}{\sqrt{m_B}}\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = f_1(v\cdot k_\pi)v^\mu + f_2(v\cdot k_\pi)\frac{k_\pi^\mu}{v\cdot k_\pi}, \qquad (1)$$

where k_π and p_B are four-momenta of π and B mesons in the external states, and $v^\mu = p_B^\mu/m_B$ is a four-velocity of the B meson. The form factors $f_1(v \cdot k_\pi)$ and $f_2(v \cdot k_\pi)$ are well-defined in the $m_B \to \infty$ limit, and it is natural to consider a $1/m_B$ expansion around that limit. We also note that $f_1(v \cdot k_\pi)$ and $f_2(v \cdot k_\pi)$ are implicit functions of light quark mass m_q. The relation to the conventional form factors $f^+(q^2)$ and $f^0(q^2)$ is given

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by

$$f^+(q^2) = \sqrt{m_B}\left[\frac{f_2(v\cdot k_\pi)}{v\cdot k_\pi} + \frac{f_1(v\cdot k_\pi)}{m_B}\right], \qquad (2)$$

$$f^0(q^2) = \frac{2}{\sqrt{m_B}}\frac{m_B^2}{m_B^2-m_\pi^2}\Big[f_1(v\cdot k_\pi)+f_2(v\cdot k_\pi) - \frac{v\cdot k_\pi}{m_B}\left(f_1(v\cdot k_\pi)+\hat{k}_\pi^2 f_2(v\cdot k_\pi)\right)\Big], \qquad (3)$$

where we define $\hat{k}_\pi^\mu = k_\pi^\mu/(v\cdot k_\pi)$ and $q^2 = m_B^2 + m_\pi^2 - 2m_B(v\cdot k_\pi)$.

2. Lattice calculations

We have performed quenched simulations at β=5.9 on a $16^3\times 48$ lattice, for which the lattice scale determined from the string tension is $1/a = 1.64$ GeV. We use the NRQCD action including $O(1/m_Q)$ terms for heavy quark and the $O(a)$-improved light quark action with c_{SW} calculated at one-loop $c_{\mathrm{SW}} = 1.58$. For the heavy quark mass we take $aM_0 = 5.0$, 3.0, 2.1 and 1.3, which cover the b quark mass. The light quark mass parameters, and some of the heavy quark mass, are the same as in our previous study for f_B [5]. We accumulated 2150 independent gauge configurations to reduce statistical error for the signals with finite spatial momenta. Even with this large number of statistics, the signal for heaviest heavy or lightest light quark is not clean enough to extract ground state.

The matching of the heavy-light vector current $V^\mu = \bar{q}\gamma^\mu Q$ is done at one-loop by Morningstar and Shigemitsu [6] and by Ishikawa *et al.* [7], in which the lattice operators involved are $\bar{q}\gamma_0 Q$, $-\frac{1}{2M_0}\bar{q}\gamma_0\gamma\cdot\nabla\mathbf{Q}$, and $-\frac{1}{2M_0}\bar{q}\gamma\cdot\overleftarrow{\nabla}\mathbf{Q}$ for the temporal component, and $\bar{q}\gamma_k Q$, $-\frac{1}{2M_0}\bar{q}\gamma_k\gamma\cdot\nabla\mathbf{Q}$, $-\frac{1}{2M_0}\bar{q}\gamma\cdot\overleftarrow{\nabla}\gamma_\mathbf{0}\gamma_\mathbf{k}\mathbf{Q}$, $-\frac{1}{2M_0}\bar{q}\gamma_0\nabla_k Q$, and $\frac{1}{2M_0}\bar{q}\overleftarrow{\nabla}\gamma_0 Q$ for the spatial components. In this work, we use the V-scheme coupling $\alpha_V(q^*)$ with $q^* = 1/a$ for the coupling constant.

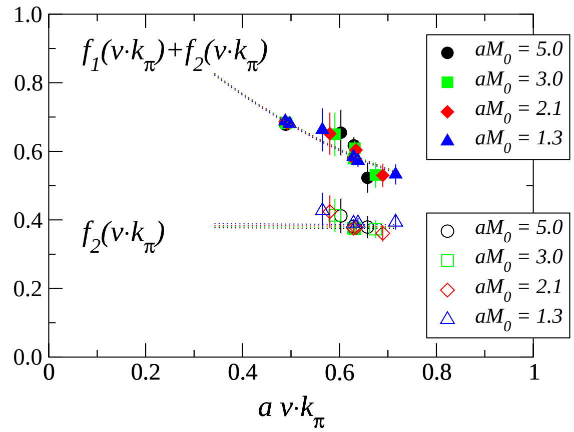

Figure 1. The HQET motivated form factors in unit of $a^{1/2}$ for four different heavy quark mass.

In the measurement of the matrix element we take several combinations of initial and final meson momenta. The maximum momentum we can measure is (1,0,0) for both initial and final momenta in the unit of $2\pi/(16a)$. As a result we can roughly cover a region 0.5~0.7 for $av\cdot k_\pi$, where the lower limit is given by a pion mass am_π. By taking an inner product with v^μ of both sides of (1), we extract a linear combination of form factors $f_1(v\cdot k_\pi)+f_2(v\cdot k_\pi)$. Another convenient basis is $f_2(v\cdot k_\pi)$, which is obtained by taking a inner product with an unit vector perpendicular to v^μ.

3. HQET form factor results

For the HQET motivated definition of the form factors (1), it is natural to expand the heavy quark mass dependence in terms of $1/m_B$. The light quark mass dependence can be expressed by a Taylor expansion in m_q when $v\cdot k_\pi$ is fixed. Thus, we take the following form to fit the matrix elements

$$f_1(v\cdot k_\pi)+f_2(v\cdot k_\pi) = A_0 + m_q A_1 + \frac{1}{m_B}A_2, \qquad (4)$$

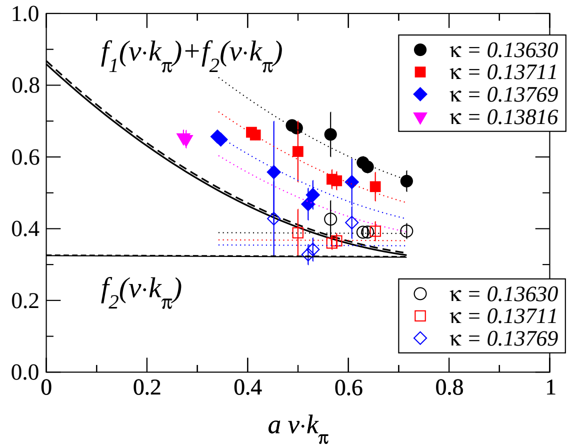

Figure 2. The HQET motivated form factors in unit of $a^{1/2}$ for four different light quark mass.

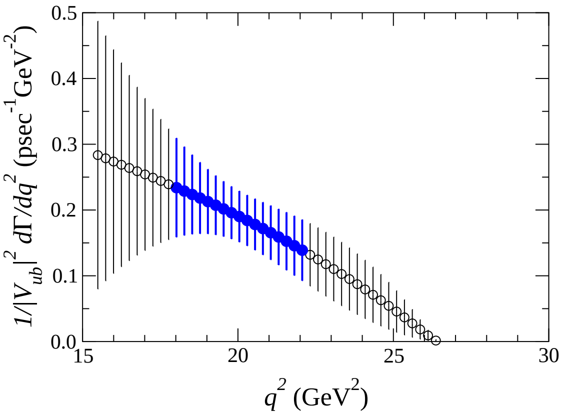

Figure 3. Differential decay rate for $B \to \pi l \nu$.

$$f_2(v \cdot k_\pi) \quad = \quad B_0 + m_q B_1 + \frac{1}{m_B} B_2, \qquad (5)$$

where the expansion coefficients A_i and B_i are functions of $v \cdot k_\pi$. The expansions are truncated at first order, since we find only mild mass dependences, as we shall see. In practice, we parametrize $v \cdot k_\pi$ dependence of the parameters by a Taylor expansion around $v \cdot k_\pi = E_0$ to a certain order. We take aE_0 to be 0.5, and expand A_0 through quadratic term, and A_1, B_0 through linear term. Others are taken to be constant.

In Figure 1 the form factors $f_1(v \cdot k_\pi) + f_2(v \cdot k_\pi)$ and $f_2(v \cdot k_\pi)$ are plotted for four different values of aM_0. We find that $f_1(v \cdot k_\pi) + f_2(v \cdot k_\pi)$ (filled symbols) has a significant negative slope in $v \cdot k_\pi$, while $f_2(v \cdot k_\pi)$ (open symbols) is consistent with constant. Dotted curves in the plot represent the fit (4) and (5). It is interesting that there is almost no heavy quark mass dependence, and therefore the $1/m_B$ correction to these form factors is very small in contrast to the decay constant, for which a large slope in $1/m_B$ was found in lattice calculations.

At a fixed heavy quark mass ($aM_0 = 1.3$), the form factors are plotted for different values of light quark mass in Figure 2, where the fit curves are also shown by dotted curves. We observe a mild dependence on the light quark mass. For $f_1(v \cdot k_\pi) + f_2(v \cdot k_\pi)$, a large fraction of the dependence comes from a trivial shift in $v \cdot k_\pi$, which is $E_\pi = \sqrt{m_\pi^2 + \mathbf{k_\pi^2}}$ in the B meson rest frame. An extrapolation to the physical or chiral light quark limit using the fit formula (4) and (5) is shown by dashed and solid curves respectively.

4. Differential decay rate

From the fit results for the HQET form factors we may obtain the usual form factors $f^+(q^2)$ and $f^0(q^2)$ using (2) and (3). The q^2 region where our lattice data is available is 18–22 GeV^2. The differential decay rate $d\Gamma/dq^2$ is then obtained through

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |\mathbf{k}_\pi|^3}{24\pi^3} |V_{ub}|^2 |f^+(q^2)|^2, \qquad (6)$$

which is plotted in Figure 3. We note that only the results in the region $q^2 = 18$–22 GeV^2, which is given by filled symbols, are obtained by interpolating the simulation data in $v \cdot k_\pi$, while the results outside the region involve an extrapolation of A_i and B_i in $v \cdot k_\pi$. Therefore, we should always keep in mind that the latter are subject to systematic errors from the choice of fitting functions.

The error bars in the plot show the statistical error only. Systematic errors, such as the dis-

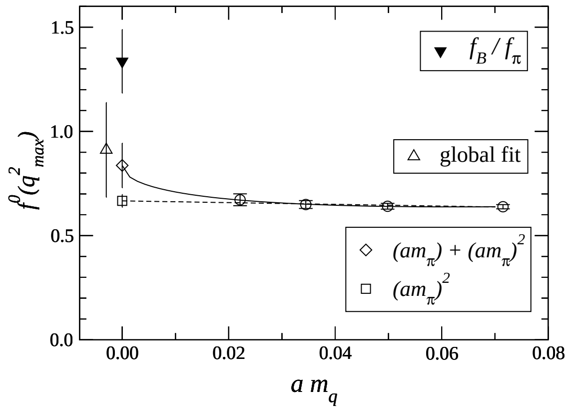

Figure 4. Soft pion limit of $f_0(q^2_{max}) = 2/\sqrt{m_B}[f_1(v\cdot k_\pi)+f_2(v\cdot k_\pi)]$. The dashed line is a linear fit in $(am_\pi)^2$, while the solid curve includes the term (am_π). A result of the fit (4) is given by an open triangle, which should be equal to f_B/f_π (filled triangle) in the soft pion theorem.

cretization effect $O((a\mathbf{k}_\pi)^\mathbf{2})$ or the perturbative error $O(\alpha_s^2)$, are under investigation.

5. Form factors in the chiral limit

Despite the uncertainties in the extrapolation $v\cdot k_\pi \to 0$, it is important to study the form factor in the soft pion limit (m_π, $k_\pi \to 0$), since the HQET and chiral symmetry predict a relation

$$f_1(0) + f_2(0) = \frac{\sqrt{m_B}}{2} f^0(q^2_{max}) = \frac{\sqrt{m_B}}{2}\frac{f_B}{f_\pi}, \quad (7)$$

where f_B is the heavy-light decay constant. In Figure 4 we compare the result of the fit (4) shown by an open triangle with the lattice calculation of f_B/f_π (filled triangle) [5]. Since the systematic errors, such as the uncertainty from $v\cdot k_\pi$ extrapolation and higher order perturbative corrections, are not yet included in the error bar, we consider that they are in reasonable agreement. We also plot two extrapolations of $f_0(q^2_{max}) = 2/\sqrt{m_B}\,[f_1(v\cdot k_\pi) + f_2(v\cdot k_\pi)]$ with the usual linear form in $(am_\pi)^2$ (dashed line) and with a quadratic fit in (am_π) (solid curve). Since the value of $v\cdot k_\pi$ varies in the extrapolation, linear dependence in m_π appears implicitly and the usual linear extrapolation in m_π^2 (or m_q) is no longer justified [1]. Although the effect of the linear term in am_π is very small and is only seen at the lightest quark mass, it raises the soft pion limit for the quadratic fit and makes it consistent with the result from extrapolation using the fit (4).

Near the zero recoil limit, the HQET predicts the B^* pole dominance [4]

$$f_2(v\cdot k_\pi) \to g_{BB^*\pi}\frac{f_{B^*}\sqrt{m_{B^*}}}{2f_\pi}\frac{v\cdot k_\pi}{v\cdot k_\pi + m_{B^*} - m_B}, \quad (8)$$

where $g_{BB^*\pi}$ denotes the $B^*B\pi$ coupling in the heavy-light meson chiral effective theory. Since the hyperfine splitting $m_{B^*} - m_B$ is small compared to $v\cdot k_\pi$, we can approximate its functional form by a constant in our data region. Then, we obtain $g_{BB^*\pi}(f_{B^*}\sqrt{am_{B^*}}/2f_\pi) = 0.32(18)$, which gives $g_{BB^*\pi} = 0.27(15)$. It agrees with the phenomenological value extracted from $D^* \to D\pi$ decay 0.27(6) [8].

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