Chiral extrapolations with small sea quark mass data in two-flavor lattice QCD [*]

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We present results on the light hadron spectrum and quark mass in two-flavor QCD calculated with small sea quark masses down to $m_{PS}/m_V = 0.35$. The configurations are generated using the RG improved gauge and tadpole-improved clover quark action at $\beta = 1.8$, where $a^{-1} \simeq 1$ GeV. Chiral extrapolations are made using not only polynomials and ChPT in the continuum but also formulae of Wilson chiral perturbation theory (WChPT) including $O(a^2)$ chiral breaking terms. We examine the viability of WChPT and its influence on quark masses.

1. Introduction

Systematic errors associated with chiral extrapolations present a serious obstacle to precise QCD calculations [1]. A rapid increase of the computational cost toward light quarks limits quark masses to the values larger than their physical ones, especially for dynamical Wilson-type quarks. This limitation causes sizable ambiguities in the extrapolation to the physical point.

In this report, we analyze issues related to small quark masses and finite lattice spacings in the chiral extrapolation using several chiral extrapolation functions. Our study is based on hadron mass data for small sea quark masses in the range $m_{PS}/m_V = 0.60 - 0.35$ [2], combined with our previous data at heavier quark masses at $m_{PS}/m_V = 0.80 - 0.55$ [3]. These hadron mass data are generated with an RG-improved gauge action and a meanfield-improved clover quark action at $\beta = 1.8$ ($a \simeq 0.2$ fm). We use $12^3 \times 24$ and $16^3 \times 24$ lattices with the spatial size of 2.4 and 3.2 fm. Our simulation parameters are summarized in Table 1. For further details, see Ref. [2].

2. Polynomial extrapolation

The conventional polynomial extrapolation has a problem that polynomials are not consistent with the logarithmic singularity expected in the chiral limit. It is imperative to estimate the systematic errors due to higher order contributions when the data are extrapolated using a low-order polynomial. Figure 1 shows that our new data at small sea quark masses deviate systematically from the quadratic fit of old data. The deviation is significant; applying higher order polynomial extrapolations to the whole data, κ_c differs by 10σ, and the lattice spacing by $6.5\sigma(7\%)$.

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κ_{sea}	$(m_{PS}/m_V)_{sea}$	N_{traj}	
		$12^3 \times 24$	$16^3 \times 24$
0.1409	0.807(1)	6250	
0.1430	0.753(1)	5000	
0.1445	0.694(2)	7000	
0.1464	0.547(4)	5250	
0.14585	0.609(2)	4000	
	0.604(3)		2000
0.14660	0.509(5)	4000	
	0.509(4)		2000
0.14705	0.413(8)	4000	
0.14720	0.349(19)	1400	

Table 1
Simulation parameters at $\beta = 1.8$. The top four runs are from Ref. [3].

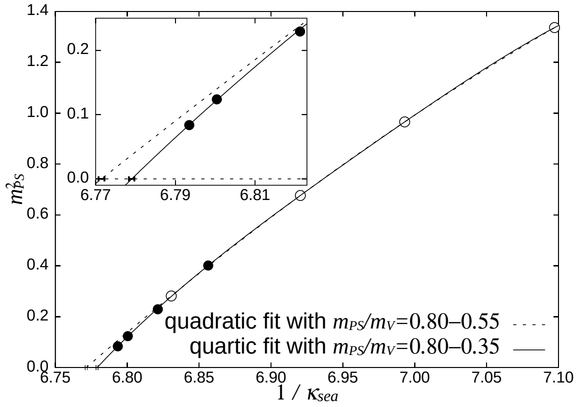

Figure 1. Chiral extrapolation of pseudoscalar meson mass. Open symbols show the results obtained in our previous calculation [3].

3. Chiral extrapolations based on ChPT

A choice for controlled chiral extrapolations is to incorporate chiral perturbation theory (ChPT).However, the present lattice data are not quite consistent with ChPT in the continuum [4], which predict the following quark mass dependence to one-loop order:

$$\frac{m_{PS}^2}{2B_0 m_{quark}} = 1 + \frac{1}{2}\frac{2B_0 m_{quark}}{(4\pi f)^2}\log\frac{2B_0 m_{quark}}{\Lambda_3^2},$$

where B_0, f and Λ_3 are parameters to be obtained by fits. Fitting the data over the whole range $m_{PS}/m_V = 0.80$–0.35 yields a large $\chi^2/dof \sim 70$. We have to restrict the fitting interval to $m_{PS}/m_V = 0.60$–0.35 to obtain a reasonable fit with $\chi^2/dof = 1.9$, as represented in Fig. 2. This may be an indication that the chiral logarithm is visible only at $m_{PS}/m_V \lesssim 0.40$.

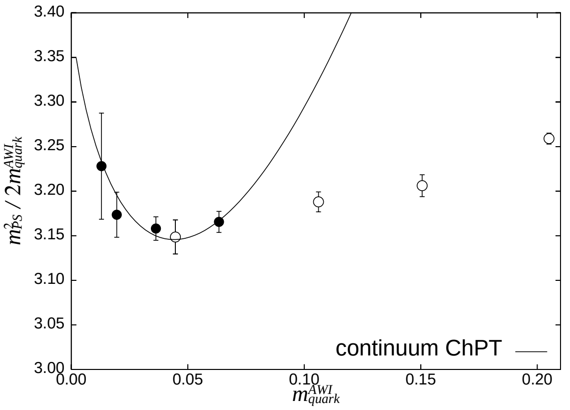

Figure 2. Simultaneous continuum ChPT fit to pseudoscalar meson mass and decay constant.

Another possibility that we do not clearly observe chiral logarithm in our data is the suppression by explicit chiral symmetry breaking of the Wilson quark action; modifications of ChPT due to finite lattice spacings may be needed for analyses of data obtained on coarse lattices.

Recently studies were made to adapt ChPT to the Wilson-type fermion at finite lattice spacings (WChPT) [5]. Care has to be exercised in the order counting since formulae for observables are different. We employ the WChPT formulae which treat the $O(a^2)$ effects at the leading order [6], because the existence of parity-broken phase and vanishing of pion mass depend on them in a critical way.

There are two features in these formulae worth emphasizing: (i) the coefficients of chiral logarithm terms receive $O(a)$ contributions, modifying the chiral behavior at finite lattice spacings, (ii) terms of the form $a^2 \log m_{quark}$ appear, which are more singular than the $m_{quark} \log m_{quark}$ terms toward the chiral limit at a finite lattice spacing. The $a^2 \log m_{quark}$ terms must be resummed. The resummed WChPT formulae (RWChPT) proposed by Aoki [6] read,

$$m_{PS}^2 \;=\; A m_{quark}^{VWI}\left(-\log\left(\frac{A m_{quark}^{VWI}}{\Lambda_0^2}\right)\right)^{\omega_0}$$

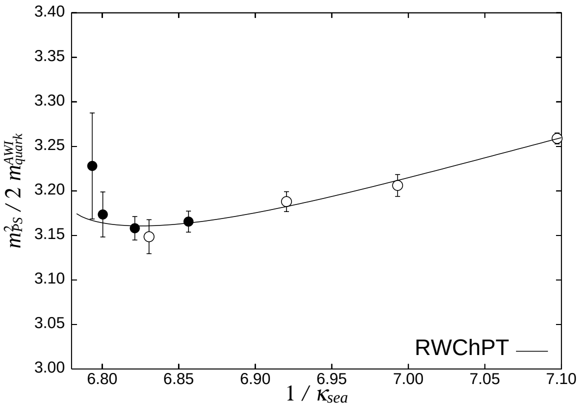

Figure 3. The resummed WChPT fit to pseudoscalar meson mass and AWI quark mass.

$$\left(1+\omega_1^{PS} m_{quark}^{VWI} \log\left(\frac{A m_{quark}^{VWI}}{\Lambda_3^2}\right)\right),$$

$$m_{quark}^{AWI} = m_{quark}^{VWI}\left(-\log\left(\frac{A m_{quark}^{VWI}}{\Lambda_0^2}\right)\right)^{\omega_0} \left(1+\omega_1^{AWI} m_{quark}^{VWI} \log\left(\frac{A m_{quark}^{VWI}}{\Lambda_{3,AWI}^2}\right)\right),$$

where the fit parameters are κ_c in m_{quark}^{VWI}, A, ω_0, ω_1^{PS}, ω_1^{AWI}, Λ_0, Λ_3 and $\Lambda_{3,AWI}$. As we show in Fig. 3, the one-loop RWChPT formulae is capable of fitting our data over the whole range $m_{PS}/m_V = 0.80$–0.35. The fit result shows that the leading and the one-loop contributions in RWChPT converges well and the parameter values are comparable with phenomenological estimates. It is also found that if we make a RWChPT extrapolation using our previous data at $m_{PS}/m_V = 0.80$–0.55, the new data at $m_{PS}/m_V = 0.60$–0.35 lie well on the extrapolation curve.

Encouraged by the features above of the WChPT fit, we reanalyzed the previous data at $m_{PS}/m_V = 0.80$–0.55 on finer lattices with $a = 0.16 (\beta = 1.95)$ and 0.11 fm ($\beta = 2.1$) using WChPT and performed the continuum extrapolation. For the quark mass in the continuum limit, we find $m_{ud}^{\overline{\rm MS}}(\mu = 2$ GeV$) = 3.11(17)$ [MeV], which is smaller than the previous result [3] based on the quadratic chiral extrapolation by approximately 10%.

4. Conclusions

We have extended the study of the light hadron spectrum and the quark mass in two-flavor QCD[3] to smaller sea quark masses corresponding to $m_{PS}/m_V = 0.60$–0.35. The new data clearly show systematic deviations from the previous quadratic chiral extrapolation using the data in the range $m_{PS}/m_V - 0.80$–0.55. Whereas fits with either polynomial or continuum chiral perturbation theory (ChPT) fails, the Wilson ChPT (WChPT) that includes a^2 effects associated with explicit chiral symmetry breaking successfully fits the whole data. In particular, WChPT correctly predicts the light quark mass behavior from data at medium heavy quark masses such as $m_{PS}/m_V \gtrsim 0.5$.

While our initial WChPT analyses gave encouraging results, further studies at smaller lattice spacings are needed to establish that the WChPT parameters show the expected lattice spacing dependence.

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