# Calculation of $K \to \pi\pi$ decay amplitudes from $K \to \pi$ matrix elements in quenched domain-wall QCD *

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We present a calculation of the $K \to \pi\pi$ decay amplitudes from the $K \to \pi$ matrix elements using leading order relations derived in chiral perturbation theory. Numerical simulations are carried out in quenched QCD with the domain-wall fermion action and the renormalization group improved gluon action. Our results show that the $I = 2$ amplitude is reasonably consistent with experiment whereas the $I = 0$ amplitude is sizably smaller. Consequently the $\Delta I = 1/2$ enhancement is only half of the experimental value, and $\varepsilon'/\varepsilon$ is negative.

## 1. INTRODUCTION

Quantitative understanding of the $K \to \pi\pi$ decay including the $\Delta I = 1/2$ rule and the value of $\varepsilon'/\varepsilon$ has been a long-standing issue in lattice QCD [1]. Here we present a summary of our work [2] based on (i) the leading order chiral perturbation theory (ChPT) formula relating the $K \to \pi\pi$ decay amplitudes to the $K \to \pi$ matrix elements [3], (ii) domain-wall fermion action to ensure chiral symmetry at finite lattice spacings in principle, and (iii) the renormalization group (RG) improved gauge action for suppression of chiral breaking effects for moderate fifth dimensional lattice size over those for the standard plaquette gauge action [4]. For a similar work using the plaquette gauge action we refer to Ref. [5].

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Table 1
Numbers of configurations generated in our calculation for each combination of $m_f$ and volume.

| $m_f$ | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 |
|---|---|---|---|---|---|
| $16^3 \times 32$ | 407 | 406 | 406 | 432 | 435 |
| $24^3 \times 32$ | 432 | 200 | 200 | 200 | 200 |

## 2. NUMERICAL SIMULATION

The numerical task at hand is to calculate the matrix elements $\left\langle \pi^+ \left| Q_i^{(I)} \right| K^+ \right\rangle$ and $\left\langle 0 \left| Q_i^{(0)} \right| K^0 \right\rangle$ where $Q_i^{(I)} (i = 1, \cdots, 10)$ are the local 4-quark operators of isospin $I = 0, 2$ that appear in the effective weak Hamiltonian.

This calculation is carried out with the RG-improved gauge action at $\beta = 2.6$, which corresponds to the scale $1/a = 1.94$ GeV determined from the string tension $\sqrt{\sigma} = 440$ MeV. For the domain-wall fermion action, we use the domain wall height $M = 1.8$ and the fifth dimensional size $N_s = 16$. We consider the case of degen-

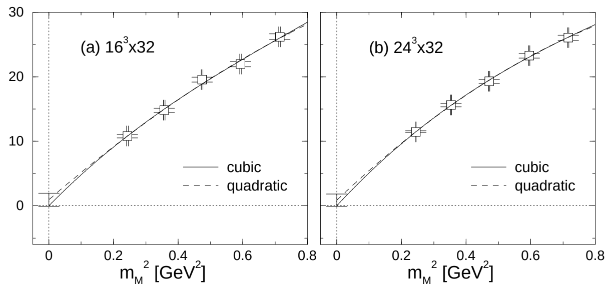


Figure 1. $K^+ \to \pi^+$ matrix element of $Q_6^{(0)}$ for (a) $16^3$ and (b) $24^3$. Solid and dashed curves show chiral fits as described in text.

erate $u$-$d$ and $s$ quark with a common mass $m_f$. Gauge configurations are generated *independently* for each value of $m_f$. Our statistics are summarized in Table 1 for the two lattice volumes and $m_f$ used in the calculation.

## 3. CHIRAL PROPERTIES

Our representative results for the $K \to \pi$ matrix elements are shown in Figs. 1 and 2. In terms of the lattice pion mass squared, $m_M^2$, the $K \to \pi$ matrix elements are expanded as

$$\left\langle \pi^+ \left| Q_i^{(I)} \right| K^+ \right\rangle = a_0 + a_1 m_M^2 + a_2 (m_M^2)^2 + a_3 (m_M^2)^2 \ln m_M^2 + a_4 (m_M^2)^3 + \cdots . \quad (1)$$

Checking the chiral property with a quadratic fit with the fit parameters $(a_0, a_1, a_2)$, we find the constant $a_0$ to be consistent with zero within statistical errors for most of matrix elements. The only exception is $Q_1^{(2)}$ as seen in Fig. 2. However, fits with the constraint $a_0 = 0$ such as a cubic one using $(a_1, a_2, a_4)$ or a form with the logarithm $(a_1, a_2, a_3)$ yield acceptable $\chi^2$/DOF. We therefore consider that the chiral properties required for viability of the ChPT relations between the $K \to \pi\pi$ and $K \to \pi$ amplitudes hold well.

An open problem is a small value of the coefficient of the chiral logarithm term obtained in the fits, *e.g.,* $a_3/a_1 = -0.58$ GeV$^{-2}$ for $Q_1^{(2)}$, which is only about 25% of the prediction of chiral perturbation theory [6]: $-3/(8\pi^2 f_\pi^2)$.

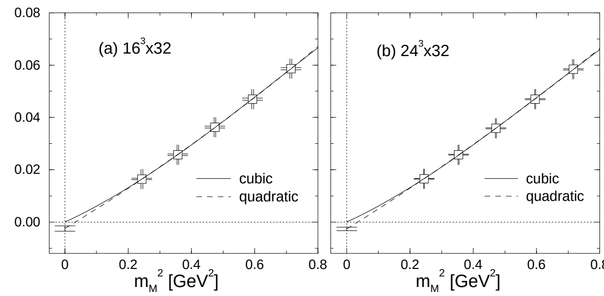


Figure 2. Same as Fig. 1 but for $Q_1^{(2)}$.

## 4. PHYSICAL RESULTS

We construct the $K \to \pi\pi$ decay amplitudes from the $K \to \pi$ matrix elements as described in Ref. [2]. Renormalization of the lattice matrix elements is made with tadpole-improved perturbation theory at one-loop level [7] at the matching point $q^* = 1/a$. The renormalization group running from $\mu = 1/a$ to $m_c$ (=1.3 GeV) is carried out for the $N_f = 3$ theory with $\Lambda_{\overline{\rm MS}} = 325$ MeV [8]. Combining these results with the Wilson coefficients lead to the physical results for the $K \to \pi\pi$ decay amplitudes $A_I$ $(I = 0, 2)$.

In Fig. 3 we present Re$A_2$, Re$A_0$ and $\omega^{-1}$ = Re$A_0$/Re$A_2$ as a function of $m_M^2$. We observe that the data from the volumes $16^3$ (open symbols) and $24^3$ (filled symbols) are consistent. Lines drawn are the fit curves for the quadratic (solid) function in $m_M^2$ or a form including chiral logarithm (dashed). The $I = 0$ amplitude reaches only half of the experimental value in the chiral limit, and hence the $\Delta I = 1/2$ rule, *i.e.,* $\omega^{-1} \approx 22$, is reproduced only by about 50% in the chiral limit.

In Fig. 4 we plot $P^{(3/2)}$ and $P^{(1/2)}$, which are related to $\varepsilon'/\varepsilon$ as [8] as

$$\varepsilon'/\varepsilon = {\rm Im}(V_{\rm ts}^* V_{\rm td})[P^{(1/2)} - P^{(3/2)}]. \quad (2)$$

Here we use our results only for the factor Im$A_{0,2}$. substituting the experimental value for the real part of the amplitudes. Our results for $\varepsilon'/\varepsilon$ is negative since $P^{(3/2)} > P^{(1/2)}$.

## 5. PENGUIN OPERATORS

Recently, Golterman and Pallante [9] pointed out a necessity of new operators to represent the

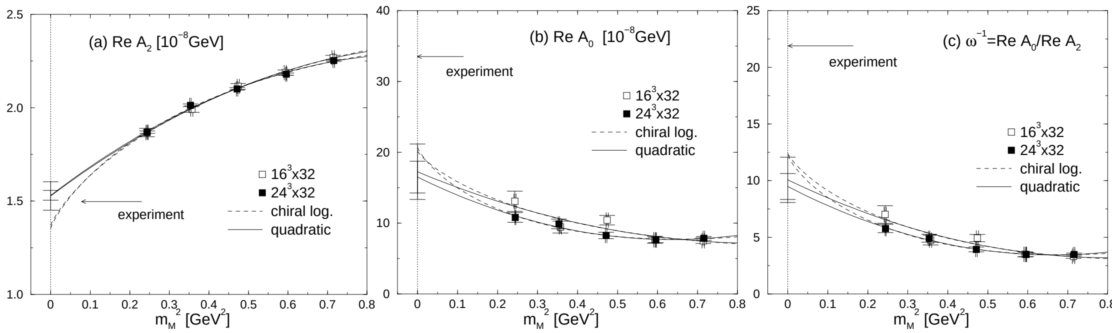


Figure 3. Real parts of the decay amplitude Re$A_I$ [$10^{-8}$GeV] and the ratio $\omega^{-1}$ = Re$A_0$/Re$A_2$ as a function of $m_M^2$ [GeV$^2$]. Open (filled) symbols are from the volume $16^3$ ($24^3$), and solid (dashed) curve show quadratic (chiral logarithm) fit result.

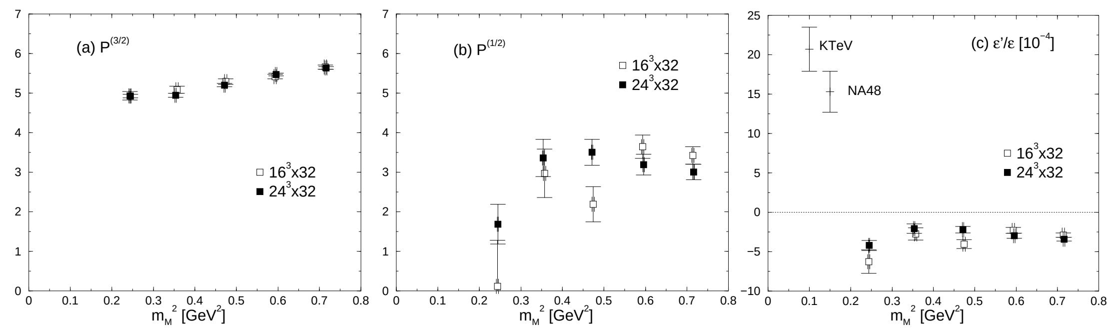


Figure 4. Imaginary parts of the decay amplitude $P^{(3/2)}$, $P^{(1/2)}$ and $\varepsilon'/\varepsilon$. Meaning of symbols is same as in Fig. 3.

penguin operators in (partially) quenched chiral perturbation theory, and gave a simple prescription to extract the physical contribution of $Q_{5,6}$ to the $K \to \pi$ and $K \to 0$ matrix elements in this framework. Our preliminary analyses show that this modification, if employed, increases $P^{(1/2)}$ by about 10–50% depending on $m_f$, but this effect is too small to affect the negative value of $\varepsilon'/\epsilon$ we have obtained.

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