B meson decay constant with the Wilson and Clover heavy quark actions*

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We present results of our quenched study of the B meson decay constant obtained with a parallel set of simulations with the Wilson and Clover actions at β=5.9, 6.1 and 6.3. Systematic errors associated with the large b-quark mass are analyzed within the Fermilab non-relativistic formalism. As our best estimate in the continuum limit we obtain f_B=163±16 MeV and f_{B_s}=175±18 MeV with the Clover action.

1. Introduction

A reliable determination of the B meson decay constant is a subject yet to be completed in lattice QCD. We have been pursuing this goal employing the relativistic formalism for heavy quark. Our results for the Wilson action has been reported in Refs. [1,2]. Since Lattice'96 we have carried out a parallel set of simulations with the $O(a)$-improved Clover action. We have analyzed the results for the two actions within the Fermilab non-relativistic formalism[3] with the view to understand the systematic error due to a large value of b-quark mass. In this article we report a summary of results from the simulations and analyses.

2. Simulation

The parameters of our simulations are listed in Table 1. For the clover coefficient we use the tadpole-modified one-loop value[5] given by $c_{sw}=P^{-3/4}[1+0.199\alpha_V(1/a)]$ with P the average plaquette. Heavy quarks are simulated for 7 values of the hopping parameter κ covering the c and b quark masses, and 4 values of κ are employed for light quark.

Table 1
Simulation parameters. The lattice scale quoted is fixed by the string tension $\sqrt{\sigma}$=427 MeV[4].

action	β	5.9	6.1	6.3
	size	$16^3\times40$	$24^3\times64$	$32^3\times80$
	$1/a$ (GeV)	1.60(1)	2.29(1)	3.05(2)
	L (fm)	2.0	2.1	2.1
Wilson	N_{conf}	150	100	100
Clover	N_{conf}	540	200	166
	c_{sw}	1.580	1.525	1.484

The heavy-light decay constant f_P is extracted from the correlators of the axial vector current A_4 and a smeared pseudoscalar density $P^S(x)=\sum_{\vec{r}}\phi(|\vec{r}|)\bar{Q}(x+r)\gamma_5 q(x)$ on the Coulomb gauge fixed gluon configurations, where $\phi(|\vec{r}|)$ is the pseudoscalar wave function measured for each heavy and light quark masses.

A new perturbative ingredient in our work is the recent one-loop result[6] for the pole (m_1^Q) and kinetic (m_2^Q) masses of heavy quark and the renormalization factor $Z_A(am_Q)$ of the axial vector current for finite bare heavy quark mass m_Q

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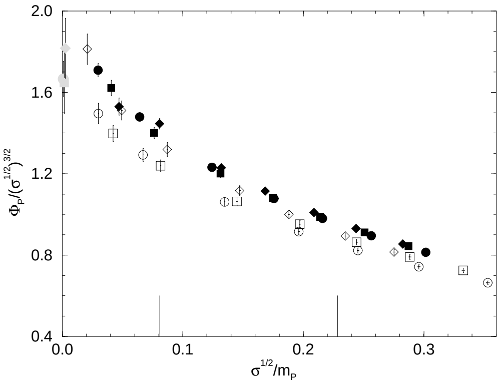

Figure 1. $\Phi_P/\sqrt{\sigma}^{3/2}$ as a function of $\sqrt{\sigma}/m_P$. Filled (open) symbols are Clover (Wilson) results. Circles, squares and diamonds correspond to values at β=5.9, 6.1 and 6.3.

(see Refs. [7,8] for previous calculation for the Wilson case). Effects of finite values of am_Q in Z_A is significant, reducing f_B by 5–2% for the Wilson action and increasing it by a similar magnitude for the Clover case compared to the value obtained with $Z_A(am_Q = 0)$.

We define the heavy-light meson mass by[9,2] $m_P = m_{P1} + m_2^Q - m_1^Q$ with m_{P1} the measured meson pole mass and the one-loop perturbative result[6] applied for $m_2^Q - m_1^Q$. This definition does not have the problem of the measured kinetic mass that the b quark mass can not be determined consistently from heavy-light and heavy-heavy mesons [10,11,2].

3. Results

We plot the quantity defined by $\Phi(m_P) = (\alpha_s(m_P)/\alpha_s(m_B))^{2/\beta_0} f_P\sqrt{m_P}$ in Fig. 1 as a function of $1/m_P$ for the the Clover(filled symbols) and Wilson (open symbols) actions. The light quark mass is linearly extrapolated to the chiral limit, and $\alpha_s(\mu)$ is calculated with the standard 2-loop definition with $\Lambda_{QCD} = 295$ MeV. We normalize the results by the string tension σ[4] since we primarily wish to examine the question of large-am_Q errors in this figure. Vertical lines indicate the position of B and D mesons for $\sqrt{\sigma} = 427$ MeV. Data points at $1/m_P = 0$ are the static results[12], to which our results for the same set of β values converge.

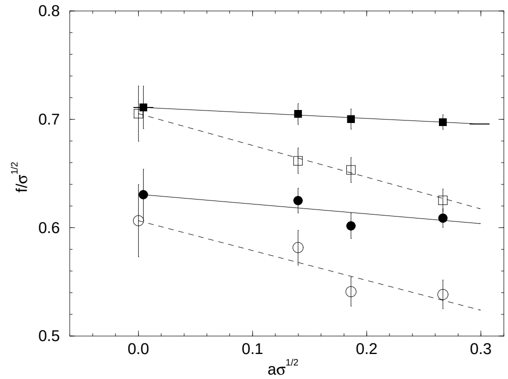

Figure 2. Continuum extrapolation of f_B (circles) and f_D (squares). Filled (open) symbols are Clover (Wilson) results.

In Fig. 2 we plot the continuum extrapolation of f_B and f_D. The Wilson results exhibit a scaling violation of 11-5% in our range of lattice spacing $a^{-1} \approx 1.6$–3 GeV, while the Clover results show a significantly reduced variation of 4–2%. These magnitudes are common to f_B and f_D. Furthermore the continuum values obtained with the two actions by a linear extrapolation agree within the statistical error of about 5%.

We emphasize that this agreement does not necessarily mean that systematic errors due to heavy quark are negligibly small. In the non-relativistic interpretation, the equivalent Hamiltonian for Wilson-type actions has the form

$$H = \bar{Q}\left[m_1 - \frac{\vec{D}^2}{2m_2} - \frac{i\vec{\sigma}\cdot\vec{B}}{2m_B} + O(1/m_Q^2)\right]Q. \quad (1)$$

For the Wilson action for which $m_B \neq m_2$, the leading error in the decay constant due to heavy quark is $O((c_B-1)\Lambda_{QCD}/m_Q)$ with $c_B = m_2/m_B$. For the B meson an examination c_B at the tree level shows that a linear extrapolation of c_B from our range of a^{-1} leads to a value $|c_B - 1| \approx 0.4$ at a=0. We should therefore allow an $O(3\%)$ error unremoved in the continuum limit where we used Λ_{QCD}=0.3 GeV. The same magnitude of error also remains for f_D.

There are two more sources of systematic error we need to consider. One is m_Q-independent scaling violation of $O(a\Lambda_{QCD})$, which we estimate to be 10% at our smallest $a^{-1} \approx 3$ GeV. The other is $O(\alpha_s^2)$ uncertainty due to the use of one-loop

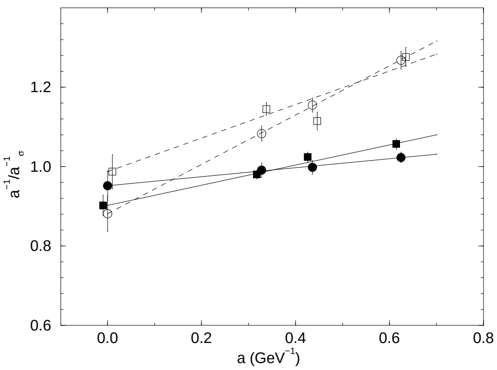

Figure 3. Ratio of lattice scale obtained from m_ρ (circles) and from f_π (squares) to that from the string tension. Filled (open) symbols are for Clover (Wilson) action.

value for Z_A, which is $O(4\%)$ with $\alpha_s \approx 0.2$. Adding all the errors by quadrature leads to a combined systematic error of $O(11\%)$ in the decay constant for the Wilson case.

For the Clover action for which $m_B = m_2$ to $O(\alpha_s)$, the large-am_Q errors have the form $O(\alpha_s\Lambda_{QCD}/m_Q)$ and $O(\Lambda_{QCD}^2/m_Q^2)$. We estimate their magnitude to be $O(1\%)$ incorporating the effect of coefficients, similar to c_B, that vanish in the continuum limit. The scaling violation errors are $O(\alpha_s a\Lambda_{QCD}, a^2\Lambda_{QCD}^2)$ which are small at $O(2\%)$. With the 2-loop error of $O(4\%)$ from Z_A and an additional error of $O(2\%)$ arising from the field rotation ignored in the present calculation, the combined systematic error amounts to $O(5\%)$ for the Clover case.

So far we have used the string tension σ to set the scale. In Fig. 3 we plot the ratio of a^{-1} obtained with m_ρ and f_π to that with σ. We use the variation of the ratio to estimate the uncertainty in setting the scale, which we take to be 10% for the Wilson case and 5% for the Clover case. This uncertainty includes possible quenching error as the ratio need not converge to unity in the continuum limit.

Our final result for the decay constant is tabulated in Table 2. To obtain the values we take the continuum extrapolation of $f_P/\sqrt{\sigma}$ and convert it f_P/m_ρ with the value of $\sqrt{\sigma}/m_\rho$ at a=0 in Fig. 3. A direct continuum extrapolation of f_P/m_ρ yields consistent results. The quoted errors are statistical, systematic and due to scale setting as estimated above in this order. We take the result for the Clover action to be our best estimate primarily because scaling violation is smaller and also since the statistical ensemble is larger compared to the Wilson action. Combining errors by quadrature we obtain the results quoted in the abstract.

Table 2
Results for the decay constant in MeV unit.

	Wilson	Clover
f_B	140(11)(15)(24)	163(9)(8)(11)
f_{B_s}	159(10)(17)(27)	175(9)(9)(13)
f_D	163(13)(18)(28)	184(9)(9)(12)
f_{D_s}	180(11)(20)(31)	203(9)(10)(14)

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REFERENCES

1. JLQCD Collaboration (S. Aoki *et al.*), Nucl. Phys. (Proc. Suppl.) **47** (1996) 433.
2. JLQCD Collaboration (S. Aoki *et al.*), Nucl. Phys. (Proc. Suppl.) **53** (1997) 355.
3. A.X. El-Khadra *et al.,* Phys. Rev. **D55** (1997) 3933.
4. G. S. Balli and K. Schilling, Phys. Rev. **D46** (1992) 2636.
5. M. Lüscher and P. Weisz, Nucl. Phys. **B479** (1996) 429.
6. S. Aoki, S. Hashimoto, K.-I. Ishikawa and T. Onogi, in preparation.
7. A.S. Kronfeld and B.P. Mertens, Nucl. Phys. B (Proc. Suppl.) **34** (1994) 495.
8. Y. Kuramashi, hep-lat/9705036.
9. C.W. Bernard, J.N. Labrenz and A. Soni, Phys. Rev. **D49** (1994) 2536.
10. S. Collins *et al.*, Nucl. Phys. B (Proc. Suppl.) **47** (1996) 455.
11. A.S. Kronfeld, Nucl. Phys. (Proc. Suppl.) **53** (1997) 401.
12. A. Duncan *et. al.*, Phys. Rev. **D51** (1995) 5101.