B meson B-parameters and the decay constant in two-flavor dynamical QCD *

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We present a two-flavor dynamical QCD calculation of the B meson B parameters and decay constant. We use NRQCD for heavy quark and the nonperturbatively $O(a)$-improved Wilson action for light quark at $\beta = 5.2$ on a $20^3 \times 48$ lattice. We confirm that the sea quark effect increases the heavy-light decay constant, while estimate of its magnitude depends significantly on the fitting form in the chiral extrapolation. For the B parameters, on the other hand, we do not find a significant sea quark effect. The chiral extrapolation with logarithmic term is examined for both quantities and compared with the prediction of ChPT.

1. Introduction

In recent lattice calculations, there have been several indications that the presence of sea quark increases the B meson decay constant f_B significantly, *i.e.* by about 10–15% [1]. Then, the study of the sea quark effect on the B parameters has a great phenomenological importance, because the relevant physical quantity to the $B - \bar{B}$ mixing is the combination $f_B^2 B_B$.

In this work we perform unquenched lattice calculations of both f_B and B_B using a consistent formulation. We use the nonperturbatively $O(a)$-improved Wilson quark action for both sea and valence light quarks, and the NRQCD action for heavy quark. Using the same dynamical QCD configurations, a search for sea quark effects in the light hadron spectrum is also being performed [2]. Quenched calculations of f_B and B_B using the NRQCD action have been presented previously [3,4].

In full QCD, the effect of virtual pion loops leads to chiral logarithms, a specific dependence of physical quantities on the sea quark mass. Hence, it provides a non-trivial test of the sea quark effect in lattice simulations. For f_B and B_B chiral perturbation theory (ChPT) is extensively studied in [5,6] including its extension to the quenched and the partially quenched cases, where sea and valence quark masses can be different. We test the chiral logarithm using our numerical data.

2. Simulation details

We generate a set of unquenched gauge configurations with the plaquette gauge action and two-flavors of nonperturbatively $O(a)$-improved Wilson fermions [7]. Simulations are performed on a $20^3 \times 48$ lattice at β=5.2, which roughly corresponds to the lattice spacing $a \sim 0.1$ fm. We take five different sea quark masses covering m_{PS}/m_V=0.6–0.8. For comparison we also perform a quenched simulation at β=6.0 on the same lattice size. Further details of our simulations are given in [2].

We use the NRQCD action for heavy quark including all $1/m_Q$ corrections consistently in the action and operators. In the calculations of f_B and B_B we follow the methods adopted in the previous works [3,4]. In particular, the B parameters
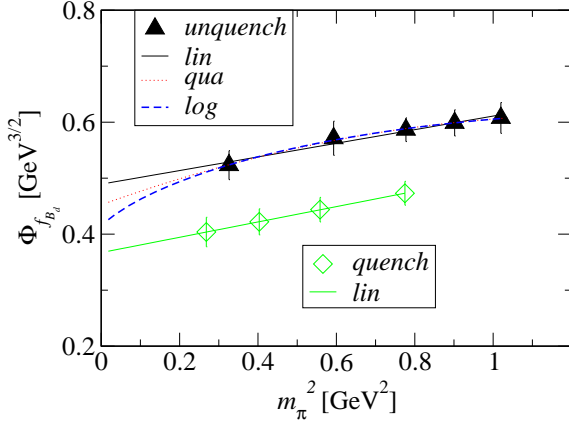
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*Talk presented by N. Yamada.

Figure 1. Chiral extrapolation of $\Phi_{f_{B_d}}$.

Figure 2. The $1/M_P$ dependence of $\Phi_{f_{B_d}}$.

are computed with four different prescriptions, all of which have equivalent accuracy up to higher order contributions we neglect. We may estimate the systematic uncertainty taking the differences among them [4].

3. Decay constant

Figure 1 shows the chiral behavior of the heavy-light decay constant $\Phi_{f_B} \simeq f_B\sqrt{m_B}$ with $m_Q \sim m_b$. Both horizontal and vertical axes are normalized with a proper power of r_0 and converted to the physical unit using the lattice result for $r_0 m_\rho$ combined with physical input for m_ρ. If we compare the unquenched result (filled triangles) with the quenched data (open diamonds), we clearly find the increase of about 15% with the unquenching as found previously [1]

We consider the following three forms for the chiral extrapolation

$$
\begin{array}{lll}
\text{"lin"} & : & a_0\,(1 + a_1 X), \\
\text{"qua"} & : & b_0\,(1 + b_1 X + b_2 X^2), \\
\text{"log"} & : & c_0\,(1 + c_1 X + c_2 X \ln X),
\end{array}
\tag{1}
$$

with $X = r_0^2 m_\pi^2$. The "log" form is motivated by the ChPT result for heavy-light mesons [5,6]. For two-flavor QCD, it predicts

$$
r_0^2 c_2 = -\frac{3}{4}\frac{1 + 3g^2}{(4\pi f)^2},
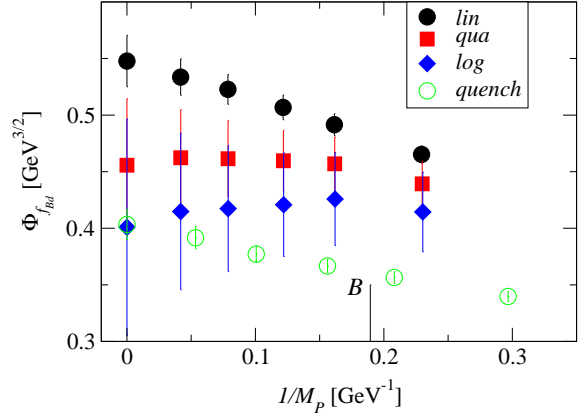\tag{2}
$$

with f the pion decay constant. The coupling g

describes $B^*B\pi$ interaction, which may be evaluated from D^* decays using heavy quark symmetry. If we take a range $g=0.2$–0.7 rather conservatively covering $D^* \to D\pi$ decay [8] and the recent D^* width experiment [9], then we obtain $r_0^2 c_2 = -0.5(2)$ GeV^{-2}.

We find that both "qua" and "log" fits describe our unquenched data well, while the linear fit gives somewhat poorer χ^2 than the other two. For the "log" fit we obtain $r_0^2 c_2 = -0.4(3)$ GeV^{-2}, which is consistent with the above phenomenological estimate.

On the other hand, in quenched QCD, no deviation from linearity is observed in our data, and the three fit forms lead to almost identical result in the chiral limit. This behavior contradicts the prediction of quenched ChPT (QChPT) [5,6], which gives even stronger logarithmic dependence. There are, however, many unknown parameters in QChPT, so it is still possible to find a parameter region which mimics the linear behavior in our light quark mass regime.

Since we are not able to decide the fit form with our current unquenched data, the uncertainty in the chiral limit is quite large. Figure 2 shows the $1/M_P$ dependence of $\Phi_{f_{B_d}}$ at the physical light quark mass. At the physical B meson mass, we observe that the two-flavor result (filled symbols) is 14–25% larger than the quenched result (open circle), depending on the fit form.
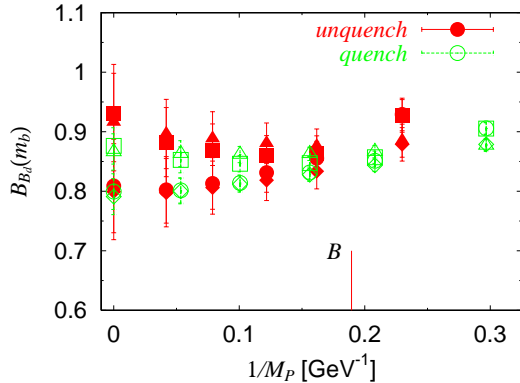
Figure 3. The $1/M_P$ dependence of $B_{B_d}(m_b)$.

4. B parameters

We also examine the chiral behavior of B_B by applying the fit forms in (1), but, because the light quark mass dependence itself is quite small, we do not observe any significant dependence on the fit functions. A weak dependence on sea quark mass is also suggested by ChPT, which gives

$$r_0^2 c_2 = -\frac{3}{2}\frac{1-3g^2}{(4\pi f)^2} = -0.1(4) \text{ GeV}^{-2} \qquad (3)$$

for B_B.

Figure 3 shows the $1/M_P$ dependence of $B_B(m_b)$ for both quenched (open symbols) and unquenched QCD (filled symbols). Four different symbols correspond to different prescriptions defined in [4]. Within the uncertainty indicated by the different prescriptions, we find no significant effect of unquenching. $B_{S_s}(m_b)$ is also analyzed in a similar manner and we arrive at the same conclusion.

5. Conclusions

Our preliminary results in two-flavor QCD are summarized as follows,

$$
\begin{aligned}
f_{B_d} &= 190(14)(07)(19)\,\text{MeV}, \\
f_{B_s}/f_{B_d} &= 1.184(26)(20)(15), \\
B_{B_d}(m_b) &= 0.872(39)(04)(73), \\
B_{B_s}/B_{B_d} &= 0.999(12)(04), \\
B_{S_s}(m_b) &= 0.858(33)(07)(72),
\end{aligned}
$$

where errors are from statistics, chiral extrapolation, and systematics. We discard the linear fit from the final results because of its poor χ^2 value. The central value is obtained by taking an average over "qua" and "log" results, and their difference is taken as a systematic error from the chiral extrapolation. Some of the systematic errors in the B parameters are estimated by the scatter with different prescriptions as in [4]. Other uncertainties are estimated by a naive dimensional counting, assuming Λ_{QCD}=500 MeV and α_s=$\alpha_V(1/a)$=0.267.

Our data for f_B and B_B are consistent with the prediction of ChPT. This is different from the light hadron sector where we find that ChPT fails to reproduce the lattice data for pion mass and decay constant [2]. To investigate these issues we are currently accumulating further statistics, which also helps to reduce the statistical error in our results.

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