

A scaling study of the step scaling function of quenched QCD with improved gauge actions *

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We study the scaling behavior of the step scaling function for SU(3) gauge theory, employing the Iwasaki gauge action and the Luescher-Weisz gauge action. In particular, we test the choice of boundary counter terms and apply a perturbative procedure for removal of lattice artifacts for the simulation results in the extrapolation procedure. We confirm the universality of the step scaling functions at both weak and strong coupling regions. We also measure the low energy scale ratio with the Iwasaki action, and confirm its universality.

1. Introduction

Recently CP-PACS and JLQCD Collaborations have started a project for $N_f = 3$ QCD simulations. One of the targets of the project is to evaluate the strong coupling constant $\alpha_{\overline{\text{MS}}}$ in the $N_f = 3$ QCD using the Schrödinger functional(SF) scheme[1]. In the project Iwasaki gauge action[2] has been employed to avoid the strong lattice artifacts of the plaquette gauge action found in $N_f = 3$ simulations[3].

In a previous study [4], as our first step toward evaluation of $\alpha_{\overline{\text{MS}}}$ for $N_f = 3$, $O(a)$ boundary improvement coefficients in the SF scheme have been determined for the various improved gauge actions up to one-loop order in the perturbation theory. As the next step, we investigate the lattice cut off dependence of the step scaling func-

tion(SSF) non-perturbatively in quenched lattice QCD simulations with the improved gauge actions for some choices of the improvement coefficients. We also confirm the universality of SSF and the low energy scale ratio, by comparing our results with the previous ones obtained by ALPHA Collaboration[5,6,7]. We refer to ref. [8] for unexplained notations in this report.

2. Setup

We follow the same SF setup as in the case of the Wilson plaquette action[5], but employ the improved gauge actions including the plaquette(\mathcal{S}_0) and rectangle(\mathcal{S}_1) loops:

$$S_{\text{imp}}[U] = \frac{1}{g_0^2} \sum_{i=0}^1 \sum_{\mathcal{C} \in \mathcal{S}_i} W_i(\mathcal{C}, g_0^2) 2\mathcal{L}(\mathcal{C}). \quad (1)$$

The assignment of the weight factor $W_i(\mathcal{C}, g_0^2)$ near the boundary in the time direction is important to achieve $O(a)$ improvement in the SF scheme. We employ the weights,

$$c_0 c_t^P(g_0^2) = c_0 (1 + c_t^{P(1)} g_0^2 + O(g_0^4)), \quad (2)$$

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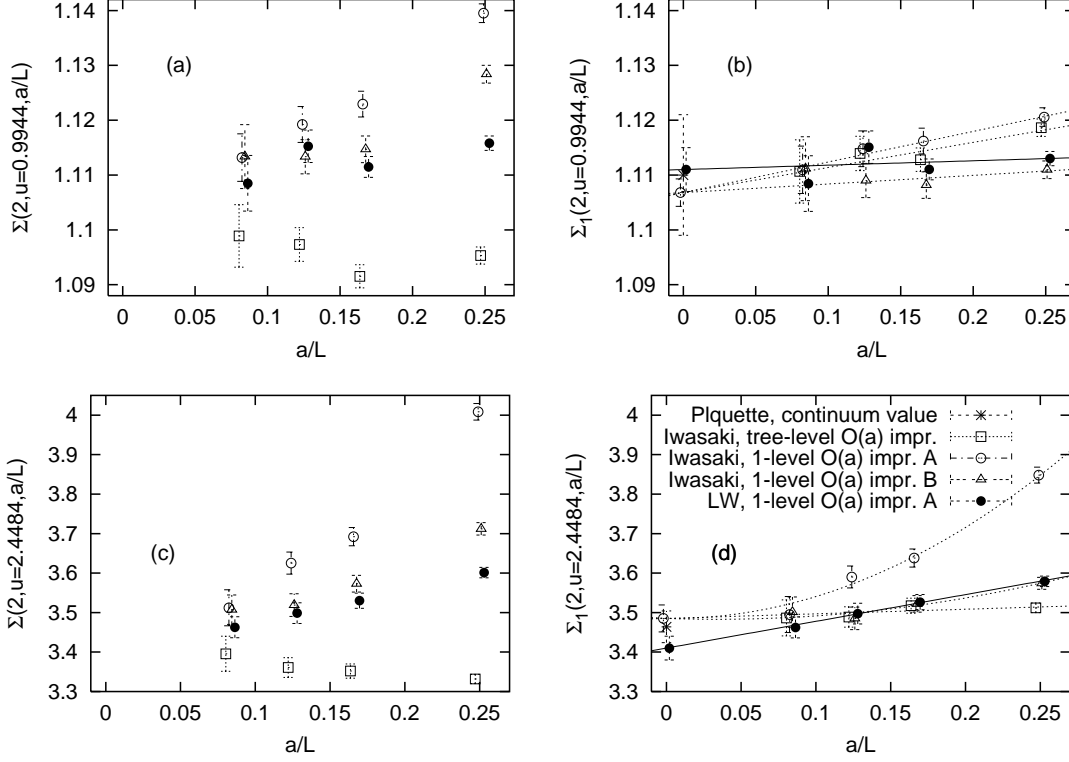


Figure 1. The cut off dependence of Σ (left-hand side) and Σ_1 (right-hand side) at the weak $u = 0.9944$ (upper) and the strong $u = 2.4484$ (bottom) couplings for the various improved gauge actions. The points at $a/L = 0$ in (b) and (d) represent continuum values for the Iwasaki action(open circle), the LW action(fuled circle) and also the plaquette action(cross)[5,6]. The dotted lines and the straight lines represent the combined fit function for the Iwasaki action and the liner fit function for the LW action respectively.

$$c_1 c_t^R(g_0^2) = c_1(3/2 + c_t^{R(1)} g_0^2 + O(g_0^4)), \quad (3)$$

where $c_0 + 8c_1 = 1$, for the plaquette and rectangular loops(having 2-link for the spatial direction) touching the boundary respectively. The leading terms necessary for the tree-level $O(a)$ improvement is taken from Ref.[9].

The one-loop terms[4] have to satisfy the following relation to achieve one-loop $O(a)$ improvement: $c_0 c_t^{P(1)} + 4c_1 c_t^{R(1)} = A_1/2$ where A_1 is a coefficient of $g_0^4 a/L$ term in the SF coupling. We consider two choices: $c_t^{R(1)} = 2c_t^{P(1)}$ (condition A), and $c_t^{R(1)} = 0$ (condition B). While the difference between the conditions A and B is an

$O(a^5)$ contribution at one-loop level, it may become larger at higher-loop orders.

Let us introduce quantities which we calculate. The SSF describes the evolution of the running coupling under a finite scaling factor(say $s = 2$). Σ denotes the SSF calculated on the lattice. In a step to fix the energy scale, one have to set the low energy scale ratio L_{\max}/r_0 at the continuum limit where L_{\max} is defined as $\bar{g}^2(L_{\max}) = 3.480$ and r_0 is the Sommer's scale. Later on, we shall show the simulation results of their cut off dependence and take the continuum limit.

3. Results

Simulations for SSF and r_0/a are performed on CP-PACS. We refer to Ref.[8] for details of simulation parameters and the continuum extrapolation procedure.

In Fig. 1, we show the cut off dependence of the lattice SSF for the improved gauge actions with some choices of the boundary counter terms. The panels (a) (for weak coupling $u = 0.9944$) and (c) (for strong coupling $u = 2.4484$) show the raw data for Σ , while a perturbatively improved observable Σ_1 [10] is plotted in (b) (for weak) and (d) (for strong). The latter is defined as

$$\Sigma_1(2, u, a/L) = \Sigma(2, u, a/L)/(1 + \delta_1(a/L)u), \quad (4)$$

where $\delta_1(a/L)$ is the one-loop relative deviation[4]. We observe that Σ_1 shows a better scaling behavior at both coupling regions than Σ . It is particularly so for the case of the Iwasaki action with the tree level $O(a)$ improvement and the condition B. Therefore, we use Σ_1 in the continuum extrapolation. For the Iwasaki action the continuum value is obtained by a joint fit of the three sets of data. For the LW action, we take the value obtained by a linear fitting of the data with the condition A. The continuum values are plotted at $a/L = 0$ in Fig. 1 (b) and (d). We observe that the three values obtained with the Iwasaki and LW actions in the present work and that of ALPHA Collaboration [6] are consistent within 1σ (2.3σ) at the weak (strong) coupling.

We also show the low energy scale ratio for the Iwasaki action with both tree-level and one-loop $O(a)$ improvements in Fig. 2. To extrapolate to the continuum limit, we use the same fitting form as in the case of SSF at strong coupling. We do not include the point $L/a = 4$ for the tree level $O(a)$ improved case. We observe that the extrapolated value agrees with the one obtained by the plaquette action[7] within errors.

4. Conclusions

We have confirmed the universality of the SSF and the low energy scale ratio. In the extrapolation procedure, the perturbative removal of lattice artifacts well reduces the scaling violation

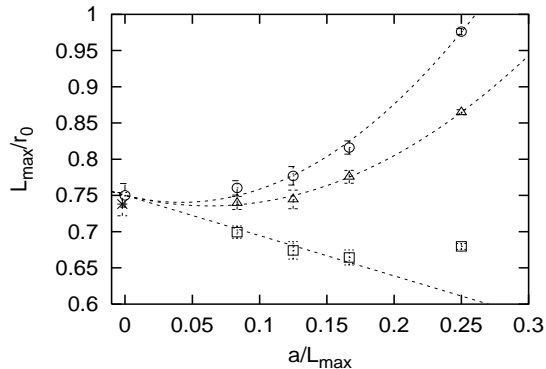


Figure 2. The cut off dependence of the low energy scale ratio for the Iwasaki action. The symbols are the same as the case of the SSF.

of the SSF for the Iwasaki action with the tree level $O(a)$ improvement and the one-loop $O(a)$ improvement with the condition B.

As mentioned in the introduction, this work is the second step toward unquenched simulations. The present study shows that we should use the above two choices in the future simulations with dynamical quarks.

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