Upper critical fields of quasi-low-dimensional superconductors with coexisting singlet and triplet pairing interactions in parallel magnetic fields

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Quasi-low-dimensional type-II superconductors in parallel magnetic fields are studied when singlet pairing interactions and relatively weak triplet pairing interactions coexist. Singlet and triplet components of order parameter are mixed at high fields, and at the same time an inhomogeneous superconducting state called a Fulde-Ferrell-Larkin-Ovchinnikov state occurs. As a result, the triplet pairing interactions enhance the upper critical field of superconductivity remarkably even at temperatures far above the transition temperature of parallel spin pairing. It is found that the enhancement is very large even when the triplet pairing interactions are so weak that a high-field phase of parallel spin pairing may not be observed in practice. A possible relevance of the result in organic superconductors and a hybrid-ruthenate-cuprate superconductor is discussed.

Anisotropic superconductivities have been studied extensively in organic, oxide, and heavy fermion superconductors. For example, a triplet pairing is confirmed by Knight shift measurements in heavy fermion superconductors UPt_3, while NMR experiments suggest a singlet pairing with a line node gap in UPd_2Al_3. On the other hand, in a quasi-one-dimensional organic superconductor $(TMTSF)_2ClO_4$, a line node gap is supported by NMR data by Takigawa *et al.*,[1,2] while thermal conductivity measurements by Belin and Behnia suggest a full gap superconductivity.[3,4] In $(TMTSF)_2PF_6$, triplet pairing superconductivity is supported by recent Knight shift measurements.[5]

Pairing symmetries can be different in $(TMTSF)_2ClO_4$ and $(TMTSF)_2PF_6$, but from the similarity of crystal structures and electronic states in these compounds, probably the pairing interactions have the same origin. From the phase diagram in pressure and temperature plane, *d*-wave-like singlet superconductivity due to pairing interactions induced by the antiferromagnetic fluctuations have been discussed.[6,7] Further, it has been discussed recently that such pairing interactions contain both singlet and triplet channels as attractive interactions.[7,8] Pairing interactions induced by antiferromagnetic fluctuations have been discussed also in high-T_c oxide superconductors for the proximity to the antiferromagnetic phase.[9]

In this paper, we examine quasi-low-dimensional superconductors in which singlet and triplet pairing interactions coexist. In particular, we concentrate on systems in which the singlet pairing interactions dominate at zero field. We calculate critical fields of superconductivity in directions parallel to the highly conductive layers. Because of the parallel direction, we assume that our system is strongly Pauli limited and the orbital pair-breaking effect can be ignored as a first approximation.

A similar problem was studied in our previous paper in a three-dimensional system with a spherical symmetric Fermi surface in which *s*-wave pairing interactions and weaker *p*-wave pairing interactions coexist.[10] We found a remarkable enhancement of the critical field due to a mixing of order parameters of *s*-wave and *p*-wave symmetries. The order parameter mixing occurs due to appearance of nonzero center-of-mass momentum of Cooper pairs stabilized by Zeeman energy. Such an inhomogeneous superconducting state is called a Fulde-Ferrell-Larkin-Ovchinnikov state (FFLO or LOFF state). It should be noted that the enhancement occurs far above a transition temperature of the pure triplet pairing superconductivity that is estimated in the absence of the singlet pairing interactions. We discussed this effect in connection with the phase diagram of a heavy fermion superconductor.[11] The order parameter mixing in the FFLO state has been pointed out also by Psaltakis and Fenton and by Schopohl and Tewordt in an *s*-wave superconductor and in a *p*-wave superfluid ^{3}He.[12,13]

In this paper, we extend our previous study to the two-dimensional systems and to the anisotropic singlet pairing. We assume interlayer interactions implicitly so that the BCS-like mean-field approximation is justified, while they are weak enough to be neglected in resultant mean-field equations. In this sense, our systems are quasi-two-dimensional. We find that even very weak triplet pairing interactions enhance the critical fields remarkably also in the present systems.

The pairing interactions are expanded as

$$V(\mathbf{k},\mathbf{k}') = -\sum_{\alpha} g_{\alpha}\gamma_{\alpha}(\mathbf{k})\gamma_{\alpha}(\mathbf{k}'), \tag{1}$$

where $\gamma_\alpha(\mathbf{k})$ are defined by $\gamma_{d_{x^2-y^2}}(\mathbf{k}) = \hat{k}_x^2 - \hat{k}_y^2$, $\gamma_{p_x}(\mathbf{k}) = \hat{k}_x$ and so forth in cylindrically symmetric systems. We take units with $\hbar = 1$ and $k_B = 1$ in this paper. We consider two cases: (i) $g_s > g_p > 0$ and (ii) $g_d > g_p > 0$, where we have defined $g_p \equiv g_{p_x} = g_{p_y}$ and $g_d \equiv g_{d_{x^2-y^2}} = g_{d_{xy}}$. In each case, the other coupling constants are assumed to be zero. The gap function is expanded as

$$\Delta(\mathbf{k}) = \sum_{\alpha} \Delta_{\alpha}\gamma_{\alpha}(\mathbf{k}). \tag{2}$$

The gap equations in the vicinity of the second-order transition are written as

$$\Delta_\alpha = g_\alpha \sum_\beta K_{\alpha\beta}\Delta_\beta \tag{3}$$

with

$$K_{\alpha\beta} = \frac{1}{N}\sum_{\mathbf{k}'} \gamma_\alpha(\mathbf{k}')\gamma_\beta(\mathbf{k}')\frac{1}{2}\sum_\sigma \frac{\tanh\dfrac{\epsilon_{\mathbf{k}'}+\zeta\sigma}{2T}}{2\epsilon_{\mathbf{k}'}}, \tag{4}$$

where we have defined $\zeta = h(\bar{q}\cos\theta+1)$, the angle θ between $\mathbf{k}'$ and $\mathbf{q}$, $\bar{q}=v_F q/2h$ and $h=|\mu_e \mathbf{H}|$ with the electron magnetic moment μ_e. It is apparent from the above equations that the mixing of the order parameter components of odd and even parities occurs only when both conditions of $\mathbf{q}\neq 0$ and $h\neq 0$ are satisfied, which is expected from a symmetry consideration in momentum and spin spaces.

In the weak-coupling limit, Eq. (3) is rewritten in the form

$$\Delta_\alpha \ln\frac{T}{T^0_{c\alpha}} = -\sum_\beta M_{\alpha\beta}\Delta_\beta \tag{5}$$

with $T^{(0)}_{c\alpha} = 2e^\gamma \omega_c/\pi e^{-1/g_\alpha N_\alpha(0)}$ and

$$M_{\alpha\beta} \equiv \int \frac{d\varphi}{2\pi}\frac{\rho_{\alpha\beta}(0,\varphi)}{N_\alpha(0)}\sinh^2\frac{\beta\zeta}{2}\Phi(\varphi),$$

$$\Phi(\varphi) \equiv \int_0^\infty dy \ln y\left[\frac{2\sinh^2 y}{\left(\cosh^2 y + \sinh^2\dfrac{\beta\zeta}{2}\right)^2} - \frac{1}{\cosh^2 y\left(\cosh^2 y+\sinh^2\dfrac{\beta\zeta}{2}\right)}\right], \tag{6}$$

where

$$\rho_{\alpha\beta}(0,\varphi) = \gamma_\alpha(\varphi)\gamma_\beta(\varphi)\rho(0,\varphi),$$

$$N_\alpha(0) = \int \frac{d\varphi}{2\pi}\rho_{\alpha\alpha}(0,\varphi). \tag{7}$$

Here, φ is the angle between $\mathbf{k}'$ and the k_x axis, and $\rho(0,\varphi)$ is the angle-dependent density of states at the Fermi energy. Equation (5) gives the upper critical fields $h=h(T,\mathbf{q})$ for a given $\mathbf{q}$. The final result of the critical fields is obtained by maximizing $h(T,\mathbf{q})$ with respect to $\mathbf{q}$.

For the case of *s*-*p* mixing (i), we can choose the vector $\mathbf{q}$ in an arbitrary direction from the symmetry of the system. Thus, let us assume the direction of $\mathbf{q}$ in the k_x axis. Then, the *p*-wave component with $\Delta_{p_y}\sim\hat{k}_y$ is not mixed with the *s*-wave component, while $\Delta_{p_x}\sim\hat{k}_x$ can be mixed. Thus, we only need to calculate the smallest eigenvalue λ of the 2×2 matrix

$$\begin{pmatrix} M_{ss} & M_{sp} \\ M_{ps} & M_{pp}+G_p \end{pmatrix}, \tag{8}$$

where we have defined

$$G_p \equiv \ln\frac{T^{(0)}_{cs}}{T^{(0)}_{cp}} = \frac{1}{g_p N_p(0)} - \frac{1}{g_s N_s(0)}. \tag{9}$$

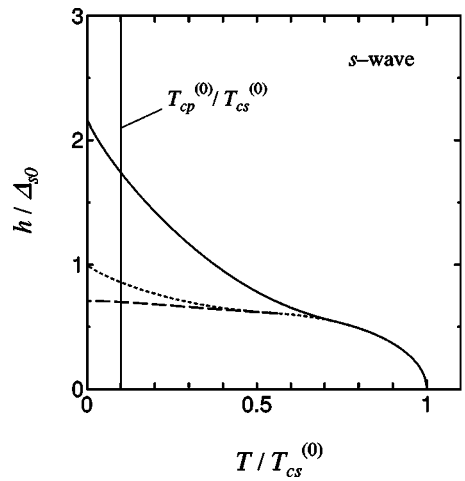

FIG. 1. The upper critical fields of superconductivity in the case (i) in the presence (solid line) and the absence (dotted line) of the *p*-wave pairing interactions. $T^{(0)}_{cp}/T^{(0)}_{cs}=0.1$ is assumed. The vertical thin solid line is the transition temperature of the pure *p*-wave superconductivity of parallel spin pairing. The broken line shows the Pauli paramagnetic limit when the FFLO state is ignored. Here, we have defined $\Delta_{s0}\equiv 2\omega_c e^{-1/g_s N_s(0)}$.

The transition temperature is given by $T_c = T^{(0)}_{cs}e^{-\lambda}$.

For the case of *d*-*p* mixing (ii), we can use the symmetry to fix the orientation of the *d*-wave order parameter. Thus, let us assume $\Delta_d \sim \hat{k}_x^2 - \hat{k}_y^2$. In the absence of *p*-wave pairing interactions, it is known that the optimum $\mathbf{q}$ is in the direction of the k_x axis (or equivalently the k_y axis) at low temperatures, while it is in the direction of the line of $k_y=\pm k_x$ at high temperatures.[14,15] In the presence of the *p*-wave pairing interactions, the direction of $\mathbf{q}$ is not known *a priori*. Thus, we should assume that the two *p*-wave components ($\hat{k}_x$ and $\hat{k}_y$) of order parameter can be mixed with the *d*-wave components. Therefore, we calculate the smallest eigenvalue λ of a 3×3 matrix with elements $M_{dd}, M_{dp_x}, M_{p_x p_x}+G_p, \ldots$ and so forth, which gives the transition temperature by $T_c = T^{(0)}_{cd}e^{-\lambda}$. Here, the parameter G_p is defined by

$$G_p \equiv \ln\frac{T^0_{cd}}{T^{(0)}_{cp}} = \frac{1}{g_p N_p(0)} - \frac{1}{g_d N_d(0)}, \tag{10}$$

similarly to Eq. (9).

Numerical results are drawn in Figs. 1–4. Figure 1 shows the results for the *s*-*p* mixing, while Figs. 2–4 show those for the *d*-*p* mixing. In both cases, remarkable enhancements of the critical field are found at temperatures far above the transition temperature of the pure *p*-wave superconductivity of the parallel spin pairing. In particular, Fig. 4 shows that the critical field is enhanced considerably even when $T^{(0)}_{cp}/T^{(0)}_{cd}=0.01$.

For the *d*-*p* mixing case, the optimum $\mathbf{q}$ is in the direction of the k_x axis at low temperatures, while it is along the line of $k_y=\pm k_x$ at high temperatures, as shown in Figs. 2–4. In these figures, the results for $\varphi_q=0$ and $\varphi_q=\pi/4$ are drawn by the solid lines. For each direction of $\mathbf{q}$, the magnitude $q=|\mathbf{q}|$ is optimized to obtain the critical field. It is confirmed by numerical calculations that $\mathbf{q}$ with the other directions are not optimum.

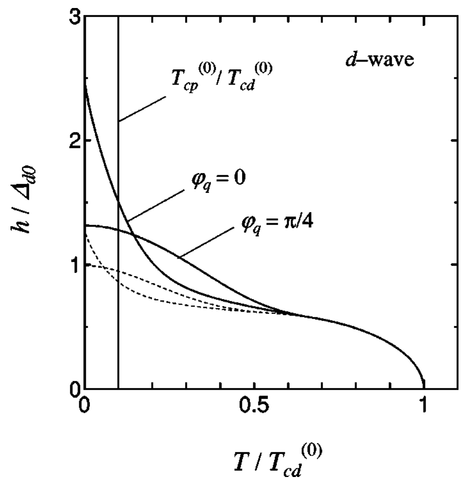

FIG. 2. The upper critical fields in the case (ii) in the presence (solid line) and the absence (dotted line) of the p-wave pairing interactions. $T_{cp}^{(0)}/T_{cd}^{(0)}=0.1$ is assumed. The vertical thin solid line is the transition temperature of the pure p-wave superconductivity of parallel spin pairing. φ_q is the angle between $\mathbf{q}$ and the k_x axis. At each temperature, the higher h on the solid line is the final result of the critical field in the presence of the p-wave interactions. Here, we have defined $\Delta_{d0} \equiv 2\omega_c e^{-1/g_d N_d(0)}$.

In Fig. 5, the optimum value of $q=|\mathbf{q}|$ along the second-order transition line is drawn. The temperature T^* at which q vanishes is the temperature of the tricritical point of the normal phase, the BCS superconductivity $(\mathbf{q}=0)$, and the FFLO state $(\mathbf{q}\neq 0)$. It is found that the temperature T^* increases due to the order parameter mixing in the presence of the p-wave interactions from the value $T^* \approx 0.561 \times T_{cd}^{(0)}$ in the absence of the p-wave interactions. For example, $T^* \approx 0.668 \times T_{cd}^{(0)}$ is estimated for $T_{cp}^{(0)}/T_{cd}^{(0)}=0.1$.

In conclusion, we examined the upper critical field of the superconductivity when the singlet and triplet pairing interactions coexist. We extended our previous study in a spherical symmetric system[10] to the quasi-two-dimensional systems in parallel magnetic fields and to the anisotropic singlet (d-wave) pairing. We obtained similar results to our previous results, and find that even very weak triplet interactions enhance the critical fields remarkably, and the temperature of the tricritical point of the normal, BCS, and the FFLO states is also enhanced. The present phase diagrams coincide with one predicted in our previous paper[8] except that the orbital pair-breaking effect is not taken into account in the present paper.

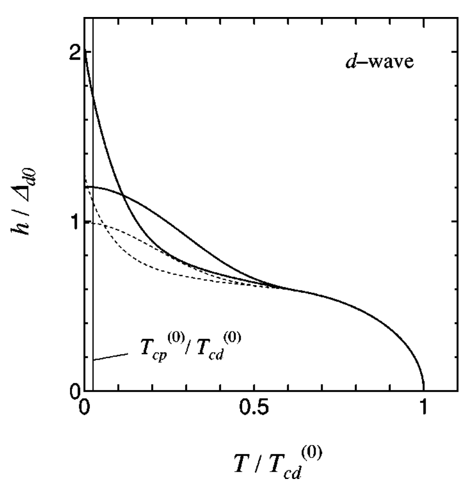

FIG. 3. The upper critical fields for the case of d-p mixing. $T_{cp}^{(0)}/T_{cd}^{(0)}=0.025$ is assumed. The definitions of the lines are the same as in Fig. 2.

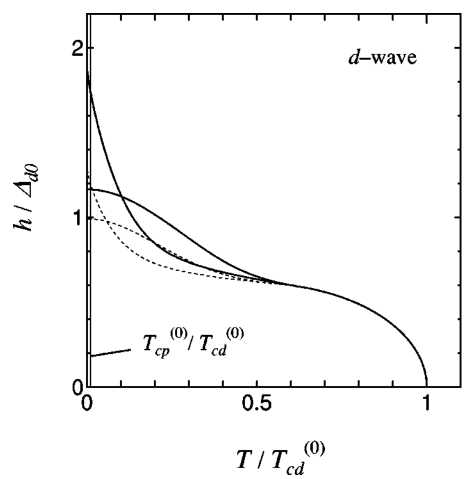

FIG. 4. The upper critical fields for the case of d-p mixing. $T_{cp}^{(0)}/T_{cd}^{(0)}=0.01$ is assumed. The definitions of the lines are the same as in Fig. 2.

In a high-T_c superconductor $RuSr_2GdCu_2O_8$, for the coexistence of the superconductivity and the ferromagnetism (or canted ferromagnetism[16]), the FFLO state has been discussed.[17,18] Probably, d-wave pairing interactions are dominant in this system, but weak triplet pairing interactions must coexist because of the proximity to the magnetic phase. Therefore, the present mechanism that enhances the critical fields and T^* may play a role in stabilizing a bulk superconductivity in this system.[18]

In the organic superconductors, the FFLO state has been discussed by many authors[19–21] to explain the high upper critical fields that exceed a conventional estimation of Pauli paramagnetic limit (Chandrasekar-Clogston limit). As we have discussed above, there is a possibility that the antifer-

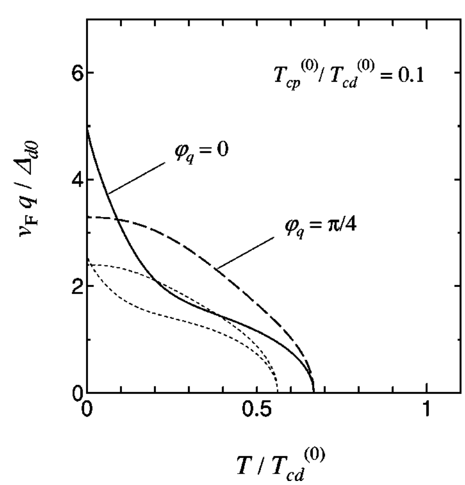

FIG. 5. The magnitude of the optimum wave vector $\mathbf{q}$ at the upper critical fields for $T_{cp}^{(0)}/T_{cd}^{(0)}=0.1$ when $\varphi_q=0$ (solid line) and $\varphi_q=\pi/4$ (broken line). The dotted lines are the result for pure d-wave pairing.

romagnetic fluctuations contribute to the pairing interactions in these systems, and they should contain both singlet and triplet channels as attractive interactions.[8] Therefore, the present mechanism may contribute to stabilizing the superconducting phase at high fields. As shown in Fig. 4, the enhancement can be very large even when the triplet pairing interactions are so small that the pure triplet superconductivity of parallel spin pairing is not observed in practice.

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