Rolling-Based Manipulation for Multiple Objects

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***Abstract*—This paper discusses the manipulation of multiple objects under rolling contacts. For manipulating multiple objects, the following two key issues do not arise in the manipulation of a single object: 1) each object's motion is restricted by the other objects and 2) the contact force among objects is not controlled directly. As for 1), we first formulate the motion constraint for the whole grasp system, and then provide a necessary condition for manipulating multiple objects uniquely. As for 2), we provide a condition for determining the contact forces among objects uniquely. We further show a sufficient condition for manipulating multiple objects within the object motion constraint. Under this sufficient condition, we propose a control scheme for object motion by taking the motion constraint into account. Simulation and experimental results are provided to confirm our idea.**

***Index Terms*—Manipulation, multiple objects, rolling contact.**

I. NOMENCLATURE

We use the following nomenclature in this paper.

Σ_R	Coordinate frame fixed at the base.
Σ_{Bi}	Coordinate frame fixed at the center of gravity of the object $i(i = 1, \ldots, m)$.
Σ_{Fj}	Coordinate frame fixed at the end link of the finger j $(j = 1, \ldots, n)$.
Σ_{CFj}	Contact frame of the finger j whose origin is always at the point of contact.
Σ_{CBij}	Contact frame of the object i whose origin is always at the point of contact with the finger j.
Σ_{LFj}	Local frame fixed relative to Σ_{Fj} which coincides with Σ_{CFj} at time t.
Σ_{LBij}	Local frame fixed relative to Σ_{Bi} which coincides with Σ_{CBij} at time t.
Σ_{COit}	Contact frame of the tth contact between objects $(t = 1, \ldots, r)$.
Σ_{LOit}	Local frame fixed relative to Σ_{Bi} which coincides with Σ_{COit} at time t.
$\boldsymbol{p}_{Bi} \in R^3$	position vector of Σ_{Bi} with respect to Σ_R.
$\boldsymbol{R}_{Bi} \in R^{3\times3}$	Rotation matrix of Σ_{Bi} with respect to Σ_R.
$\boldsymbol{p}_{Fj} \in R^3$	Position vector of Σ_{Fj} with respect to Σ_R.
$\boldsymbol{R}_{Fj} \in R^{3\times3}$	Rotation matrix of Σ_{Fj} with respect to Σ_R.
${}^{Bi}\boldsymbol{p}_{CBij} \in R^3$	Position vector of contact point of the finger j with respect to Σ_{Bi}.
${}^{Fj}\boldsymbol{p}_{CFj} \in R^3$	Position vector of contact point of the finger j with respect to Σ_{Fj}.
${}^{Bi}\boldsymbol{p}_{COit} \in R^3$	Position vector of common contact point between the object i and the object l with respect to Σ_{Bi}.

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II. INTRODUCTION

MULTIFINGERED robot hands have potential advantages in performing various tasks with the dexterity of human hands. While much research has been done on multifingered robot hands, most of works have implicitly assumed that a multifingered hand manipulates only one object. Under such a condition, several grasping issues such as the stability of grasp [1], [2], the analysis of contact force [3]–[5], and the manipulation of an object [6]–[10] have been studied.

Now, let us consider a case where a multifingered hand approaches and envelopes a cylindrical object placed on a table. For an object with small friction, we can make most use of wedge-effect, where the object receives a strong lifting force produced by fingertips inserted into a narrow gap between the table and the object. As a result, the object will be automatically lifted up by slipping over the link surface [11]. For an object with significant friction, however, it is hard to expect the wedge-effect since any slipping motion is avoided. A scheme based on a rolling motion may be a good candidate for lifting up an object and for achieving an enveloping grasp [11]. In such a case, one finger continuously pushes the object so that it may be rolled up over the surface of the other fingers. Generally, this motion planning is too complicated to be easily implemented to the actual system. Let us now consider the case where a multifingered hand approaches and envelopes two cylindrical objects with significant friction, as shown in Fig. 1. When two objects satisfy the rolling contact, we can expect that a multifingered hand can easily achieve an enveloping grasp by simply pushing two links contacting with the objects. During the lifting phase, links and two objects behave as if they were just connected by mechanical gears. Due to these mechanical properties, achieving an enveloping grasp for two objects seems to be even easier than for a single object under significant friction. This is a potential advantage for manipulating two objects, simultaneously.

We can find another advantage for manipulating multiple objects. Let us consider the case where a human picks up and transfers small objects such as coins, beans, or such, for example, from a table. In such a case, a human often grasps more than one object and manipulates them case by case. Generally, we can expect that treating multiple objects simultaneously makes it possible to achieve a handling task efficiently. These are motivations to start this work.

For manipulating multiple objects, there are a couple of questions. Can we apply the same control used for a single object?

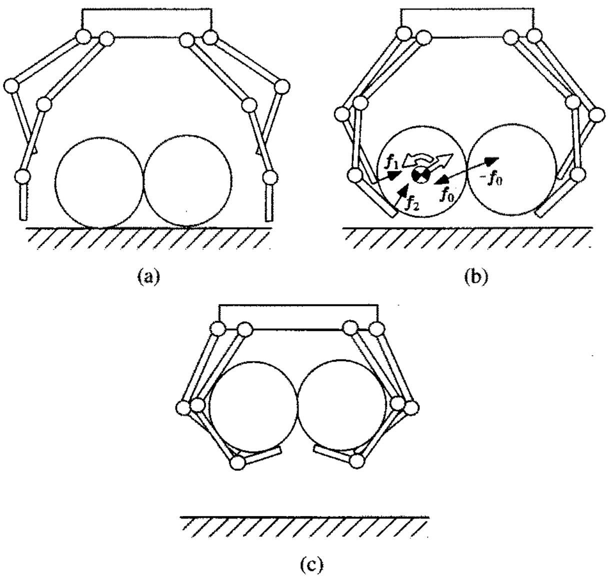

Fig. 1. Multiple objects grasped by inner-link contacts.

If this is not the case, what are the major differences between the manipulation of a single object and that of multiple objects? The goal of this work is to answer these questions and to extend the manipulation theory from single objects to multiple objects. Since the manipulation theory for multiple objects includes that for a single object, and since the manipulation theory under rolling contacts includes that under point contacts, rolling-based manipulation of multiple objects can be regarded as an extension of the manipulation theory.

By observing a series of motions shown in Fig. 1, we can roughly divide the grasp into two types. Fig. 1(b) shows a kind of inner-link-based grasp where one finger contacts an object only at the end link. Although many fingers are needed to grasp objects firmly, we can expect that the freedom of manipulation will increase since a finger can exert an arbitrary contact force at the contact point. On the other hand, Fig. 1(c) shows the final state of the grasp where one finger contacts an object at multiple points. In such a grasp, although a robot hand can grasp objects firmly with a small number of fingers, the freedom to manipulate the objects is strongly limited since a finger cannot exert an arbitrary contact force at the contact points. Due to the large potentiality of object manipulation with the end link, we focus on the grasp style as shown in Fig. 1(b).

When multiple objects are manipulated simultaneously by rolling contact at each contact point, we have to take note of two factors. One is the object motion constraint. For the manipulation of two objects as shown in Fig. 1(b), even if the left-hand object moves in an arbitrary direction, the right-hand object has to maintain contact with the left-hand object and, therefore, cannot move arbitrarily. For multiple objects, we have to carefully choose the control parameters so that they may not interfere with each other. The other factor is the dependency of contact force. Suppose that each finger exerts contact force onto the objects, as shown in Fig. 1(b). Even if an arbitrary force ($\boldsymbol{f}_1$ and $\boldsymbol{f}_2$) is exerted at the contact point between the finger link and the object, the contact force ($\boldsymbol{f}_0$) at the contact point between the two objects depends on both $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$. Therefore, we have to consider the dependency of the contact force as well as the constraint on the object motion when manipulating multiple objects.

In this paper, we discuss the manipulation of multiple objects under rolling contacts by taking the above two issues into account. As for the discussion of the object motion constraint, we first formulate the motion constraint of the grasp, and introduce the matrix $\boldsymbol{L}$ denoting the velocity relationship between finger contact points and objects. By utilizing $\boldsymbol{L}$, we show a necessary condition for manipulating multiple objects uniquely. As for the dependency of the contact force, we formulate the equation for computing the contact force among objects for a given set of finger forces by considering the dynamics of the system, and provide a condition for determining the contact force among objects uniquely. Under the condition for generating unique contact force among objects, we show a sufficient condition that ensures that each object can generate an arbitrary linear and rotational acceleration under the motion constraint caused by multiple contacts. Under the unique contact force among objects and the motion constraint caused by multiple contacts, we show a control scheme for the manipulation of multiple objects. Trajectory tracking of the object motion is experimentally performed by using a three-fingered hand to verify our idea. We believe that this experiment is the first attempt at making multiple objects follow along desired trajectories.

III. Revelant Work

Grasp of Multiple Objects

Dauchez and Delebarre [14] used two manipulators holding two objects independently and tried to apply them to an assembly task. Kosuge *et al.* [15] also used two manipulators holding two objects where each manipulator was controlled by its own controller. Aiyama *et al.* [16], [17] studied a scheme for grasping multiple box type objects stably by using two manipulators. For an assembly task, Mattikalli *et al.* [18], [19] proposed a method of finding stable alignments of multiple objects in a gravitational field. While these works treated multiple objects, they have considered neither the motion of the objects within the hand nor any manipulation of objects based on rolling contacts. The authors in [12] and [13] are the first to study the enveloping grasp for multiple objects. They have shown a condition to judge the rolling contact at each contact point and have shown the rolling-up condition.

Manipulation by Rolling Contacts

Kerr and Roth [3] formulated the kinematics of rolling contact. They first introduced the rolling contact model into manipulation by robotic hand. Montana [20] formulated the general relation of contact states including the sliding contact and the pure rolling contact that assumes that the linear velocity of each object is same at the contact point and that the relative rotational velocity about the contact normal becomes zero. Li and Canny [21] discussed the controllability of an object under rolling contact, and proposed a motion planning method taking the nonholonomy into account. Howard and Kumar [2] and Maekawa *et al.* [22] studied the effect of surface curvature on the stability of a grasped object under rolling contacts. Cole *et al.* [6] proposed the simultaneous controller of the object motion and the

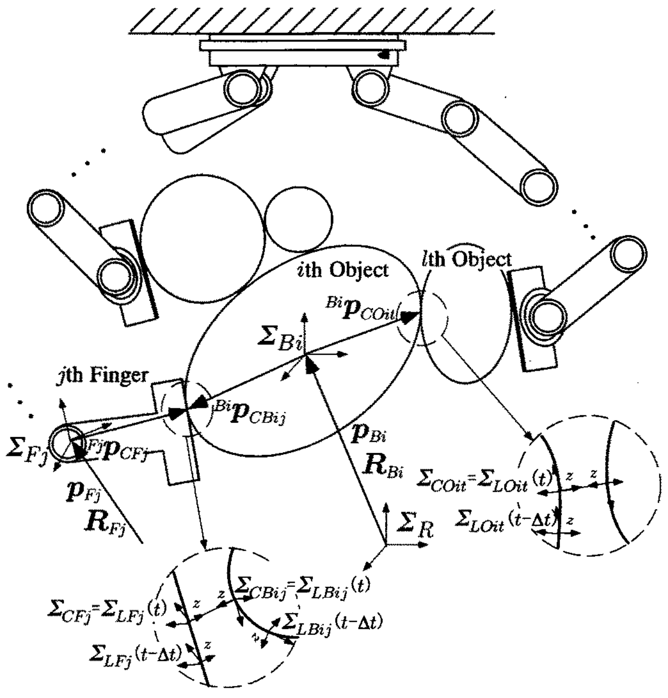

Fig. 2. Model of the system.

internal force taking the rolling constraints into account. Paljug *et al.* [7] proposed a simultaneous controller of the 2-D motion of the object and the contact states. Sarkar *et al.* [8] provided the second-order rolling constraints and simultaneously controlled the 3-D motion of an object and the contact states. Bicchi and Sorrentio [9] and Han *et al.* [10] proposed a control scheme for the full contact states taking the nonholonomy into account.

While there have been a number of works concerning grasp and manipulation under rolling contacts, we believe that this is the first work discussing the manipulation of multiple objects under rolling contacts.

IV. Object Motion Constraint

Fig. 2 shows the grasp of m objects by n fingers, where the finger j contacts the object i, and additionally the object i has a common contact point with the object l.We also assume that the fingers have enough degrees of freedom to exert an arbitrary contact force and an arbitrary moment around the contact normal ($s_j \geq 4$), where s_j denotes the number of joints of the finger j.

We assume the pure rolling at each contact point. In case of pure rolling contact, there are four constraints at each contact point. The linear velocity of the object i should coincide with the linear velocity of the finger j at the contact point, and the rotational velocity of the object i relative to the finger j about the contact normal should equal zero. Similarly, the linear velocity of the object i should coincide with the linear velocity of the object l at the contact point, and the rotational velocity of the object i relative to the object l about the contact normal should equal zero. These relationships are expressed as follows:

$$\boldsymbol{D}_{Bij}\begin{bmatrix}\dot{\boldsymbol{p}}_{Bi}\\ \boldsymbol{\omega}_{Bi}\end{bmatrix} = \boldsymbol{D}_{Fj}\begin{bmatrix}\dot{\boldsymbol{p}}_{Fj}\\ \boldsymbol{\omega}_{Fj}\end{bmatrix} \tag{1}$$

$$\boldsymbol{D}_{Oit}\begin{bmatrix}\dot{\boldsymbol{p}}_{Bi}\\ \boldsymbol{\omega}_{Bi}\end{bmatrix} = \boldsymbol{D}_{Olt}\begin{bmatrix}\dot{\boldsymbol{p}}_{Bl}\\ \boldsymbol{\omega}_{Bl}\end{bmatrix} \tag{2}$$

$$\boldsymbol{D}_{Bij} = \begin{bmatrix}\boldsymbol{I}_3 & -\left(\left(\boldsymbol{R}_{Bi}\,{}^{Bi}\boldsymbol{p}_{CBij}\right)\times\right)\\ \boldsymbol{o} & \boldsymbol{e}_3^T\boldsymbol{R}_{LBij}^T\end{bmatrix} \in R^{4\times 6}$$

$$\boldsymbol{D}_{Fj} = \begin{bmatrix}\boldsymbol{I}_3 & -\left(\left(\boldsymbol{R}_{Fj}\,{}^{Fj}\boldsymbol{p}_{CFj}\right)\times\right)\\ \boldsymbol{o} & \boldsymbol{e}_3^T\boldsymbol{R}_{LFj}^T\end{bmatrix} \in R^{4\times 6}$$

$$\boldsymbol{D}_{Oit} = \begin{bmatrix}\boldsymbol{I}_3 & -\left(\left(\boldsymbol{R}_{Bi}\,{}^{Bi}\boldsymbol{p}_{COit}\right)\times\right)\\ \boldsymbol{o} & \boldsymbol{e}_3^T\boldsymbol{R}_{LOit}^T\end{bmatrix} \in R^{4\times 6}$$

where $\boldsymbol{I}_3$ denotes the 3×3 identity matrix, $((\boldsymbol{R}_{Bi}\,{}^{Bi}\boldsymbol{p}_{CBij})\times)$, $((\boldsymbol{R}_{Fj}\,{}^{Fj}\boldsymbol{p}_{CFj})\times)$, and $((\boldsymbol{R}_{Bi}\,{}^{Bi}\boldsymbol{p}_{COit})\times)$ denote the skew-symmetric matrices equivalent to the vector product, $\boldsymbol{\omega}_{Bi}$ and $\boldsymbol{\omega}_{Fj}$ denote the rotational velocity vectors of Σ_{Bi} and Σ_{Fj} with respect to Σ_R, respectively, and $\boldsymbol{e}_3 = [0\,0\,1]^T$. Aggregating (1) and (2) for $j = 1, \ldots, n$ and $t = 1, \ldots, l$, respectively, the equation of motion constraint is derived as follows:

$$\boldsymbol{D}_L\dot{\boldsymbol{p}}_{LF} = \boldsymbol{D}_B\dot{\boldsymbol{p}}_B \tag{3}$$

where

$$\dot{\boldsymbol{p}}_{LF} = \left[\dot{\boldsymbol{p}}_{LF1}^T\omega_{LF1}\cdots\dot{\boldsymbol{p}}_{LFn}^T\omega_{LFn}\right]^T \in R^{4n}$$

$$\begin{bmatrix}\dot{\boldsymbol{p}}_{LFj}\\ \omega_{LFj}\end{bmatrix} = \boldsymbol{D}_{Fj}\begin{bmatrix}\dot{\boldsymbol{p}}_{Fj}\\ \boldsymbol{\omega}_{Fj}\end{bmatrix} \in R^4$$

$$\dot{\boldsymbol{p}}_B = \left[\dot{\boldsymbol{p}}_{B1}^T\boldsymbol{\omega}_{B1}^T\cdots\dot{\boldsymbol{p}}_{Bm}^T\boldsymbol{\omega}_{Bm}^T\right]^T \in R^{6m}$$

$$\boldsymbol{D}_L = [\boldsymbol{I}_{4n}\boldsymbol{o}]^T \in R^{(4n+4r)\times 4n}$$

$$\boldsymbol{D}_B = \left[\boldsymbol{D}_{LB}^T\boldsymbol{D}_O^T\right]^T \in R^{(4n+4r)\times 6m}$$

$\boldsymbol{D}_{LB} \in R^{4n\times 6m}$ includes $\boldsymbol{D}_{Bij}$, and $\boldsymbol{D}_O \in R^{4r\times 6m}$ includes both $\boldsymbol{D}_{Oit}$ and $\boldsymbol{D}_{Olt}$. For more precise discussions, see Appendix I. $\dot{\boldsymbol{p}}_{LFj}$ and ω_{LFj} denote the linear velocity of the end link evaluated at the contact point and the rotational velocity of the end link about the contact normal, respectively. Since $\dot{\boldsymbol{p}}_{LF}$ denotes the velocity at the contact point of the finger where the pure rolling constraint is imposed, we have $\dot{\boldsymbol{p}}_{LF} \in R^{4n}$. In (3), the matrix $\boldsymbol{D}_B$ is a function of both $\boldsymbol{p}_B$ and vectors ${}^{B1}\boldsymbol{p}_{CB11}, \ldots,$ and ${}^{Bi}\boldsymbol{p}_{COir}$. ${}^{B1}\boldsymbol{p}_{CB11}, \ldots,$ and ${}^{Bi}\boldsymbol{p}_{COir}$ are derived by utilizing the method proposed by Montana [20] and also shown in Appendix II.

Now, we consider the object motion constraint. For a grasp composed of multiple objects, the objects cannot move in an arbitrary direction due to (3). Since $[-\boldsymbol{D}_L\,\boldsymbol{D}_B] \in R^{(4n+4r)\times(4n+6m)}$, $4n + 6m > \mathrm{rank}[-\boldsymbol{D}_L\,\boldsymbol{D}_B]$ has to be satisfied so that (3) has the solution except for $\dot{\boldsymbol{p}}_{LF} = \boldsymbol{o}$ and $\dot{\boldsymbol{p}}_B = \boldsymbol{o}$. The dimension of the solution depends on $4n + 6m - \mathrm{rank}[-\boldsymbol{D}_L\,\boldsymbol{D}_B]$. Moreover, since $\boldsymbol{D}_L$ is composed of $\boldsymbol{I}_{4n}$ in the upper side and $\boldsymbol{o} \in R^{4r\times 4n}$ in the lower side, $4n + 6m - \mathrm{rank}[-\boldsymbol{D}_L\,\boldsymbol{D}_B] = 6m - \mathrm{rank}\boldsymbol{D}_O$ is always satisfied. Based on these discussions, we now define the dimension of object motion as follows.

Definition 1 (Dimension of Object Motion): For the grasp of multiple objects, the grasped objects have

$$I_M = 4n + 6m - \mathrm{rank}[-\boldsymbol{D}_L\,\boldsymbol{D}_B] = 6m - \mathrm{rank}\boldsymbol{D}_O \tag{4}$$

dimensional motion.

It should be noted that, since the terms with respect to finger motion ($4n$ and $\boldsymbol{D}_L$) disappear in the second row of (4), I_M shows the pure dimension of the motion of grasped objects. Moreover, when $\boldsymbol{D}_O$ is a full-rank matrix, we have $I_M = 6m - 4r$. Since $4r$ shows the sum of constraint between two objects, the definition of I_M coincides with the definition of degrees of freedom introduced by Hunt [23]. The physical interpretation of the dimension of object motion is shown for a planar grasp of two objects in Section VIII.

We now introduce a new vector $\boldsymbol{\zeta} \in R^{I_M}$ whose dimension is same as that of object motion. It should be noted that $\boldsymbol{\zeta}$ includes the independent variables controlling the motion of the grasped objects. Now, let us define $\boldsymbol{\zeta}$ as $\dot{\boldsymbol{\zeta}} = \boldsymbol{E}_L \dot{\boldsymbol{p}}_{LF} + \boldsymbol{E}_B \dot{\boldsymbol{p}}_B \in R^{I_M}$, where the matrices $\boldsymbol{E}_L$ and $\boldsymbol{E}_B$ are defined in such a way that these matrices have the minimum size making

$$\begin{bmatrix} -\boldsymbol{D}_L & \boldsymbol{D}_B \\ \boldsymbol{E}_L & \boldsymbol{E}_B \end{bmatrix}$$

full-column rank in the following equation:

$$\begin{bmatrix} -\boldsymbol{D}_L & \boldsymbol{D}_B \\ \boldsymbol{E}_L & \boldsymbol{E}_B \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{p}}_{LF} \\ \dot{\boldsymbol{p}}_B \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \dot{\boldsymbol{\zeta}} \end{bmatrix}. \tag{5}$$

We note that, since $\boldsymbol{D}_L$ is composed of $\boldsymbol{I}_{4n}$ in the upper side, we can always make the above matrix full-column rank even when $\boldsymbol{E}_L = \boldsymbol{o}$. Assuming $\boldsymbol{E}_L = \boldsymbol{o}$, since $\dot{\boldsymbol{\zeta}}$ becomes a function of $\dot{\boldsymbol{p}}_B$ as $\dot{\boldsymbol{\zeta}} = \boldsymbol{E}_B \dot{\boldsymbol{p}}_B$, $\dot{\boldsymbol{\zeta}}$ can express the motion of the grasped objects. We also note that the selection of $\boldsymbol{\zeta}$ is not unique.

For a grasp system with a single object, Bicchi *et al.* [24] have studied the mobility which is composed of the independent variables of the grasp system under rolling motion. For a single object, since $m = 1$, $\text{rank}\boldsymbol{D}_O = 0$ and $I_M = 6$, we can choose three parameters from the position and three from the orientation of the object as a component of $\boldsymbol{\zeta}$. Therefore, irrespective of the number of objects, we can regard $\boldsymbol{\zeta}$ as an extended parameter expressing the dimension of object motion. $\boldsymbol{\zeta}$ is similar to the connectivity defined in [24]. The connectivity includes the effect of degrees of freedom of the finger, while $\boldsymbol{\zeta}$ does not, since we assumed that the finger can exert an arbitrary contact force. Also, the variables of the connectivity are automatically determined by utilizing the row-reduced echelon form of the constraint matrix, while we can choose the variables of $\boldsymbol{\zeta}$ arbitrarily. For multiple objects, it may be convenient for us to manipulate a particular object precisely and to make others simply follow it. The arbitrariness makes it possible to achieve such a requirement. The only restriction is choosing variables that can make

$$\begin{bmatrix} -\boldsymbol{D}_L & \boldsymbol{D}_B \\ \boldsymbol{E}_L & \boldsymbol{E}_B \end{bmatrix}$$

full-column rank.

By solving (5) with respect to $\dot{\boldsymbol{p}}_{LF}$ and $\dot{\boldsymbol{p}}_B$, the following equation is derived:

$$\begin{bmatrix} \dot{\boldsymbol{p}}_{LF} \\ \dot{\boldsymbol{p}}_B \end{bmatrix} = \begin{bmatrix} -\boldsymbol{D}_L & \boldsymbol{D}_B \\ \boldsymbol{E}_L & \boldsymbol{E}_B \end{bmatrix}^+ \begin{bmatrix} \boldsymbol{0} \\ \dot{\boldsymbol{\zeta}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{L} \\ \boldsymbol{B} \end{bmatrix} \dot{\boldsymbol{\zeta}} \tag{6}$$

where $*^+$ denotes the pseudo-inverse of $*$. It should be noted that, since

$$\begin{bmatrix} -\boldsymbol{D}_L & \boldsymbol{D}_B \\ \boldsymbol{E}_L & \boldsymbol{E}_B \end{bmatrix}$$

is full-column rank, the null space does not exist. While the selection of $\dot{\boldsymbol{\zeta}}$ is not unique, we have the following theorem for the uniqueness of $\dot{\boldsymbol{p}}_{LF}$ and $\dot{\boldsymbol{p}}_B$.

Theorem 1 (Uniqueness of Object Motion): Although the selection of $\dot{\boldsymbol{\zeta}}$ is not unique, $\dot{\boldsymbol{p}}_{LF}$ and $\dot{\boldsymbol{p}}_B$ are determined uniquely and are independent of the selection of $\dot{\boldsymbol{\zeta}}$.

Proof: Suppose that two selections of $\dot{\boldsymbol{\zeta}}$ ($\dot{\boldsymbol{\zeta}}_1$ and $\dot{\boldsymbol{\zeta}}_2$) exist for a given $\dot{\boldsymbol{p}}_{LF}$ and $\dot{\boldsymbol{p}}_B$ as follows:

$$\begin{aligned} \boldsymbol{E}_{L1}\dot{\boldsymbol{p}}_{LF} + \boldsymbol{E}_{B1}\dot{\boldsymbol{p}}_B &= \dot{\boldsymbol{\zeta}}_1 \\ \boldsymbol{E}_{L2}\dot{\boldsymbol{p}}_{LF} + \boldsymbol{E}_{B2}\dot{\boldsymbol{p}}_B &= \dot{\boldsymbol{\zeta}}_2. \end{aligned}$$

Since both

$$\begin{bmatrix} -\boldsymbol{D}_L & \boldsymbol{D}_B \\ \boldsymbol{E}_{L1} & \boldsymbol{E}_{B1} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -\boldsymbol{D}_L & \boldsymbol{D}_B \\ \boldsymbol{E}_{L2} & \boldsymbol{E}_{B2} \end{bmatrix}$$

are full-column rank and do not have the null space, $\dot{\boldsymbol{p}}_{LF}$ and $\dot{\boldsymbol{p}}_B$ can be derived uniquely for two selections of $\dot{\boldsymbol{\zeta}}$ as

$$\begin{bmatrix} \dot{\boldsymbol{p}}_{LF} \\ \dot{\boldsymbol{p}}_B \end{bmatrix} = \begin{bmatrix} -\boldsymbol{D}_L & \boldsymbol{D}_B \\ \boldsymbol{E}_{L1} & \boldsymbol{E}_{B1} \end{bmatrix}^+ \begin{bmatrix} \boldsymbol{0} \\ \dot{\boldsymbol{\zeta}}_1 \end{bmatrix} = \begin{bmatrix} -\boldsymbol{D}_L & \boldsymbol{D}_B \\ \boldsymbol{E}_{L2} & \boldsymbol{E}_{B2} \end{bmatrix}^+ \begin{bmatrix} \boldsymbol{0} \\ \dot{\boldsymbol{\zeta}}_2 \end{bmatrix}.$$

Since we have the same $\dot{\boldsymbol{p}}_{LF}$ and $\dot{\boldsymbol{p}}_B$ for two selections of $\dot{\boldsymbol{\zeta}}$, we hold the theorem. ∎

To have the unique object motion, $\dot{\boldsymbol{\zeta}}$ has to be also uniquely determined for a given fingertip motion $\dot{\boldsymbol{p}}_{LF}$; otherwise, there exists an arbitrary motion for at least one object even when the finger motion is determined. By using (6), the unique determination of $\dot{\boldsymbol{\zeta}}$ is guaranteed by the following condition.

Condition 1 (Kinematic Condition for Manipulation): A necessary condition for a robot hand to uniquely determine the object motion is

$$\text{Ker}(\boldsymbol{L}) = \emptyset \tag{7}$$

where $\text{Ker}(*)$ denotes the null space of $*$.

For a single object, $\text{Ker}(\boldsymbol{L}) = \emptyset$ when a three-fingered hand grasps an object, while $\text{Ker}(\boldsymbol{L}) \neq \emptyset$ when an object is simply placed on a palm or a table. Condition 1 (kinematic condition for manipulation) is a necessary condition since we do not consider the contact force applied to the objects. Since $\text{size}\boldsymbol{L} = 4n \times (6m - \text{rank}\boldsymbol{D}_O)$,[1] the above condition is satisfied when $\text{rank}\boldsymbol{L} = 6m - \text{rank}\boldsymbol{D}_O$. In case that $\boldsymbol{L}$ is a full-rank matrix, $6m - \text{rank}\boldsymbol{D}_O \leq 4n$. This condition can be finally achieved when the number of fingers increases.

V. Dependency of Contact Force

A. Equation of Dependency

We now make clear the dependency of contact force among objects. Since the contact force and moment are applied to the

[1] To express the size of a matrix, we define a function $\text{size}A = m \times n$ for a matrix $A \in R^{m \times n}$.

objects by the finger links, the equation of motion of the grasped objects is given by

$$\boldsymbol{M}_B\ddot{\boldsymbol{p}}_B + \boldsymbol{h}_B = \boldsymbol{D}_{LB}^T\boldsymbol{f}_{CB} + \boldsymbol{D}_O^T\boldsymbol{f}_{CO} \tag{8}$$

where

$$\boldsymbol{f}_{CB} = \left[\boldsymbol{f}_{CB1}^T\, n_{CB1}\ \cdots\ \boldsymbol{f}_{CBn}^T\, n_{CBn}\right]^T \in R^{4n}$$

$$\boldsymbol{f}_{CO} = \left[\boldsymbol{f}_{CO1}^T\, n_{CO1}\ \cdots\ \boldsymbol{f}_{COr}^T\, n_{COr}\right]^T \in R^{4r}$$

$\boldsymbol{f}_{CBj}$ and n_{CBj} $(j = 1, \ldots, n)$ denote the contact force and the moment about the contact normal applied by the finger j, respectively, and $\boldsymbol{f}_{COt}$ and n_{COt} $(t = 1, \ldots, r)$ denote the contact force and the moment about the contact normal at the tth contact between objects, respectively, where we assume that the object l can apply the contact force to the object i when $i < l$. $\boldsymbol{M}_B = \mathrm{diag}[m_{B1}\boldsymbol{I}_3\, \boldsymbol{H}_{B1} \cdots m_{Bm}\boldsymbol{I}_3\, , \boldsymbol{H}_{Bm}]$, m_{Bi}, $\boldsymbol{H}_{Bi}$, and $\boldsymbol{h}_B$ denote the inertia matrix, the mass of the object i, the inertia tensor of the object i, and the vector with respect to the centrifugal and the Coriolis' force, respectively. From (3), the constraint condition among objects is expressed as

$$\boldsymbol{D}_O\dot{\boldsymbol{p}}_B = \boldsymbol{0}. \tag{9}$$

By using (8) and the differentiation of (9), the following relation is derived:

$$\boldsymbol{A}\boldsymbol{f}_C = \boldsymbol{b} \tag{10}$$

where $\boldsymbol{A} = [\boldsymbol{D}_O\boldsymbol{M}_B^{-1}\boldsymbol{D}_{LB}^T \quad \boldsymbol{D}_O\boldsymbol{M}_B^{-1}\boldsymbol{D}_O^T]$ and $\boldsymbol{b} = \boldsymbol{D}_O\boldsymbol{M}_B^{-1}\boldsymbol{h}_B - \dot{\boldsymbol{D}}_O\dot{\boldsymbol{p}}_B$. Equation (10) shows the dependency of the contact force, namely $\boldsymbol{f}_O$ is dependent on $\boldsymbol{f}_{CB}$.

B. Uniqueness of Contact Force

Now, we have a couple of questions. Can we always find the unique contact force $\boldsymbol{f}_{CO}$? If this is not the case, under what condition can we have the unique $\boldsymbol{f}_{CO}$? Let us now answer these questions. If $\boldsymbol{D}_O\boldsymbol{M}_B^{-1}\boldsymbol{D}_O^T$ in (10) is nonsingular, $\boldsymbol{f}_{CO}$ can be expressed uniquely in the following form:

$$\boldsymbol{f}_{CO} = (\boldsymbol{D}_O\boldsymbol{M}_B^{-1}\boldsymbol{D}_O^T)^{-1} \cdot \left(-\boldsymbol{D}_O\boldsymbol{M}_B^{-1}\boldsymbol{D}_{LB}^T\boldsymbol{f}_{CB} + \boldsymbol{D}_O\boldsymbol{M}_B^{-1}\boldsymbol{h}_B - \dot{\boldsymbol{D}}_O\dot{\boldsymbol{p}}_B\right). \tag{11}$$

Therefore, the nonsingularity of $\boldsymbol{D}_O\boldsymbol{M}_B^{-1}\boldsymbol{D}_O^T$ is the necessary condition for finding the unique $\boldsymbol{f}_{CO}$. Taking the nonsingularity of $\boldsymbol{M}_B$ into account, the nonsingularity of $\boldsymbol{D}_O\boldsymbol{M}_B^{-1}\boldsymbol{D}_O^T$ is equivalent to the following condition.

Condition 2 (Uniqueness of Contact Force): The necessary condition for the contact force among objects to be uniquely determined is given by

$$\mathrm{rank}\boldsymbol{D}_O = 4r. \tag{12}$$

This condition can be satisfied under $4r \leq 6m$ if we assume that $\boldsymbol{D}_O$ is a full-rank matrix.

Now, suppose that a contact force among objects is not uniquely determined. Under such a condition, it is not ensured whether or not the contact force always exists within the friction cone at the point of contact. The contact force components in the null space produce a slipping motion at the contact point. To avoid such a complicated situation, we set condition 2 (uniqueness of contact force) in this work.

C. Internal Forces

Now, we discuss the relationship between the dependency of the contact force and the internal force. By the relationship of duality between force and infinitesimal displacement, we can obtain the force balance equation for multiple objects as follows:

$$\boldsymbol{f}_B = \boldsymbol{D}_B^T\boldsymbol{f}_C = \boldsymbol{D}_{LB}^T\boldsymbol{f}_{CB} + \boldsymbol{D}_O^T\boldsymbol{f}_O \tag{13}$$

where

$$\boldsymbol{f}_B = \left[\boldsymbol{f}_{B1}^T\boldsymbol{n}_{B1}^T \cdots \boldsymbol{f}_{Bm}^T\boldsymbol{n}_{Bm}^T\right]^T \in R^{6m}$$

$\boldsymbol{f}_{Bi}$ and $\boldsymbol{n}_{Bi}(i = 1, \ldots, m)$ denote the force and the moment at the center of gravity of the object i, respectively.

Since $\boldsymbol{D}_B \in R^{(4n+4r)\times 6m}$, $\boldsymbol{f}_C$ can be obtained uniquely when $\mathrm{rank}\boldsymbol{D}_B^T = 4n+4r$. However, when $\mathrm{rank}\boldsymbol{D}_B^T < 4n+4r$, a homogeneous solution exists, which means that there exists internal force according to $4n + 4r - \mathrm{rank}\boldsymbol{D}_B^T$. Similar discussions can be applied for $\boldsymbol{D}_O \in R^{4r\times 6m}$. Here, we define the dimensions of the internal forces as follows.

Definition 2 (Dimension of Internal Force): For a grasp of multiple objects, the grasped objects have the following internal forces:

$$I_I = 4n + 4r - \mathrm{rank}\boldsymbol{D}_B^T \tag{14}$$

$$I_{IO} = 4r - \mathrm{rank}\boldsymbol{D}_O^T \tag{15}$$

where I_I and I_{IO} denote the dimension of the total internal force and the internal force among objects, respectively.

For the grasp satisfying $I_{IO} > 0$, the internal force occurs passively even if all the contacts with fingers are released and if all the contacts among objects are maintained. This internal force among objects is peculiar to multiple objects and is not affected by the finger forces. By comparing Definition 2 (dimension of internal force) with Condition 2 (uniqueness of contact force), since we can see that Condition 2 is the same as $I_{IO} = 0$, the contact force can be determined uniquely when the internal force among objects does not exist. Moreover, since $\mathrm{rank}\boldsymbol{D}_B \leq \mathrm{rank}\boldsymbol{D}_{LB} + \mathrm{rank}\boldsymbol{D}_O$ is satisfied, the internal force among objects is included in the total internal force. Therefore, we cannot apply an arbitrary internal force for the grasp without satisfying Condition 2 (uniqueness of contact force).

VI. A Sufficient Condition for Manipulation

A. Friction Constraint

Each contact force should exist within the friction cone as long as each contact avoids a slipping motion. Such a friction constraint results in a nonlinear constraint and makes the problem complicated. To release us from such nonlinear constraint, we approximate the friction cone by the h faced polyhedral convex cone, as taken in many conventional works such as [3]. By utilizing such polyhedral convex cones, we can express each contact force and moment as follows:

$$\begin{bmatrix} f_{CBj} \\ n_{CBj} \end{bmatrix} = \sum_{k=1}^{h} v_{jk}\lambda_{jk}, \qquad \lambda_{jk} \geq 0 \tag{16}$$

$$\begin{bmatrix} f_{COt} \\ n_{COt} \end{bmatrix} = \sum_{k=1}^{h} v_{Otk}\lambda_{Otk}, \qquad \lambda_{Otk} \geq 0 \tag{17}$$

where v_{jk} and v_{Otk} denote the span vectors. We choose v_{jk} and v_{Otk} so that they may coincide with the boundary surface of the actual friction cone. This means that the actual friction cone always includes the approximated polyhedral convex cone. Such an approximation of friction cone enables us to conservatively evaluate the contact force from the viewpoint of bringing a slipping motion at the contact point. Aggregating (16) and (17) for all contact points, we obtain the following relationship:

$$f_C = V\lambda, \qquad \lambda \geq 0 \tag{18}$$

where

$$V = \begin{bmatrix} V_1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & V_n & 0 & \cdots & 0 \\ 0 & \cdots & 0 & V_{O1} & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & V_{Or} \end{bmatrix}$$

$$V_j = [v_{j1}, \ldots, v_{jh}], \qquad j = 1, \ldots, n$$

$$V_{Ot} = [v_{Ot1}, \ldots, v_{Oth}], \qquad t = 1, \ldots, r$$

$$\lambda = [\lambda_{11}, \ldots, \lambda_{1h}, \lambda_{21}, \ldots, \lambda_{nh}, \lambda_{O11}, \ldots, \lambda_{Orh}].$$

B. A Sufficient Condition

We now examine a sufficient condition for robot hands to manipulate the grasped objects arbitrarily under the motion constraint while maintaining the friction constraint. We assume that Condition 2 is satisfied. Different from the condition for the manipulation of a single object, we have to take the motion constraint and the dependency of the contact force into account. For an arbitrary manipulation, we have to examine whether the objects can generate an arbitrary acceleration or not [4]. By using (6) and (8), the following equation is derived:

$$M_B B\ddot{\zeta} = D_B^T f_C - \hat{h}_B \tag{19}$$

where $\hat{h}_B = h_B + M_B\dot{B}\dot{\zeta}$. Equation (19) shows the relationship between the object acceleration and the contact force under the motion constraint among objects. Since the dependency of contact force is not taken into consideration in (19), we consider (10) and formulate the linear programming problem. To make AV_1 nonsingular, we partition V and λ as $V = [V_1\, V_2]$ and $\lambda = [\lambda_1^T\, \lambda_2^T]^T$, respectively. Since A is full-row rank under Condition 2, we can always make AV_1 nonsingular by a proper method of partition. By using this partition, (10) is rewritten as follows:

$$\lambda_1 = -(AV_1)^{-1}(AV_2\lambda_2 - b). \tag{20}$$

Substituting (20) into (19), we consider the following linear programming problem:

$$\begin{aligned} &\text{Maximize} && z = \min[\lambda_1^T\, \hat{\lambda}_1^T]^T \\ &\text{Subject to} && M_B B\ddot{\hat{\zeta}} = H\lambda_2, \qquad \lambda_2 \geq 0 \end{aligned} \tag{21}$$

where

$$\begin{aligned} M_B B\ddot{\hat{\zeta}} &= M_B B\ddot{\zeta} - D_B^T V_1 (AV_1)^{-1} b - \hat{h}_B \\ \hat{\lambda}_1 &= \lambda_1 - (AV_1)^{-1} b \\ &= -(AV_1)^{-1} AV_2\lambda_2 \\ H &= D_B^T V_2 - D_B^T V_1 (AV_1)^{-1} AV_2. \end{aligned}$$

Due to (20), we cannot impose any constraints for λ_1 such as $\lambda_1 \geq \mathbf{o}$. Therefore, we consider maximizing the minimum element of λ_1 in the objective function. Moreover, in (19) and (20), there are nonlinear terms with respect to the centrifugal and Colioris' force, $\hat{h}_B$ and b. To get rid of these terms from the formulation, we used the coordinate transformation from ζ to $\hat{\zeta}$.

We now derive a condition for generating arbitrary acceleration for multiple objects. Let $e_1, e_2, \ldots, e_{I_M} \in R^{I_M}$ be I_M number of given linearly independent vectors [4]. Let $\lambda_1 = \lambda_{1+1}, \ldots, \lambda_{1+I_M}, \lambda_{1-1}, \ldots, \lambda_{1-I_M}$, $\hat{\lambda}_1 = \hat{\lambda}_{1+1}, \ldots, \hat{\lambda}_{1+I_M}, \hat{\lambda}_{1-1}, \ldots, \hat{\lambda}_{1-I_M}$, and $\lambda_2 = \lambda_{2+1}, \ldots, \lambda_{2+I_M}, \lambda_{2-1}, \ldots, \lambda_{2-I_M}$ be the solutions of the linear programming problem (21) for $\ddot{\hat{\zeta}} = e_1, \ldots, e_{I_M}, -e_1, \ldots, -e_{I_M}$, respectively. Now, we have the following condition.

Condition 3 (Generation of Arbitrary Acceleration): Assume that Condition 2 is satisfied. A sufficient condition for the grasped objects to generate the acceleration $\ddot{\zeta}$ arbitrarily is that the linear programming problem (21) has solutions for $2I_M$ number of $\ddot{\hat{\zeta}} = \pm e_k$, $(k = 1, \ldots, I_M)$ satisfying both $\lambda_{1\pm k} \geq \mathbf{o}$ for all $\lambda_{1\pm k}$ $(k = 1, \ldots, I_M)$ and $\hat{\lambda}_{1+k} + \hat{\lambda}_{1-k} > \mathbf{o}$ for at least one pair of $\hat{\lambda}_{1+k}$ and $\hat{\lambda}_{1-k} (k = 1, \ldots, I_M)$.

Proof: See Appendix III.

To solve the linear programming problem, we can use a set of the orthonormal basis as $e_k (k = 1, \ldots, I_M)$. Condition 3 (generation of arbitrary acceleration) is a sufficient condition since we set the approximated friction cone existing inside of the actual friction cone. Diverging from the condition for the manipulation of a single object [4], we substituted vectors $e_k (k = 1, \ldots, I_M)$ into the acceleration $\ddot{\hat{\zeta}}$ taking the motion constraints into consideration. Moreover, due to the term b in (20), the condition $\hat{\lambda}_{1+k} + \hat{\lambda}_{1-k} > \mathbf{o}$ is added for the reason shown in Appendix III. To satisfy $\hat{\lambda}_{1+k} + \hat{\lambda}_{1-k} > \mathbf{o}$, we consider maximizing the minimum of $\hat{\lambda}_1$ in the objective function.

So far, we have shown three conditions for manipulating multiple objects, i.e., the kinematic condition (Condition 1), the uniqueness of contact force among objects (Condition 2), and generation of arbitrary acceleration (Condition 3). Since Condition 3 is a sufficient condition for manipulation, Condition 1 should be included in Condition 3.

VII. CONTROLLER

A. Introduction of Joint Variables

We now discuss the relationship between the end link and the joints of each finger, so that we may introduce the control scheme in joint level. Since the velocity at the end link of the finger j can be expressed by the joint velocity of the finger j, we obtain the following relationship:

$$\begin{bmatrix} \dot{\boldsymbol{p}}_{Fj} \\ \boldsymbol{\omega}_{Fj} \end{bmatrix} = \boldsymbol{J}_j \dot{\boldsymbol{\theta}}_j \tag{22}$$

where $\boldsymbol{J}_j$ and $\boldsymbol{\theta}_j$ are the Jacobian matrix of Σ_{Fj} and the vector of joint displacement of the finger j, respectively. Substituting (22) into (1), we aggregate (1) and (2) for $j = 1, \ldots, n$ and $t = 1, \ldots, r$, respectively, and we obtain the following equation:

$$\boldsymbol{J}_F \dot{\boldsymbol{\theta}} = \boldsymbol{D}_{LB} \dot{\boldsymbol{p}}_B \tag{23}$$

where

$$\boldsymbol{J}_F = \begin{bmatrix} \boldsymbol{D}_{F1}\boldsymbol{J}_1 & \mathbf{o} & \cdots & \mathbf{o} \\ \mathbf{o} & \boldsymbol{D}_{F2}\boldsymbol{J}_2 & \cdots & \mathbf{o} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{o} & \mathbf{o} & \cdots & \boldsymbol{D}_{Fn}\boldsymbol{J}_n \end{bmatrix}$$

$$\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T \cdots \boldsymbol{\theta}_n^T]^T.$$

The equation of the finger motion is derived by using the Lagrange's method as follows:

$$\boldsymbol{M}_F \ddot{\boldsymbol{\theta}} + \boldsymbol{h}_F = \boldsymbol{\tau} - \boldsymbol{J}_F^T \boldsymbol{f}_{CB} \tag{24}$$

where $\boldsymbol{M}_F$, $\boldsymbol{h}_F$, and $\boldsymbol{\tau}$ are the inertia matrix, the vector with respect to the centrifugal, and Coriolis' force and the joint torque vector, respectively.

B. Joint Torque Command

As a controller for object manipulation, we extended the trajectory controller proposed for the grasp of a single object [25]. Since the derivation of the controller is almost same as that which is described in [25], we omit the formulation in detail. Assuming that each finger does not have the redundant degrees of freedom ($s_j = 4$, $j = 1, \ldots, n$), this controller has the following form as a joint torque command:

$$\boldsymbol{\tau} = \boldsymbol{J}_F^T \left(\boldsymbol{B}^T \boldsymbol{D}_{LB}^T\right)^+ \boldsymbol{F} + \boldsymbol{J}_F^T \boldsymbol{N}_B \boldsymbol{k}_B \tag{25}$$

$$\boldsymbol{F} = \overline{\boldsymbol{M}}_B \left(\ddot{\boldsymbol{\zeta}}_d - \boldsymbol{K}_V \dot{\boldsymbol{\delta}}_\zeta - \boldsymbol{K}_P \boldsymbol{\delta}_\zeta\right) + \overline{\boldsymbol{h}}_B \tag{26}$$

where

$$\overline{\boldsymbol{M}}_B = \boldsymbol{B}^T \left(\boldsymbol{M}_B + \boldsymbol{D}_{LB}^T \boldsymbol{J}_F^{-T} \boldsymbol{M}_F \boldsymbol{J}_F^{-1} \boldsymbol{D}_{LB}\right) \boldsymbol{B}$$

$$\overline{\boldsymbol{h}}_B = \boldsymbol{B}^T \boldsymbol{h}_B + \boldsymbol{B}^T \boldsymbol{D}_{LB}^T \boldsymbol{J}_F^{-T} \left(\boldsymbol{M}_F \frac{d}{dt}\left(\boldsymbol{J}_F^{-1} \boldsymbol{D}_{LB}\right) \dot{\boldsymbol{p}}_B + \boldsymbol{h}_F\right)$$

$$\boldsymbol{\delta}_\zeta = \boldsymbol{\zeta} - \boldsymbol{\zeta}_d$$

$\boldsymbol{K}_P$ and $\boldsymbol{K}_V$ are the diagonal matrices corresponding to feedback gain, and $\boldsymbol{\zeta}_d$ is the desired value of $\boldsymbol{\zeta}$. $\boldsymbol{N}_B$ is the null space of $\boldsymbol{B}^T \boldsymbol{D}_{LB}^T$, where $\boldsymbol{B}^T \boldsymbol{D}_{LB}^T \boldsymbol{N}_B = \mathbf{o}$ is satisfied. The first term on the right-hand side of (25) controls the position of the objects to the desired trajectory, and the second term controls the internal force to the desired value.

Here, the control law itself does not ensure that the contact force caused by both finger link and other objects produces an arbitrary acceleration for the parameter $\boldsymbol{\zeta}$. Therefore, before applying the control law, we have to confirm whether an arbitrary acceleration can be achieved or not based on Condition 3 (generation of arbitrary acceleration). Moreover, to confirm whether the desired internal force can be realized or not, the grasp must satisfy Condition 2 (uniqueness of contact force).

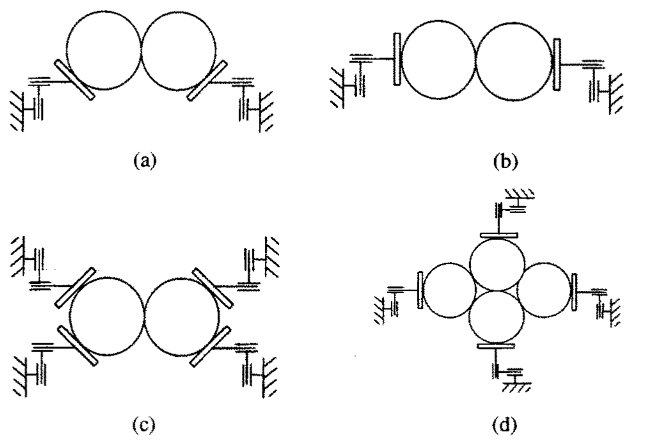

Fig. 3. Several grasp configurations.

VIII. EXAMPLES

A. Case Study

For simplicity, we consider 2-D examples. For a 2-D model, the matrices are redefined such as

$$\boldsymbol{D}_{Bij} = \begin{bmatrix} 1 & 0 & -y_{CBij} \\ 0 & 1 & x_{CBij} \end{bmatrix}$$

$\boldsymbol{R}_{Bi}\, {}^{Bi}\boldsymbol{p}_{CBij} = [x_{CBij}\; y_{CBij}]^T$ and size$\boldsymbol{D}_B = (2n+2r) \times 3m$. We need to consider neither the rotation nor the moment around the contact normal for a 2-D model.

Fig. 3 shows the grasps used for numerical examples, where size$\boldsymbol{D}_B$, rank$\boldsymbol{D}_B$, size$[-\boldsymbol{D}_L\, \boldsymbol{D}_B]$, rank$[-\boldsymbol{D}_L\, \boldsymbol{D}_B]$, I_M, I_I, I_{IO}, and Conditions 1, 2, and 3 for these grasp configurations are shown in Table I.

For the grasp as shown in Fig. 3(a), the grasped objects have 4-D motion and zero-dimensional internal force since $I_M = 4$ and $I_I = 0$. The physical interpretation of this dimension of motion is shown in Fig. 4, which depicts two dimensions for translational motion at the center of gravity between two objects [Fig. 4(a) and (b)], one dimension for rotational motion around the center of gravity [Fig. 4(c)], and one dimension for rotational motion without changing the rotation angle around the center of mass [Fig. 4(d)]. The grasp shown in Fig. 3(a) satisfies Condition 1 (kinematic condition for manipulation) because the finger can manipulate the objects arbitrarily under the motion constraint if we assume that two objects always make contact at each contact point. Condition 2 (uniqueness of contact force rank$\boldsymbol{D}_O = 2r$ for 2-D example) is also satisfied. On the other hand, Condition 3 (generation of arbitrary acceleration) is not satisfied because the contact force does not exist when two objects move in a downward direction.

Fig. 3(b) also shows the grasp of two objects by two fingers. The difference between Fig. 3(a) and (b) is the position of the contact point between a finger and an object. This configuration satisfies neither Condition 1 nor 3, which means that two objects can move freely in a vertical direction even if two finger

TABLE I
RESULT OF CALCULATION

	sizeD_B	rankD_B	size $\begin{bmatrix} -D_L^T \\ D_B^T \end{bmatrix}$	rank $\begin{bmatrix} -D_L^T \\ D_B^T \end{bmatrix}$	I_M	I_I	I_{IO}	Cond.1	Cond.2	Cond.3
(a)	6×6	6	6×10	6	4	0	0	○	○	×
(b)	6×6	5	6×10	6	4	1	0	×	○	×
(c)	10×6	6	10×14	10	4	4	0	○	○	○
(d)	18×12	12	18×20	17	3	6	1	○	×	

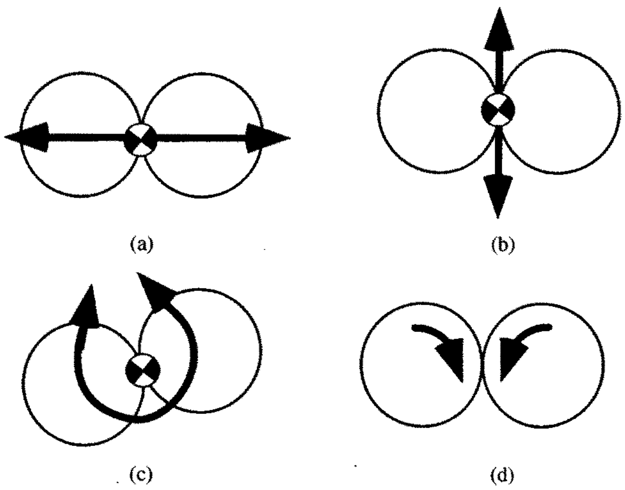

Fig. 4. Four motion degrees of freedom.

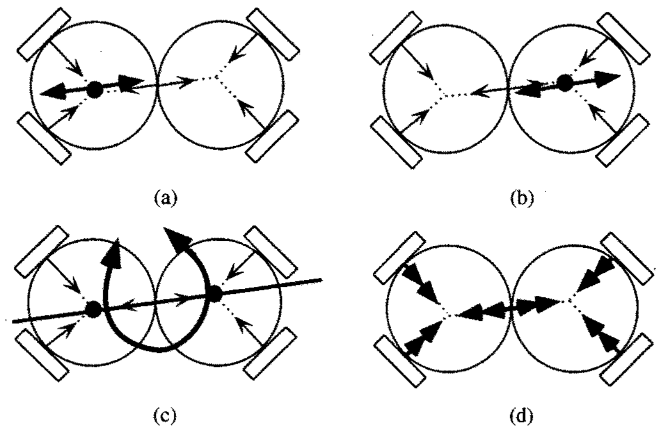

Fig. 5. Four internal force degrees of freedom.

positions are fixed. Therefore, the fingers cannot control the objects' motion in a vertical direction.

The grasp as shown in Fig. 3(c) has 4-D motion and 4-D total internal force since $I_M = 4$ and $I_I = 4$. The physical interpretation of this dimension of the total internal force is shown in Fig. 5. In this grasp configuration, the force focus exists for both objects, and these force focuses lie on the line including the contact point between the two objects. Thus, we have two dimensions for two force focuses capable of moving on the line [Fig. 5(a) and (b)], and one dimension for the rotation of the line [Fig. 5(c)]. The other one dimension is the magnitude of the internal force with the position of the force focuses unchanged [Fig. 5(d)].

The grasp of four objects as shown in Fig. 3(d) has five contact points among objects. This grasp configuration has 3-D motion ($I_M = 3$) and 6-D total internal force ($I_I = 6$). Moreover, the total internal force contains 1-D internal force among objects ($I_{IO} = 1$). Therefore, since $I_{IO} \neq 0$, Condition 2 (uniqueness of contact force) is not satisfied. Since $\boldsymbol{AV}_1$ is not invertible, (21) cannot be formulated, and we cannot judge Condition 3 (generation of arbitrary acceleration).

B. Trajectory Tracking

To show the effectiveness of the controller for object manipulation proposed in Section VII, we performed numerical simulation in which two objects were manipulated by four fingers as shown in Fig. 3(c). Since $I_M = 4$, we have $\zeta \in R^4$. Taking $I_M = 4$ into account, we control all motions (three dimensions) for the left-hand object (object 1) and the motion in the vertical direction for the right-hand object (object 2) while there are of course other variations. As a result, ζ is given as follows:

$$\zeta \triangleq \begin{bmatrix} x_{B1} \\ y_{B1} \\ \phi_{B1} \\ y_{B2} \end{bmatrix} = \boldsymbol{E}_L \boldsymbol{p}_{LF} + \boldsymbol{E}_B \boldsymbol{p}_B \tag{27}$$

where

$$\begin{aligned} \boldsymbol{p}_{LF} &= \left[\boldsymbol{p}_{LF1}^T \boldsymbol{p}_{LF2}^T \boldsymbol{p}_{LF3}^T \boldsymbol{p}_{LF4}^T\right]^T \\ \boldsymbol{p}_B &= [x_{B1}\, y_{B1}\, \phi_{B1}\, x_{B2}\, y_{B2}\, \phi_{B2}]^T \\ \boldsymbol{E}_L &= \mathbf{o} \in R^{4\times 8} \\ \boldsymbol{E}_B &= \begin{bmatrix} 1\,0\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,0 \\ 0\,0\,0\,0\,1\,0 \end{bmatrix}. \end{aligned}$$

We set $\boldsymbol{K}_P = \text{diag}[1000\ \ 1000\ \ 1000\ \ 1000]$, $\boldsymbol{K}_V = \text{diag}[20\ \ 20\ \ 20\ \ 20]$ and $\boldsymbol{k}_d = \mathbf{o}$. The mass and the radius of each object are set as 1 and 0.1, respectively. The mass of each finger link is set as 0.01. x_{B1}, y_{B1}, and ϕ_{B1} are planned to keep constant values while y_{B2} increases in 1 s. From Fig. 6, although we set the initial error, we can see that objects converge to the desired trajectories well. These results show the effectiveness of the proposed controller.

IX. EXPERIMENT

We performed the experiment of trajectory tracking by using the Hiroshima Hand [26]. The Hiroshima Hand is composed of three planar finger units, where each finger has the same structure and has three joints. Each joint is driven by the tendon-pulley transmission system making the tendon length as short as possible, so that we can avoid the nonlinear characteristics

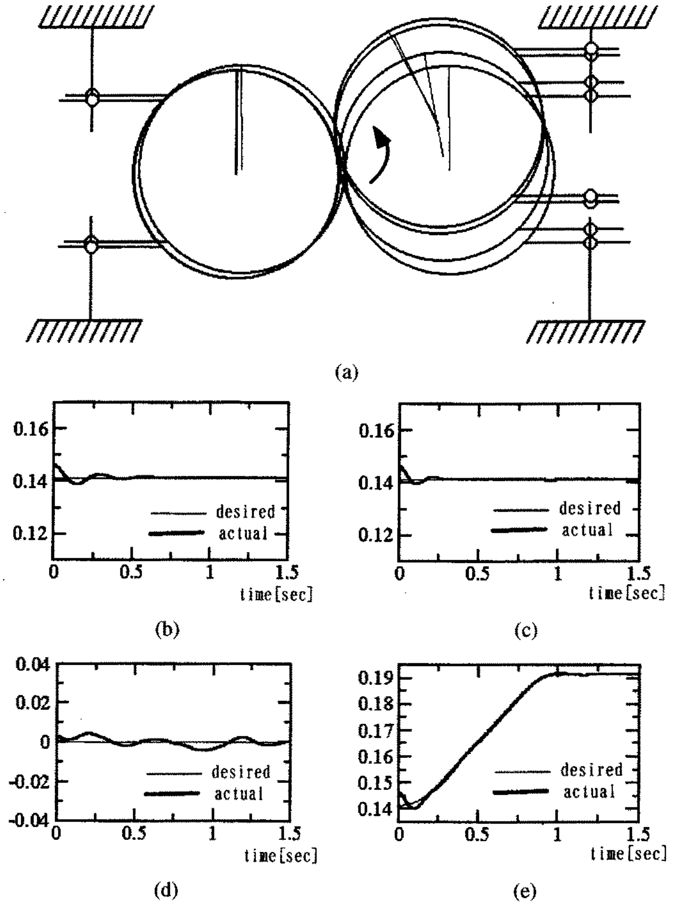

Fig. 6. Simulation results. (a) Motion of the objects. Trajectory of (b) x_{B1}, (c) y_{B1}, (d) ϕ_{B1}, and (e) y_{B2}.

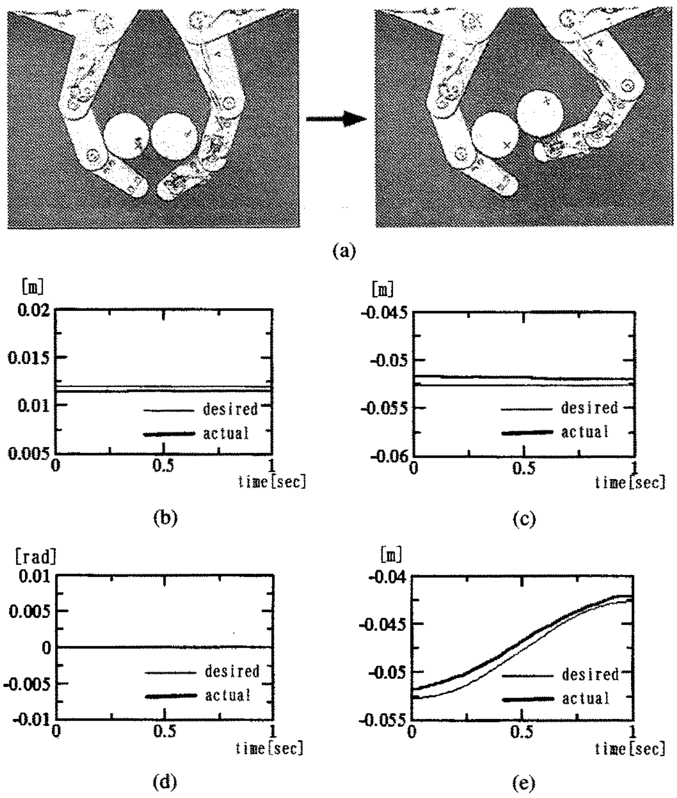

Fig. 7. Experimental results. (a) Overview of the experiment. Trajectory of (b) x_{B1}, (c) y_{B1}, (d) ϕ_{B1}, and (e) y_{B2}.

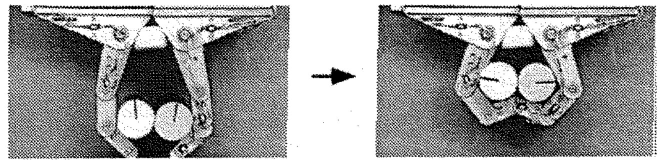

Fig. 8. Lifting up two objects by the Hiroshima hand.

stemming from the compliance and friction existing in tendons. Since the Hiroshima Hand is driven by the velocity servo, we used the following controller as a joint velocity command:

$$\dot{\boldsymbol{\theta}} = \boldsymbol{K}\boldsymbol{J}_F^{+}\boldsymbol{C}(\zeta_d - \zeta) \tag{28}$$

where $\boldsymbol{K}$ is the gain matrix. For a planar grasp, each finger must have at least two joints. Since each finger has three joints, the pseudo-inverse of $\boldsymbol{J}_F$ is used in the command. The radius of each object is 0.01 m. The objects contact the third link of each finger. In the experiment, x_{B1}, y_{B1}, and ϕ_{B1} are planned to keep their initial values while y_{B2} increases 0.01 m in 1 s. The position of the objects are measured by analyzing the image taken in a videotape. From Fig.7, we can see that the objects follow fairly well along the desired trajectory. By using (28), we believe that the performance of the position controller can be shown. However, since (28) does not have the term controlling the internal force, we cannot show the performance of the internal force controller. The experimental verification of the controller for the internal force is considered to be our future research topic.

X. Discussions

For manipulating multiple objects, we suggested a potential advantage in the introduction, where an enveloping grasp for two cylindrical objects placed on a table may be achieved by a simple pushing motion by fingers. To confirm it, we executed an experimental result using the Hiroshima Hand [26]. Fig. 8 shows two pictures before and after the lifting-up motion. With constant torque command, the robot hand can successfully lift up two objects from the table and smoothly result in an equilibrium grasp. During the lifting-up motion, two objects roll up around the surface of each link without any complicated motion planning, as if they were connected by gears. While a manipulation scheme for multiple objects is generally complicated due to both kinematic and dynamic constraints, this experiment proves that there exists an exceptional example in which manipulating multiple objects can be achieved easily.

Now, let us consider why such a manipulation can be achieved easily without any complicated motion planning. For this purpose, consider the grasp configuration as shown in Fig. 3(a), since it has the same one appeared in the lifting-up manipulation. The motion of two objects is uniquely determined by the finger motion, because the grasp satisfies Condition 1 (kinematic condition for manipulation). Since the grasp does not have the dimension of internal force ($I_I = 0$), all forces applied by the fingers is perfectly utilized for manipulating the objects without losing as any internal force. For the lifting-up motion, we can provides with a sufficient condition, where *two objects are lifted by a horizontal pushing motion of finger links if the contact points between an object and a finger link are lower* *than the horizontal line including the contact point between two objects.* We believe that these properties for the grasp configuration enable us to achieve the lifting-up motion by a surprisingly simple control scheme. We would note that although the analysis in this paper is partly too complicated, the results obtained through the work are conveniently available for explaining various issues encountered for manipulating multiple objects under rolling contact, as utilized for explaining the lifting-up motion.

XI. Conclusions

This paper discussed the manipulation of multiple objects under rolling contacts. For manipulating multiple objects, each object's motion is restricted by the other objects, and the contact force between objects is not controlled independently. Taking the above issues into account, we showed three conditions for the manipulation of multiple objects, i.e., a kinematic condition for determining object motion uniquely, a sufficient condition for generating arbitrary acceleration on the objects, and the necessary and sufficient condition for determining the contact force among objects uniquely.

We note that while we have completed our formulation for a multifingered model, the result is applicable to multiple arms. We also note that, although we assumed rolling contact, the condition for rolling contact includes the condition for point contact with friction and the condition for line or surface contact. Therefore, our approach can be extended to more general cases without difficulties.

Appendix I
Construction of Matrices

In this section, the construction of matrices $\boldsymbol{D}_{LB}$ and $\boldsymbol{D}_O$ included in (3) is discussed. Rows of $\boldsymbol{D}_{LB}$ from $(4j+1)$th to $(4j+4)$th are defined as follows:

$$\boldsymbol{D}_{LB} = \begin{array}{c} \\ \vdots \\ 4j+1 \\ 4j+4 \\ \vdots \end{array} \begin{array}{c} \cdots \quad 6i+1 \cdots 6i+6 \quad \cdots \\ \left[\begin{array}{c|c|c} \cdots & \cdots & \cdots \\ \hline \mathbf{o} & \boldsymbol{D}_{Bij} & \mathbf{o} \\ \hline \cdots & \cdots & \cdots \end{array}\right] \end{array}. \tag{29}$$

Similarly, rows of $\boldsymbol{D}_O$ from the $(4t+1)$th to $(4t+4)$th are defined as follows:

$$\boldsymbol{D}_O = \begin{array}{c} \\ \vdots \\ 4j+1 \\ 4j+4 \\ \vdots \end{array} \begin{array}{c} \cdots \quad 6i+1 \cdots 6i+6 \quad \cdots \quad 6l+1 \cdots 6l+6 \quad \cdots \\ \left[\begin{array}{c|c|c|c|c} \cdots & \cdots & \cdots & \cdots & \cdots \\ \hline \mathbf{o} & \boldsymbol{D}_{Oit} & \mathbf{o} & -\boldsymbol{D}_{Olt} & \mathbf{o} \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right] \end{array} \tag{30}$$

where $i < l$ is assumed. For an example of a grasp of three objects by two fingers as shown in Fig. 9, $\boldsymbol{D}_{LB}$ and $\boldsymbol{D}_O$ are given as follows:

$$\boldsymbol{D}_{LB} = \begin{bmatrix} \boldsymbol{D}_{B11} & 0 & 0 \\ 0 & 0 & \boldsymbol{D}_{B32} \end{bmatrix}$$
$$\boldsymbol{D}_O = \begin{bmatrix} \boldsymbol{D}_{O11} & -\boldsymbol{D}_{O21} & \mathbf{o} \\ \mathbf{o} & \boldsymbol{D}_{O22} & -\boldsymbol{D}_{O32} \\ \boldsymbol{D}_{O13} & \mathbf{o} & -\boldsymbol{D}_{O33} \end{bmatrix}.$$

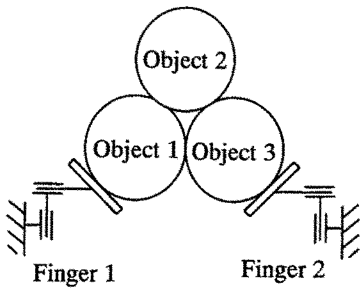

Fig. 9. Grasp of three objects by two fingers.

Appendix II
Derivation of Contact Coordinates [20], [25]

In this section, we briefly summarize the contact kinematics derived by Montana. The relative motion between the tip link of the finger j and the object i can be defined by $[v_{ijx}\, v_{ijy}\, v_{ijz}]^T$ and $[\omega_{ijx}\, \omega_{ijy}\, \omega_{ijz}]^T$, which are the components of the linear and rotational velocity of Σ_{LBij} relative to Σ_{LFj} as seen from Σ_{LBij}, respectively. There are five degrees of freedom of the evolution of the contact points defined by $\boldsymbol{u}_{CBij} \in R^2$, $\boldsymbol{u}_{CFj} \in R^2$, and $\varphi_{Cij} \in R^1$. $\boldsymbol{u}_{CBij} \in R^2$ and $\boldsymbol{u}_{CFj} \in R^2$ can express the contact point on the surface uniquely as ${}^{Bi}\boldsymbol{p}_{CBij} = \boldsymbol{f}_{Bi}(\boldsymbol{u}_{CBij})$ and ${}^{Fj}\boldsymbol{p}_{CFj} = \boldsymbol{f}_{Fj}(\boldsymbol{u}_{CFj})$, respectively. Let φ_{Cij} be the angle of contact, defined as the angle between the x axes of Σ_{CBij} and Σ_{CFj}. We choose the sign of φ_{Cij} so that a rotation of the x axis of Σ_{CBij} through an angle $-\varphi_{Cij}$ around the z axis of Σ_{CBij} aligns with the x axis of Σ_{CFj}. Let the curvature form, connection form and the metric of the finger surface and the object surface be $\mathcal{K}_{Fj}$, $\mathcal{T}_{Fj}$, $\mathcal{M}_{Fj}$ and $\mathcal{K}_{Bi}$, $\mathcal{T}_{Bi}$, $\mathcal{M}_{Bi}$, respectively. Also, let

$$\mathcal{R}_{\varphi ij} = \begin{bmatrix} \cos\varphi_{Cij} & -\sin\varphi_{Cij} \\ -\sin\varphi_{Cij} & -\cos\varphi_{Cij} \end{bmatrix}$$
$$\tilde{\mathcal{K}}_{Fj} = \mathcal{R}_{\varphi ij}\mathcal{K}_{Fj}\mathcal{R}_{\varphi ij}$$

where $(\mathcal{K}_{Bi} + \tilde{\mathcal{K}}_{Fj})$ is called the relative curvature form. By using these forms, $\boldsymbol{u}_{CBij}$, $\boldsymbol{u}_{CFj}$, and φ_{Cij} are expressed as follows:

$$\dot{\boldsymbol{u}}_{CBij} = \mathcal{M}_{Bi}^{-1}(\mathcal{K}_{Bi} + \tilde{\mathcal{K}}_{Fj})^{-1}\left(\begin{bmatrix} -\omega_{ijy} \\ \omega_{ijx} \end{bmatrix} - \tilde{\mathcal{K}}_{Fj}\begin{bmatrix} v_{ijx} \\ v_{ijy} \end{bmatrix}\right) \tag{31}$$

$$\dot{\boldsymbol{u}}_{CFij} = \mathcal{M}_{Fj}^{-1}\mathcal{R}_{\varphi ij}(\mathcal{K}_{Bi} + \tilde{\mathcal{K}}_{Fj})^{-1}\left(\begin{bmatrix} -\omega_{ijy} \\ \omega_{ijx} \end{bmatrix} + \mathcal{K}_{Bi}\begin{bmatrix} v_{ijx} \\ v_{ijy} \end{bmatrix}\right) \tag{32}$$

$$\dot{\varphi}_{Cij} = \omega_{ijz} + \mathcal{T}_{Bi}\mathcal{M}_{Bi}\dot{\boldsymbol{u}}_{CBij} + \mathcal{T}_{Fj}\mathcal{M}_{Fj}\dot{\boldsymbol{u}}_{CFj} \tag{33}$$

$$v_{ijz} = 0. \tag{34}$$

If we assume the condition for pure rolling ($v_{ijx} = v_{ijy} = \omega_{ijz} = 0$) into the above equations, the vector of contact velocity can be derived as

$${}^{Bi}\dot{\boldsymbol{p}}_{Cij} = \frac{\partial \boldsymbol{f}_{Bi}}{\partial \boldsymbol{u}_{CBij}^T}\dot{\boldsymbol{u}}_{CBij} \quad \text{and} \quad {}^{Fj}\dot{\boldsymbol{p}}_{CFj} = \frac{\partial \boldsymbol{f}_{Fj}}{\partial \boldsymbol{u}_{CFj}^T}\dot{\boldsymbol{u}}_{CFj}.$$

${}^{Bi}\boldsymbol{p}_{COt}$ can be derived similarly by following the above derivation.

APPENDIX III
PROOF FOR CONDITION 3

If the linear programming problem (21) has solutions for $2I_M$ number of $\ddot{\zeta} = \pm e_k$, $(k = 1, \ldots, I_M)$, we have the following $2I_M$ number of equations:

$$\begin{aligned} \boldsymbol{M}_B \boldsymbol{B} \boldsymbol{e}_1 &= \boldsymbol{H}\boldsymbol{\lambda}_{2+1} \\ &\vdots \\ \boldsymbol{M}_B \boldsymbol{B} \boldsymbol{e}_{I_M} &= \boldsymbol{H}\boldsymbol{\lambda}_{2+I_M} \\ -\boldsymbol{M}_B \boldsymbol{B} \boldsymbol{e}_1 &= \boldsymbol{H}\boldsymbol{\lambda}_{2-1} \\ &\vdots \\ -\boldsymbol{M}_B \boldsymbol{B} \boldsymbol{e}_{I_M} &= \boldsymbol{H}\boldsymbol{\lambda}_{2-I_M}. \end{aligned}$$

By introducing nonnegative scolors $\rho_{+1}, \ldots, \rho_{+I_M}, \rho_{-1}, \ldots, \rho_{-I_M}$, an arbitrary $\ddot{\zeta}$ can be expressed as

$$\begin{aligned} \ddot{\zeta} &= \rho_{+1}\boldsymbol{e}_1 + \cdots + \rho_{+I_M}\boldsymbol{e}_{I_M} - \rho_{-1}\boldsymbol{e}_1 - \cdots - \rho_{-I_M}\boldsymbol{e}_{I_M} \\ &= (\rho_{+1} - \rho_{-1})\boldsymbol{e}_1 + \cdots + (\rho_{+I_M} - \rho_{-I_M})\boldsymbol{e}_{I_M}. \end{aligned} \tag{35}$$

Corresponding to (35), the solutions for $\boldsymbol{\lambda}_2$ are expressed as

$$\begin{aligned} \boldsymbol{\lambda}_2 &= \rho_{+1}\boldsymbol{\lambda}_{2+1} + \cdots + \rho_{+I_M}\boldsymbol{\lambda}_{2+I_M} \\ &\quad + \rho_{-1}\boldsymbol{\lambda}_{2-1} + \cdots + \rho_{-I_M}\boldsymbol{\lambda}_{2-I_M} \\ &\geq \boldsymbol{0}. \end{aligned} \tag{36}$$

Thus, we obtain $\boldsymbol{\lambda}_2$ generating arbitrary $\ddot{\zeta}$. We now examine whether we can always make $\boldsymbol{\lambda}_1 \geq \boldsymbol{o}$ for arbitrary $\ddot{\zeta}$. From (20), the following equations are derived:

$$\begin{aligned} \boldsymbol{\lambda}_{1+1} &= -(\boldsymbol{A}\boldsymbol{V}_1)^{-1}(\boldsymbol{A}\boldsymbol{V}_2\boldsymbol{\lambda}_{2+1} - \boldsymbol{b}) \\ &\vdots \\ \boldsymbol{\lambda}_{1+I_M} &= -(\boldsymbol{A}\boldsymbol{V}_1)^{-1}(\boldsymbol{A}\boldsymbol{V}_2\boldsymbol{\lambda}_{2+I_M} - \boldsymbol{b}) \\ \boldsymbol{\lambda}_{1-1} &= -(\boldsymbol{A}\boldsymbol{V}_1)^{-1}(\boldsymbol{A}\boldsymbol{V}_2\boldsymbol{\lambda}_{2-1} - \boldsymbol{b}) \\ &\vdots \\ \boldsymbol{\lambda}_{1-I_M} &= -(\boldsymbol{A}\boldsymbol{V}_1)^{-1}(\boldsymbol{A}\boldsymbol{V}_2\boldsymbol{\lambda}_{2-I_M} - \boldsymbol{b}). \end{aligned} \tag{37}$$

By summing all equations in (37), the following equation is derived:

$$\begin{aligned} &\rho_{+1}\hat{\boldsymbol{\lambda}}_{1+1} + \cdots + \rho_{-I_M}\hat{\boldsymbol{\lambda}}_{1-I_M} + (\boldsymbol{A}\boldsymbol{V}_1)^{-1}\boldsymbol{b} \\ &\quad = -(\boldsymbol{A}\boldsymbol{V}_1)^{-1}\{\boldsymbol{A}\boldsymbol{V}_2(\rho_{+1}\boldsymbol{\lambda}_{2+1} + \cdots + \rho_{-I_M}\boldsymbol{\lambda}_{2-I_M}) - \boldsymbol{b}\}. \end{aligned} \tag{38}$$

Although the left-hand side of (38) expresses $\boldsymbol{\lambda}_1$, we cannot always insure $\boldsymbol{\lambda}_1 \geq \boldsymbol{o}$ since $\boldsymbol{b}$ is included in the left-hand side of (38). However, since $\hat{\boldsymbol{\lambda}}_{1+k}$ and $\hat{\boldsymbol{\lambda}}_{1-k}$ correspond to the acceleration in the opposite direction, $\ddot{\zeta} = \boldsymbol{e}_k$ and $\ddot{\zeta} = -\boldsymbol{e}_{-k}$, respectively, there is no effect of acceleration if we increase $\boldsymbol{\lambda}_1$ in the direction of $\hat{\boldsymbol{\lambda}}_{1+k} + \hat{\boldsymbol{\lambda}}_{1-k}$. In other words, by observing (35), we can put $\rho_{+k} = \hat{\rho}_{+1} + \alpha$ and $\rho_{-k} = \hat{\rho}_{-1} + \alpha$ for an arbitrary α without loss of the arbitrariness of $\ddot{\zeta}$, since α disappears as $\rho_{+k} - \rho_{-k} = \hat{\rho}_{+k} - \hat{\rho}_{-k}$. Since we can set α large enough, we can always make $\boldsymbol{\lambda}_1 = \rho_{+1}\hat{\boldsymbol{\lambda}}_{1+1} + \cdots + \rho_{-I_M}\hat{\boldsymbol{\lambda}}_{1-I_M} + (\boldsymbol{A}\boldsymbol{V}_1)^{-1}\boldsymbol{b} \geq 0$ if $\hat{\boldsymbol{\lambda}}_{1+k} + \hat{\boldsymbol{\lambda}}_{1-k} > 0$ is satisfied. Now, we can always satisfy $\boldsymbol{\lambda}_1 \geq \boldsymbol{0}$ and $\boldsymbol{\lambda}_2 \geq \boldsymbol{0}$ for an arbitrary $\ddot{\zeta}$. Since $\boldsymbol{M}_B\boldsymbol{B}$ is always full-column rank, the arbitrariness of $\hat{\zeta}$ is equivalent to the arbitrariness of $\ddot{\zeta}$. These discussions hold the theorem.

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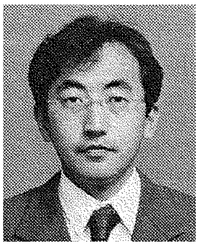

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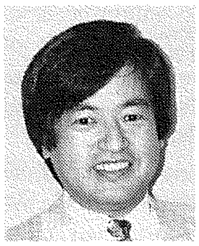

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