Thermal Restoration of Chiral Symmetry in Supersymmetric Nambu-Jona-Lasinio Model with Soft SUSY Breaking

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Abstract

The supersymmetric version of the Nambu-Jona-Lasinio model is investigated in connection with the chiral symmetry breaking induced by a soft SUSY breaking term. It is found that the broken chiral symmetry due to the soft breaking term is restored at suitably high temperature and the symmetry restoration occurs as first-order phase transitions. The critical temperature at which the chiral symmetry is restored is determined as a function of the strength of the soft breaking term and the field coupling constant. The dynamical fermion mass is calculated at finite temperature. Some possible applications to the breaking scenario of unified field theories are discussed.

In various theoretical circumstances the Nambu-Jona-Lasinio model [1] plays a role of the low energy effective theory of the more fundamental unified field theories. Its supersymmetric version is of special interest because the model is useful to study the low energy consequences of the supersymmetric grand unified theories. In particular it is important to see whether the chiral symmetry posessed by the model is broken dynamically to generate fermion masses. Unfortunately, however, it is well-known [2] that the supersymmetry of the model protects the chiral symmetry from its breaking. If one introduces a soft SUSY breaking term in the model, one finds that the chiral symmetry in the model is broken for a sufficiently large soft SUSY breaking term in comparison with the coupling constant in the interaction term .[3] This situation gives rise to a possibility of studying the phenomenological consequences of the broken chiral symmetry in the supersymmetric theory.

In connection with the early history of the universe it is especially interesting to investigate the finite temperature behavior of the broken chiral symmetry in the supersymmetric model like the one mentioned above. In the present communication we shall examine whether the thermal restoration of the chiral symmetry takes place in the supersymmetric Nambu-Jona-Lasinio model with the soft SUSY breaking term.

The Nambu-JonaLasinio model is characterized by the Lagrangian

$$\mathcal{L}_{NJL} = i\bar{\psi}_+\bar{\sigma}^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\bar{\sigma}^\mu\partial_\mu\psi_- + \frac{G}{N}\psi_+\psi_-\bar{\psi}_+\bar{\psi}_-, \tag{1}$$

where Weyl spinors $\psi_\pm$ are assumed to consist of N components and we shall work in the leading order of the $1/N$ expansion throughout the paper. A natural SUSY extension of the model is given by the Lagrangian [2]

$$\mathcal{L}_{SNJL} = \int d^4\theta \; (\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_- + \frac{G}{N} \; \Phi_+\Phi_-\bar{\Phi}_+\bar{\Phi}_-), \tag{2}$$

where $d^4\theta = d^2\theta d^2\bar{\theta}$ and chiral superfield $\Phi_\pm$ is represented by component scalar ($\phi_\pm$) and spinor ($\psi_\pm$) fields and auxiliary field $F_\pm$,

$$\Phi_\pm(y) = \phi_\pm(y) + \sqrt{2}\theta\psi_\pm(y) + F_\pm(y)\theta^2, \tag{3}$$

with $y = x+i\theta\sigma^\mu\bar{\theta}$. The chiral superfields Φ_+ and Φ_- belong to the multiplet N and $\bar{N}$ in SU(N) respectively.

In this SUSY version of the Nambu-Jona-Lasinio model we introduce the following soft SUSY breaking term [3]

$$\mathcal{L}_{SB} = -\int d^4\theta \, \Delta^2 \theta^2 \bar{\theta}^2 (\bar{\Phi}_+ \Phi_+ + \bar{\Phi}_- \Phi_-). \qquad (4)$$

It is convenient to introduce auxiliary chiral superfield H and S and use the Lagrangian equivalent to Eq.(2) with the soft breaking term:

$$\mathcal{L} = \int d^4\theta [(\bar{\Phi}_+ \Phi_+ + \bar{\Phi}_- \Phi_-)(1 - \Delta^2 \theta^2 \bar{\theta}^2) + \frac{N}{G}\bar{H}H]$$
$$+ \int d^2\theta \, (\frac{N}{G}HS - S\Phi_+\Phi_- + h.c.). \qquad (5)$$

We write the relevant Lagrangian in terms of component fields to calculate the effective potential in the leading order of $1/N$ expansion:

$$\mathcal{L} = -\bar{\phi}_+(\Box + |\phi_S|^2 + \Delta^2)\phi_+ - \bar{\phi}_-(\Box + |\phi_S|^2 + \Delta^2)\phi_-$$
$$+i\bar{\psi}_+\bar{\sigma}^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\bar{\sigma}^\mu\partial_\mu\psi_- + \psi_+\psi_-\phi_S + \bar{\psi}_+\bar{\psi}_-\bar{\phi}_S - \frac{N}{G}|\phi_S|^2. \qquad (6)$$

where we have used equations of motion for the auxiliary fields $F_\pm$ and F_H. The field ϕ_S is related to the ordinary scalar (σ) and pseudoscalar (π) field such that

$$\phi_S = \sigma + i\pi.$$

The effective potential for this Lagrangian is easily written down in the leading order of the $1/N$ expansion and reads

$$V_0(|\phi_S|^2) = \frac{|\phi_S|^2}{G} + 2i\int \frac{d^4k}{(2\pi)^4}(\ln\frac{-k^2 + |\phi_S|^2}{-k^2} - \ln\frac{-k^2 + |\phi_S|^2 + \Delta^2}{-k^2 + \Delta^2}). \qquad (7)$$

After Wick rotation we perform the k_0 integration and obtain

$$V_0(|\phi_S|^2) = \frac{|\phi_S|^2}{G} + 2\int \frac{d^3k}{(2\pi)^3}[\sqrt{k^2 + |\phi_S|^2 + \Delta^2} - \sqrt{k^2 + |\phi_S|^2}$$
$$-(\sqrt{k^2 + \Delta^2} - \sqrt{k^2})]. \qquad (8)$$

Note here that the effective potential is normalized so that $V_0(0) = 0$. By analyzing the above effective potential we recognize that the chiral symmetry is broken for sufficiently large Δ. This situation is understood more clearly by observing the gap equation which is derived from the effective potential (8). The gap equation $\partial V_0/\partial|\phi_S|^2 = 0$ is found to be

$$\frac{1}{G} = \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{\sqrt{k^2 + |\phi_S|^2}} - \frac{1}{\sqrt{k^2 + |\phi_S|^2 + \Delta^2}} \right). \tag{9}$$

Introducing the three dimensional cut-off Λ in the momentum integration we perform the integration in Eq. (9) and obtain

$$\begin{aligned}\frac{1}{2\alpha} &= \sqrt{1+x} - \sqrt{1+x+\delta} + (x+\delta)\ln\frac{1+\sqrt{1+x+\delta}}{\sqrt{x+\delta}} \\ &\quad -x\ln\frac{1+\sqrt{1+x}}{\sqrt{x}},\end{aligned} \tag{10}$$

where α, δ and x are defined by

$$\alpha = \frac{G\Lambda^2}{8\pi^2}, \quad \delta = \frac{\Delta^2}{\Lambda^2}, \quad x = \frac{|\phi_S|^2}{\Lambda^2}. \tag{11}$$

It is not difficult to show that Eq. (10) allows a nontrivial solution for x with the suitable choice of parameters α and δ. Thus the dynamical fermion mass is generated and the chiral symmetry is broken dynamically. Noting that the chiral symmetry breaking in the present case is of the second order phase transition we let the gap parameter x vanish so that we obtain the critical δ at which the chiral symmetry is broken. The critical δ is given by solving the following relation as a function of the coupling constant α:

$$\frac{1}{2\alpha} = 1 - \sqrt{1+\delta} + \delta\ln\frac{1+\sqrt{1+\delta}}{\sqrt{\delta}} \tag{12}$$

Now we introduce the temperature. We employ the Matsubara formalism [4] to introduce temperature effects in our calculation of the effective potential. We simply replace the integral in energy variable by the summation where we apply the periodic boundary condition for boson fields and the anti-periodic boundary condition for fermion fields respectively. The energy variable k_0 in the summation is replaced in the following way:

$$\int \frac{dk_0}{2\pi} f(k_0) \rightarrow \frac{1}{\beta} \sum_{n=-\infty}^{\infty} f(\omega_n), \tag{13}$$

where β is the inverse of the temperature and ω_n is given by

$$\omega_n = \begin{cases} \omega_n^B \equiv \frac{2n}{\beta}\pi & \text{(boson)}, \\ \\ \omega_n^F \equiv \frac{2n+1}{\beta}\pi & \text{(fermion)}. \end{cases}$$

The calculation is essentially the same as the one described in the previous paper.[5] We first calculate the effective potential for the supersymmetric Nambu-Jona-Lasinio model without the soft SUSY breaking term and obtain

$$V(|\phi_S|^2) = \frac{|\phi_S|^2}{G} - \frac{2}{\beta} \sum_n \int \frac{d^3k}{(2\pi)^3} \left(\ln \frac{\omega_n^{F2} + k^2 + |\phi_S|^2}{\omega_n^{F2} + k^2} - \ln \frac{\omega_n^{B2} + k^2 + |\phi_S|^2}{\omega_n^{B2} + k^2} \right).$$
(14)

The summation on n in Eq. (14) is performed easily to give

$$V(|\phi_S|^2) = \frac{|\phi_S|^2}{G} - \frac{4}{\beta} \int \frac{d^3k}{(2\pi)^3} \ln \left[\coth \frac{\beta\sqrt{k^2 + |\phi_S|^2}}{2} \tanh \frac{\beta\sqrt{k^2}}{2} \right].$$
(15)

The result reflects the well-known fact that the supersymmetry is spoiled at finite temperature according to the different distribution functions for bosons and fermions.[6] It is, however, interesting to note that the supersymmetry is preserved in the low temperature region where the Boltzmann distribution is a good approximation. In fact, if we keep only the dominant terms for small T in Eq.(15), we find that the second term in Eq.(15) disappears leaving the tree expression of the effective potential. We realize that the supersymmetry is a good symmetry only in the low temperature region.

We then take into account the soft SUSY breaking term and calculate the effective potential to obtain

$$V(|\phi_S|^2) = \frac{|\phi_S|^2}{G} - \frac{2}{\beta} \sum_n \int \frac{d^3k}{(2\pi)^3} \left(\ln \frac{\omega_n^{F2} + k^2 + |\phi_S|^2}{\omega_n^{F2} + k^2} \right.$$
$$\left. - \ln \frac{\omega_n^{B2} + k^2 + |\phi_S|^2 + \Delta^2}{\omega_n^{B2} + k^2 + \Delta^2} \right).$$
(16)

The summation on n in the above equation can be easily performed as before and results in

$$V(|\phi_S|^2) = V_0(|\phi_S|^2) - \frac{4}{\beta} \int \frac{d^3k}{(2\pi)^3} \ln \left(\frac{1 + e^{-\beta\sqrt{k^2 + |\phi_S|^2}}}{1 + e^{-\beta\sqrt{k^2}}} \frac{1 - e^{-\beta\sqrt{k^2 + \Delta^2}}}{1 - e^{-\beta\sqrt{k^2 + |\phi_S|^2 + \Delta^2}}} \right),$$
(17)

where V_0 is the effective potential for vanishing temperature already given in Eq.(8).

For almost whole range of the set of parameters α and δ we estimated the effective potential (17) by numerical integration. We show a typical example of its behavior in Fig. 1 for several values of β.

- Fig.1 -

Apparently we observe that the broken chiral symmetry at $T = 0$ is restored at critical temperature $\beta\Lambda = 13$ and the chiral symmetry restoration occurs as a first-order phase transition for which the mass gap is expected at the critical temperature. In order to see the situation more clearly we write down the gap equation

$$
\begin{aligned}
\frac{1}{G} &= \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{\sqrt{k^2 + |\phi_S|^2}} - \frac{1}{\sqrt{k^2 + |\phi_S|^2 + \Delta^2}} \right) \\
&\quad -2 \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{\sqrt{|\phi_S|^2 + k^2}} \frac{1}{e^{\beta\sqrt{|\phi_S|^2+k^2}} + 1} \right. \\
&\quad \left. - \frac{1}{\sqrt{|\phi_S|^2 + k^2 + \Delta^2}} \frac{1}{e^{\beta\sqrt{|\phi_S|^2+k^2+\Delta^2}} - 1} \right].
\end{aligned}
\tag{18}
$$

By the numerical anlysis of Eq.(18) we recognize that two nontrivial solutions for x develop as temperature goes through the critical value signaling the possibility of the first-order phase transition. The critical temperature T_C and the critical value of x are determined by solving the gap equation (18) and $V = 0$ simultaneously. The critical temperature T_C is obtained at the same time as in the above analysis and is a function of the soft breaking parameter $\delta = \Delta^2/\Lambda^2$ for a fixed $\alpha = G\Lambda^2/8\pi^2$.

In the broken phase x is non-vanishing and hence there develops the non-vanishing vacuum expectation value of the scalar field σ which is responsible for generating fermion masses. In Fig. 2 we present the dynamical fermion mass $m/\Lambda = \sqrt{x}$ as a function of $\beta\Lambda$. We clearly observe the mass gap characteristic of the first-order phase transition.

- Fig.2-

It should be pointed out here that the integration in Eq. (18) is performed rigorously in the case of the three dimensional space-time. We leave the argument in the three dimensional case in the forthcoming publication.

The supersymmetric four-fermion theory is considered to be a low-energy manifestation of the more fundamental supersymmetric unified theories. The typical example is the supersymmetric GUT which breaks down to the electroweak theory. In the early universe scenario this stage of the breaking of the supersymmetric GUT corresponds to the beginning of the inflation

era where the first-order phase transition is required. In this connection our finding of the chiral symmetry restoration as the first-order phase transition is of great interest.

References

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122(1961) 345.

[2] W. Buchmueller and S. T. Love, Nucl. Phys. B204(1982) 213;

V. Elias, D. G. C. Mckeon, V. A. Miransky, I. A. Shovkovy, Phys. Rev. D54(1996)7884;

L. L. Buchbinder, T. Inagaki and S. D. Odintsov, Mod. Phys. Lett. A12 (1997)2271.

[3] W. Buchmueller and U. Ellwanger, Nucl. Phys. B245(1984) 237.

[4] T. Matsubara, Prog. Theor. Phys. 14(1955) 351.

[5] T. Inagaki, T. Kouno and T. Muta, Intern. J. Mod. Phys. A10(1995) 2241.

[6] D. Buchholtz and I. Ojima, Nucl. Phys. B498(1997) 228 and references therein.

Figure Captions

Fig 1: Effective potential at finite temperature for $G\Lambda^2 = 7 \times 10^3$ and $\Delta/\Lambda = 0.1$

Fig 2: Dynamical mass of fermion as a function of $\beta\Lambda(\beta_C\Lambda = \Lambda/T_C = 13)$
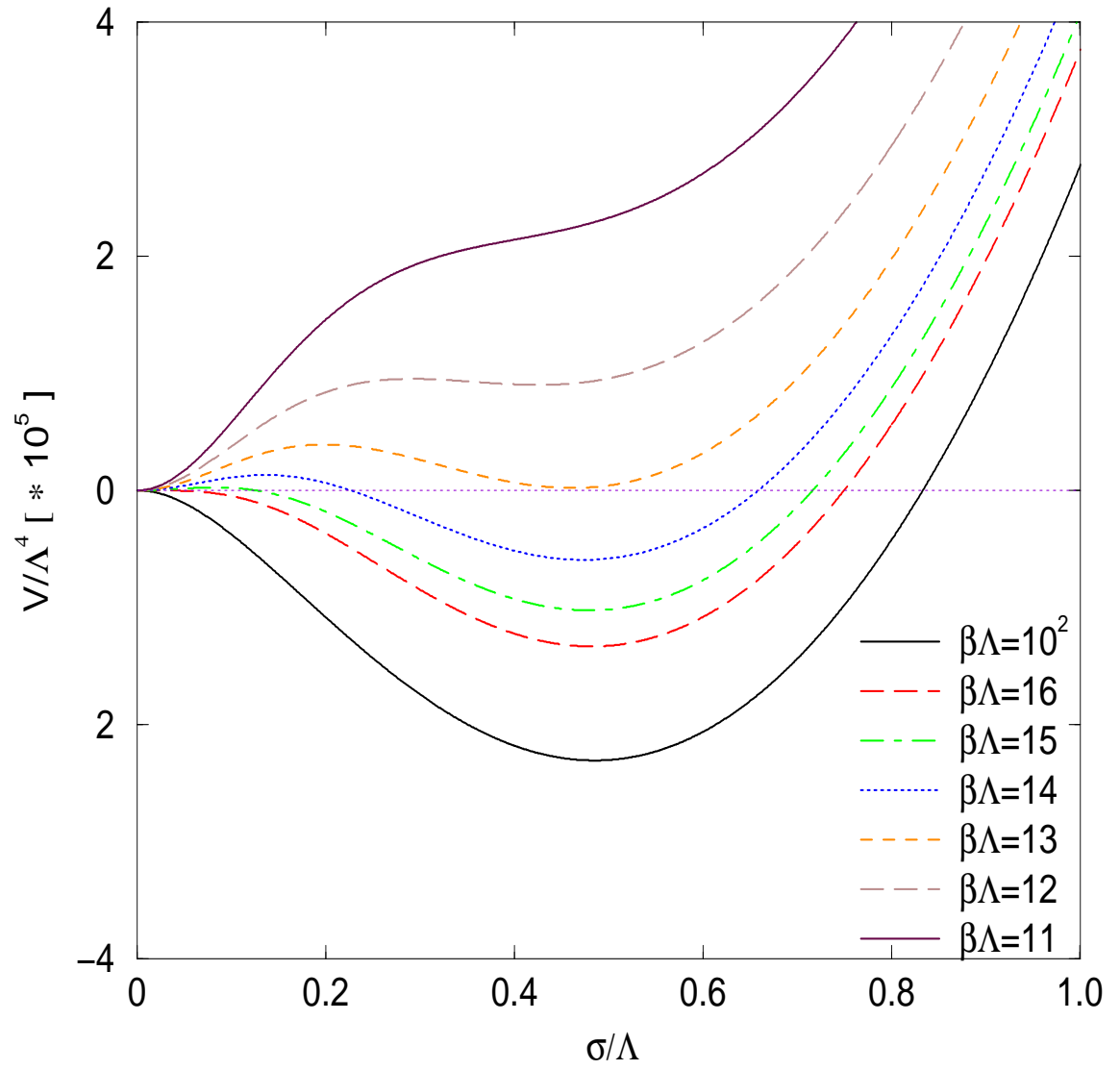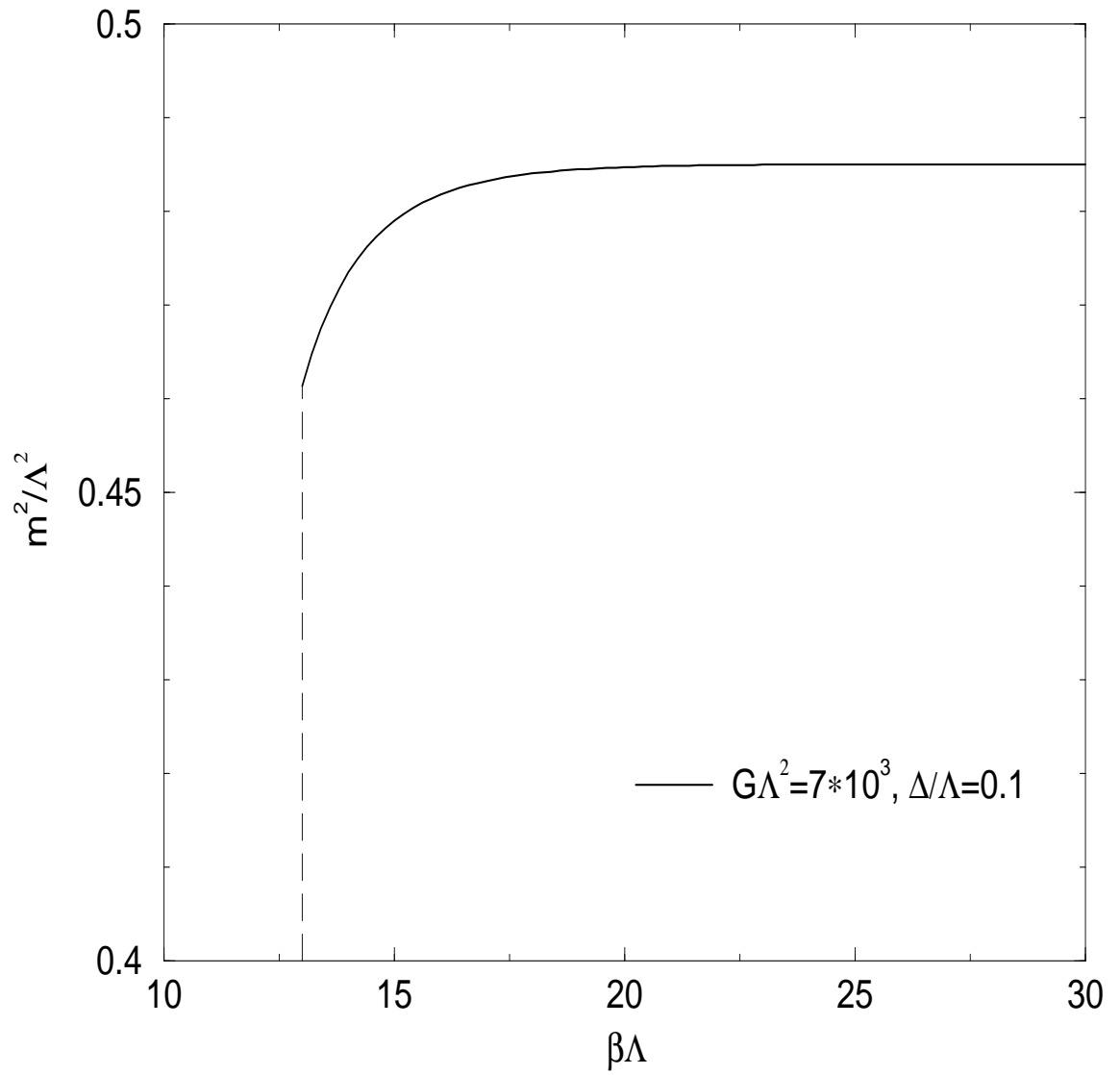
Figure 1:

Figure 2: