

Model of curvature-induced phase transitions in the inflationary universe

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Chiral phase transitions driven by space-time curvature effects are investigated in de Sitter space in the supersymmetric Nambu–Jona-Lasinio model with soft supersymmetry breaking. The model is considered to be suitable for the analysis of possible phase transitions in inflationary universe. It is found that a restoration of the broken chiral symmetry takes place in two patterns for increasing curvature: the first-order and second-order phase transition, respectively, depending on initial settings of the four-body interaction parameter and the soft supersymmetry breaking parameter. The critical curves expressing the phase boundaries in these parameters are obtained. Cosmological implications of the result are discussed in connection with bubble formations and the creation of cosmic strings during the inflationary era. [S0556-2821(99)50308-1]

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In the scenario of the early universe, the Higgs mechanism has been considered to be one of the possible candidates in explaining the onset of the inflation era. At the beginning of the inflation era, it is assumed that the grand unified theory phase is broken down to the quantum chromodynamics and electroweak theory phase through the Higgs mechanism. In this connection it is interesting to note that the Higgs fields may be composed of some fundamental fermions as in the technicolor model and to see the consequence of this idea in the scenario of inflation. On the other hand, supersymmetry is considered to be a vital nature possessed by the fundamental unified theory and, hence, the incorporation of supersymmetry in composite Higgs models seems to be of principal importance. Under these circumstances, it is natural for us to consider a supersymmetric composite Higgs model in the early stage of the universe and to see whether any remarkable effects are drawn during the inflation era. For simplicity we adopt the Nambu–Jona-Lasinio (NJL) model [1] as a prototype of the composite Higgs model in the present communication.

In the inflation era the quantum effect of gravitation is of minor importance, while the external gravitational field is non-negligible. Hence, we are naturally led to the supersymmetric NJL model in curved space. Dealing with the composite Higgs fields is essentially nonperturbative and does not accept approximate treatments. Accordingly, we try to solve the problem rigorously working in a specific space-time, de Sitter space, which possesses a maximal symmetry. de Sitter space is suitable for describing the inflationary universe. As a nonperturbative method, we rely on the $1/N$ expansion technique.

The four-fermion interaction model (which is the basis of the NJL model) in de Sitter space has been discussed by several authors [2–5] and is found to reveal the restoration of the broken chiral symmetry for increasing curvature as a second-order phase transition. The supersymmetric version of the NJL model in curved space was considered by Buchbinder, Inagaki, and Odintsov [6] in the weak curvature limit. They found that the chiral symmetry is broken as the curvature increases. Their result is in contrast with the result in the nonsupersymmetric NJL model. On the other hand, the supersymmetric NJL model in the flat space-time has been investigated by several authors [7] in the context of dynamical chiral symmetry breaking.

We start with the Lagrangian for the supersymmetric Nambu–Jona-Lasinio model in de Sitter space expressed in terms of component fields of the superfields [6]:

$$\mathcal{L} = -\nabla^\mu \phi^\dagger \nabla_\mu \phi - \rho^2 \phi^\dagger \phi - \nabla^\mu \phi^{c\dagger} \nabla_\mu \phi^c - \rho^2 \phi^{c\dagger} \phi^c - \bar{\psi}(i\nabla - \sigma + i\gamma_5 \pi)\psi - \frac{N}{2\lambda} \rho^2, \quad (1)$$

where we have kept only terms relevant to the leading order in the $1/N$ expansion. In Eq. (1) ϕ and ψ refer to the scalar and spinor component fields of the superfield, respectively, N is the number of components of these fields, ∇_μ the covariant derivative, λ the four-fermion coupling constant, and ρ the auxiliary scalar field with $\rho^2 \equiv \sigma^2 + \pi^2$. We introduce an additional term $\delta\mathcal{L}$ composed of nonminimal gravitational terms and a soft supersymmetry breaking term to the above Lagrangian (1):

$$\delta\mathcal{L} = -\xi_1 R \phi^\dagger \phi - \xi_2 R \phi^{c\dagger} \phi^c - \Delta^2 (\phi^\dagger \phi + \phi^{c\dagger} \phi^c), \quad (2)$$

where R is the space-time curvature, and ξ_1 , ξ_2 and Δ are coupling parameters. We assume that the term (2) existed already when the inflation era started.

The effective potential for the auxiliary field ρ is calculated in the leading order of the $1/N$ expansion such that [6]

$$V(\rho) = \frac{\rho^2}{2\lambda} + i \int_0^\rho ds \text{Tr} S(x, x; s) + 2i \int_0^{\rho^2} dt G(x, x; t), \quad (3)$$

where $S(x, y; s)$ and $G(x, y; t)$ represent the fermion and boson propagator in the coordinate space with mass s and \sqrt{t} , respectively. The effective potential (3) is obtained by taking the short-distance limit $y \rightarrow x$, once the full expressions of these propagators are found in de Sitter space.

The boson propagator in de Sitter space is well known [8–12] and is given for arbitrary dimension D by

$$G(x, y; t) = -i \frac{r^{2-D}}{(4\pi)^{D/2}} \frac{\Gamma(a_+) \Gamma(a_-)}{\Gamma(D/2)} \times {}_2F_1(a_+, a_-, D/2; 1-z), \quad (4)$$

with $a_{\pm} = [D - 1 \pm \sqrt{(D-1)^2 - 4tr^2}]/2$. Here $z = \sigma^2/(2r^2)$ with σ the geodesic distance and r the radius of de Sitter space which is related to the conventional Hubble parameter such that $r = 1/H$. The scalar propagator for Lagrangian $\mathcal{L} + \delta\mathcal{L}$ is obtained simply by replacing t by $t + \xi R + \Delta^2$ with $\xi = \xi_1 = \xi_2$ and $R = D(D-1)/r^2$ where we have taken $\xi_1 = \xi_2$ for simplicity. The fermion propagator is given by [13,14]

$$S(x, y; s) = (A(x, y; s) + B(x, y; s) \sigma_{;\mu} \gamma^{\mu}) U, \quad (5)$$

where

$$A(x, y; s) = i \frac{sr^{2-D} \Gamma(a) \Gamma(a^*)}{(4\pi)^{D/2} \Gamma(D/2+1)} \times \sqrt{1-z_2} F_1(a, a^*, D/2+1; 1-z), \quad (6)$$

and $a = D/2 + isr$ and U is the matrix composed of the Dirac matrices. We do not present the explicit expression of the invariant function $B(x, y; s)$, although it is known analytically. The reason is that we do not need the explicit expression of this function, since the second term on the right-hand side of Eq. (5) disappears for vanishing distance. We find for small geodesic distance $z \sim 0$ that

$$\begin{aligned} \text{Tr} S(x, x; s) &= \lim_{z \rightarrow 0} A(x, y; s) \text{Tr} U \\ &= \text{Tr}[1] \lim_{z \rightarrow 0} A(x, y; s), \end{aligned} \quad (7)$$

with the normalization $\text{Tr} U = \text{Tr}[1]$. Equipped with these propagators for bosons and fermions, we are now ready to calculate the effective potential (3) in an exact form.

In order to explain our idea in a transparent way, we mainly work in 3 space-time dimensions and give a brief comment on the full extension to 4 dimensions. We use a well-known formula which is found in any mathematical table (e.g., [15]) to rewrite the boson propagator and find for small geodesic distance $z \sim 0$,

$$G(x, y; t) = \frac{-i}{8\pi r z^{1/2}} + \frac{i}{4\pi r} \nu \coth \pi \nu + \mathcal{O}(z^{1/2}), \quad (8)$$

where we have defined $\nu = \sqrt{tr^2 + 6\zeta - 1}$ and $\zeta = \xi + \Delta^2 r^2/6$. Note here that we are calculating the boson propagator for Lagrangian $\mathcal{L} + \delta\mathcal{L}$. The fermion propagator at vanishing distance in 3 dimensions is obtained in a similar way. Using the relation (7), we have

$$\begin{aligned} \text{Tr} S(x, x; s) &= \text{Tr}[1] \left[\frac{is}{8\pi r z^{1/2}} - \frac{is}{4\pi r} \right. \\ &\quad \left. \times \left(\frac{1}{4} + s^2 r^2 \right) \frac{\tanh \pi s r}{s r} + \mathcal{O}(z^{1/2}) \right]. \end{aligned} \quad (9)$$

Substituting propagators (8) and (9) into effective potential (3), we find

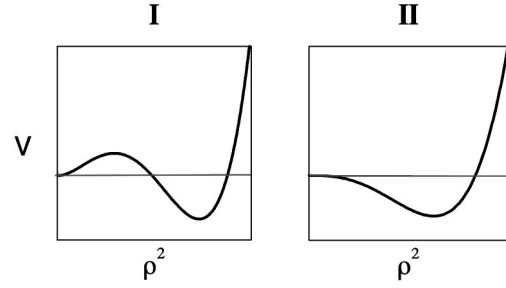


FIG. 1. Typical behaviors I and II of the effective potential.

$$\begin{aligned} V(\rho) &= \frac{\rho^2}{2\lambda} + \frac{1}{\pi r} \int_0^\rho ds s \left(\frac{1}{4} + s^2 r^2 \right) \frac{\tanh \pi s r}{s r} \\ &\quad - \frac{1}{2\pi r} \int_0^{\rho^2} dt \nu \coth \pi \nu, \end{aligned} \quad (10)$$

where we have set $\text{Tr}[1] = 4$ (we adopt the reducible representation of the Clifford algebra of Dirac matrices to afford the existence of γ_5). It is important to note here that the divergence present in both expressions (8) and (9) cancel out in the effective potential (10). The origin of this cancellation may be traced back to the supersymmetry of our model.

The gap equation is given by

$$\begin{aligned} V'(\rho^2) &= \frac{1}{2\pi r} \left[\frac{\pi r}{\lambda} + \left(\frac{1}{4} + \rho^2 r^2 \right) \frac{\tanh \pi \rho r}{\rho r} \right. \\ &\quad \left. - \sqrt{\rho^2 r^2 + 6\zeta - 1} \coth(\pi \sqrt{\rho^2 r^2 + 6\zeta - 1}) \right] = 0, \end{aligned} \quad (11)$$

where by $V'(\rho^2)$ we mean the differentiation of the effective potential with respect to ρ^2 . Just by observing the left-hand side of the gap equation (11), we find that it is a function only of ρr with parameters $\pi r/\lambda$ and ζ . Thus the solution ρr of this gap equation is completely specified by two parameters $\pi r/\lambda$ and ζ , respectively. This fact suggests that the phase diagram of the model will be given on a plane specified by these two parameters.

The direct observation of Eq. (11) shows that it has at most two solutions for ρr and so the shape of the effective potential (10) is of three types: the symmetric type S, and type I and II which are shown in Fig. 1. The phase diagram of the model is given by the numerical analysis of the effective potential (10) and the gap equation (11), and is given in Fig. 2. The region above the dashed line and the right half of the solid line in Fig. 2 represents a broken phase with the effective potential of the shape II given in Fig. 1. The small region between the solid line and the dashed line in Fig. 2 corresponds to a broken phase with the effective potential of the shape I in Fig. 1. The region below the whole solid line represents a symmetric phase where the shape of the effective potential is of the single-well type S.

The boundaries which separate the above three phases are determined as follows: The dashed line and the right half of

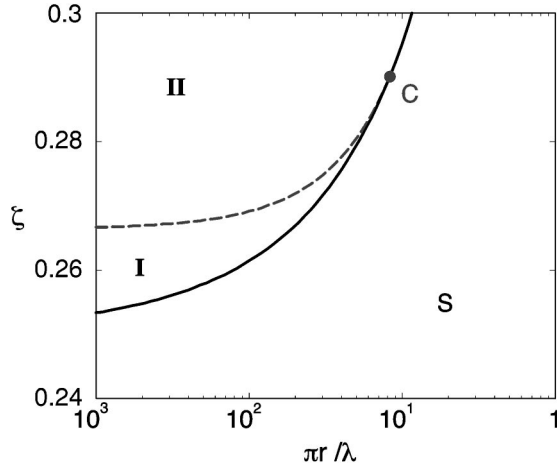


FIG. 2. The phase diagram on the parameter plane in ζ and $\pi r/\lambda$. The point denoted by C is the branching point where $(\pi r/\lambda, \zeta) = (\eta_*, \zeta_*)$ with $\eta_* = 0.083063$ and $\zeta_* = 0.290138$.

the solid line constitute the boundary of the region characterized by the potential of the shape II. The boundary is determined by the condition

$$V'(0) = 0. \quad (12)$$

The above equation reduces to

$$\frac{\pi r}{\lambda} + \frac{\pi}{4} - \sqrt{6\zeta - 1} \coth \pi \sqrt{6\zeta - 1} = 0. \quad (13)$$

The condition for the line separating the phases of type I and type S is given by $V(\rho_*) = 0$ where $\rho_* = \max\{\rho_1, \rho_2\}$ with ρ_1 and ρ_2 two solutions of the gap equation (11).

The branching point C in Fig. 2 is of special interest. It is a critical point which divides the broken phase into type I and type II. At the branching point C, the following conditions are found to be satisfied simultaneously:

$$V'(0) = 0, \quad V''(0) = 0. \quad (14)$$

The condition (14) is explicitly given by Eq. (13) and

$$1 - \frac{\pi^2}{12} + \frac{1}{2 \sinh^2 \pi \sqrt{6\zeta - 1}} - \frac{\coth \pi \sqrt{6\zeta - 1}}{2 \pi \sqrt{6\zeta - 1}} = 0. \quad (15)$$

From Eq. (15) we find that $\zeta = 0.290138 (\equiv \zeta_*)$ at the branching point C. By substituting this value for ζ in Eq. (13), we obtain $\pi r/\lambda = 0.083063 (\equiv \eta_*)$ at the branching point C.

Now let us discuss the time-evolution of the chiral structure of the model assuming that the inflationary era is well described by the effective potential in de Sitter space. It is natural to assume that the curvature slowly decreases (r increases) as the universe evolves. First, let us consider the case where the soft supersymmetry breaking term is not included, i.e., $\Delta = 0$. In this case it is easily seen in Fig. 2 that we move from left to right by increasing radius r with $\zeta (= \xi)$ and λ fixed. By direct observation of the gap equation, one can easily show that, if the parameter ξ is kept

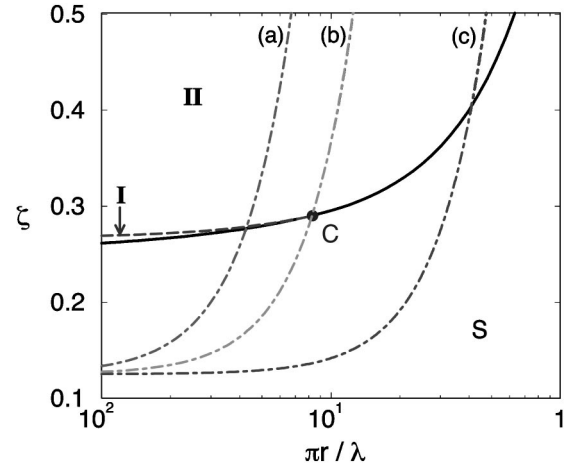


FIG. 3. Typical trajectories for varying r with Δ and λ fixed, (a) $\Delta\lambda/\pi = 22.3$; (b) $\Delta\lambda/\pi = 12.0$; (c) $\Delta\lambda/\pi = 3.2$. Here ξ is fixed so that $\xi = 1/8$.

below $\xi = 1/4$ which is the value of ξ corresponding to $\pi r/\lambda = 0$, the effective potential stays in the type S as r increases and, hence, the chiral symmetry is preserved (there is no phase transition). If the parameter ξ is kept in the region $1/4 < \xi < \zeta_*$, the effective potential changes its shape from type I to type S as r increases. Hence, the broken chiral symmetry is restored as r increases through the first-order phase transition. If the parameter ξ is kept above ζ_* , the effective potential changes its shape from type II to type S and so the transition is of the second order. Thus, in the case with $\Delta = 0$ the chiral symmetry restoration occurs as the universe evolves. This situation reflects the fact that the non-minimal gravitational coupling terms, which break the supersymmetry that protects the chiral symmetry for any value of λ and r , are effective only at large curvature. It is also interesting to examine the pattern of the phase transitions when the parameter ξ is changed with r and λ fixed. In this case we easily conclude that the chiral symmetry is broken by increasing ξ through the first order phase transition for $\pi r/\lambda < \eta_*$ while the transition is of the second order for $\pi r/\lambda > \eta_*$.

We next focus our attention on the role of the soft supersymmetry breaking terms. Here we assume the conformal gravitational coupling in 3 dimension, i.e., $\xi = 1/8$, for simplicity. This assumption is not essential for discussions in the following as long as $\xi < 1/4$. In Fig. 3 we show trajectories on the phase diagram as the radius r increases with Δ and λ fixed [see dot-dashed lines (a)–(c) in Fig. 3]. The trajectories are specified by the equation $\zeta = 1/8 + (\Delta\lambda/\pi)^2 \eta/6$ with $\eta = \pi r/\lambda$ for each value of $\Delta\lambda/\pi$. Increasing the radius r along the curve (a), one experiences a first-order phase transition and along the curve (c) a second-order one. The critical case is shown by the curve (b). These curves show that the chiral symmetry breaking can occur in the model with the soft supersymmetry breaking terms when the radius r increases or when the curvature of the universe decreases during the inflation. By observing the behaviors of these curves, we find that the types of the phase transitions are classified as follows: If $\Delta\lambda/\pi > \alpha (\equiv 29.5\sqrt{\zeta_*} - 1/8 = 12.0)$

is satisfied, the phase transition is of the first order. The case $1 < \Delta\lambda/\pi < \alpha$ corresponds to the second-order phase transition, and the phase transition does not occur if $\Delta\lambda/\pi < 1$ is satisfied.

In summary we have investigated the supersymmetric NJL model with the nonminimal gravitational coupling terms and soft supersymmetry breaking terms in de Sitter space. The phase structure is completely clarified in the model of 3 dimensions. We have found that both the nonminimal gravitational terms and the soft supersymmetry breaking terms lead to the phase transition phenomena as radius r increases or the curvature of the universe decreases. These two terms work in different ways. The soft supersymmetry breaking terms lead to the chiral symmetry breaking as the curvature decreases, while the nonminimal gravitational terms lead to the restoration of the chiral symmetry. The extension of our work to the 4 dimensional case is straightforward although we have to rely on the full numerical estimates. The result of the analysis will be presented elsewhere.

The cosmological consequences of the phase transitions found here seems to be quite interesting. A production of cosmic strings is expected because the $U(1)$ -symmetry is broken in the model. The formation of topological defects in the inflationary universe has been studied in many references (e.g., [16]). It is well known that the cosmic strings are constrained observationally as $G\mu \lesssim \mathcal{O}(10^{-5})$ where G is the

gravitational constant and μ is the line density of the cosmic string [17]. This condition sets a constraint on our model and it is an interesting problem to investigate whether the condition is satisfied in the model. In order to apply our model to a more realistic inflationary scenario, we need to deal with the model in 4 dimensions. At the moment we have found that the phase structure in 4 dimensions is very similar to that in 3 dimensions. The expression of the effective potential in 4 dimensions is rather cumbersome and the detailed analysis is still in progress.

Another comment is directed to the possibility of creation of an open universe in our model. Inflation models for an open universe have been investigated by using a bounce solution for a bubble formation [18]. The trajectory (a) in Fig. 3 indicates that bubble nucleations may occur in the region I. It is worthy to investigate whether our model can provide a successful model for an open universe or not.

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