

Dynamical fermion masses under the influence of Kaluza-Klein fermions in extra dimensions

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Abstract

The dynamical fermion mass generation in the 4-dimensional brane is discussed in a model with 5-dimensional Kaluza-Klein fermions in interaction with 4-dimensional fermions. It is found that the dynamical fermion masses are generated beyond the critical radius of the compactified extra dimensional space and may be made small compared with masses of the Kaluza-Klein modes.

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I. INTRODUCTION

It is an interesting idea to assume an existence of the extra-dimensional space which eventually compactifies leaving our 4-dimensional space-time as a real world [1]. The recent proposal [2,3] for the mass scale of the compactified space to be much smaller than the Planck scale gave a strong impact on the onset of studying phenomenological evidences of extra-dimensional effects. In recent approaches with extra dimensions it is usually assumed that the standard model particles reside in the 4-dimensional brane while the graviton may move around the bulk, the space-time with extra dimensions. In our present analysis we introduce bulk fermions in addition to the graviton and see what effects could be observed on the standard model particles. The bulk fermions interact with themselves as well as with fermions in the 4-dimensional brane through the exchange of the graviton and its Kaluza-Klein excited modes [4], or through the exchange of gauge bosons which may be assumed to exist in the bulk [5]. The interactions among fermions generated as a result of the exchange of all the Kaluza-Klein excited modes of the graviton or gauge bosons may be expressed as effective four-fermion interactions [4,5]. According to the four-fermion interactions we expect that the dynamical generation of fermion masses will take place.

In the present communication we look for a possibility of the dynamical fermion mass generation under the influence of the bulk fermions through the effective four-fermion interactions. Although our argument is applicable to any higher dimensional models, we confine ourselves to the 5-dimensional space-time for the convenience of explanations. In 5 dimensions fermion mass terms are forbidden if we require the symmetry under the chiral projection. The possible source of fermion masses in 4 dimensions is two-fold, i. e. the dynamically generated fermion masses and masses of the Kaluza-Klein excited modes of the bulk fermions. The mass of the Kaluza-Klein excited modes is known to be of order $1/R$ where R is the radius of the compactified fifth dimension.

We show in Sec. III that the mixing between the brane fermion and bulk fermion does not lead to the large mass of order $1/R$ for the fermion in 4 dimensions. We then consider in Secs. IV and V whether the dynamically generated fermion mass can be made small compared with the mass of the Kaluza-Klein excited modes. We calculate an effective potential for a composite operator composed of a fermion and an anti-fermion in the leading order of the $1/N_f$ expansion with N_f the number of fermion species. We find that the mass of fermions in the 4-dimensional brane is generated dynamically if the compactification radius R passes its critical value R_C and the phase transition associated with the mass generation is of second order. This means that the fermion masses in the 4-dimensional brane is small as far as the radius of the compactified fifth dimension is close to its critical value.

II. 5-DIMENSIONAL FERMION THEORY AND TORUS COMPACTIFICATION

We assume an existence of 5-dimensional bulk fermions ψ in interaction with fermions L on the 4-dimensional brane. Effective interactions among these fermions can be given in the form of the four-fermion interaction. We imagine that such effective interactions originate from the exchange of the bulk gravitons between fermions. In fact it is known that the exchange of the Kaluza-Klein excited modes of the bulk graviton results in effective

four-fermion interactions [4]. After the Fierz transformation on the four-fermion interactions we generate the transition-type interactions. Accordingly we start with the following Lagrangian for our model

$$\mathcal{L}^{(5)} = \bar{\psi}i\gamma^M\partial_M\psi + [\bar{L}i\gamma^\mu\partial_\mu L + g^2\bar{\psi}\gamma^M L \bar{L}\gamma_M\psi]\delta(x^4), \quad (1)$$

where g is the coupling constant with mass dimension $-3/2$ and index M runs from 0 to 4 while index μ runs from 0 to 3. Fermions ψ and L are assumed to be of N_f components.

It is easy to see that the Lagrangian is symmetric under the chiral projection $x^4 \rightarrow -x^4$, $\psi(-x^4) \rightarrow -i\gamma^4\psi(x^4)$ and $L(-x^4) \rightarrow -i\gamma^4L(x^4)$. Thus, if we impose this symmetry on a Lagrangian describing our fermion system, any fermion mass terms are forbidden thus resulting in the above Lagrangian. It should be noted here that, as is easily understood by referring to the Clifford algebra, an irreducible representation of the 5-dimensional fermion field is given by a 4-component field just as in the case of 4 dimensions. Hence we can use the same field both for the 4 and 5 dimensions. In odd dimensions it is well-known that there exists no object like γ_5 in 4 dimensions which commutes with all the γ matrices. Hence we do not have such object in 5 dimensions while the fifth component of the γ matrices in 5 dimensions, γ^4 , turns out to be $i\gamma_5$ in 4 dimensions.

For the later convenience we rewrite the above Lagrangian by using auxiliary field σ_M in the following equivalent form,

$$\mathcal{L}^{(5)} = \bar{\psi}i\gamma^M\partial_M\psi + [\bar{L}i\partial L - \sigma^M\sigma_M^* + (g\sigma_M\bar{\psi}\gamma^M L + h.c.)]\delta(x^4). \quad (2)$$

Since we are mainly interested in the dynamical fermion mass generation in the leading order of the $1/N_f$ expansion, we neglect the irrelevant terms in the Lagrangian by assuming $\langle\sigma_\mu\rangle = 0$ where μ runs from 0 to 3. After chiral rotation $\psi \rightarrow e^{i\frac{\pi}{4}\gamma_5}\psi$ and $L \rightarrow e^{i\frac{\pi}{4}\gamma_5}L$ we have

$$\mathcal{L}^{(5)} = \bar{\psi}i\partial\psi - i\bar{\psi}\partial_4\psi + [\bar{L}i\partial L - |\sigma|^2 + (g\sigma\bar{\psi}L + h.c.)]\delta(x^4), \quad (3)$$

where $\sigma = -\sigma_4$. The Lagrangian (3) is considered to be a composite-Higgs version of the mixing interaction adopted in Ref. [6–8].

We now consider that the space of the fifth dimension is compactified on a circle with radius R . By adopting the periodic boundary condition at $x_4 = 0$ and $x_4 = 2\pi R$ the bulk fermion field is expressed as a Fourier series,

$$\psi(x^\mu, x^4) = N \sum_{n=-\infty}^{\infty} \psi_n(x^\mu) e^{i\frac{n}{R}x^4}, \quad (4)$$

where N is the normalization constant.

We define the 4-dimensional Lagrangian in the following way,

$$\begin{aligned} \mathcal{L}^{(4)} &\equiv \int_0^{2\pi R} dx^4 \mathcal{L}^{(5)} \\ &= \sum_{n=-\infty}^{\infty} \bar{\psi}_n i\partial\psi_n + \sum_{n=-\infty}^{\infty} \frac{n}{R} \bar{\psi}_n\psi_n + \bar{L}i\partial L \\ &\quad - |\sigma|^2 + \left(m \sum_{n=-\infty}^{\infty} \bar{\psi}_n L + h.c. \right), \end{aligned} \quad (5)$$

where $m \equiv Ng\sigma$. Note that we have normalized the kinetic terms in Eq. (5) by choosing $N \equiv 1/\sqrt{2\pi R}$.

III. MASS SPECTRUM IN 4-DIMENSIONAL SPACE-TIME

In the following arguments we employ the matrix expressions

$$\Psi^t \equiv (L, \psi_0, \psi_1, \psi_{-1}, \psi_2, \psi_{-2} \cdots), \quad (6)$$

and

$$M \equiv \begin{pmatrix} 0 & m^* & m^* & m^* & m^* & m^* & \cdots \\ m & 0 & 0 & 0 & 0 & 0 & \cdots \\ m & 0 & \frac{1}{R} & 0 & 0 & 0 & \cdots \\ m & 0 & 0 & -\frac{1}{R} & 0 & 0 & \cdots \\ m & 0 & 0 & 0 & \frac{2}{R} & 0 & \cdots \\ m & 0 & 0 & 0 & 0 & -\frac{2}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (7)$$

By using Eqs. (6) and (7) we rewrite the mass term and mixing term in the 4-dimensional Lagrangian such that

$$\mathcal{L}_{\text{mixing}}^{(4)} = \sum_{n=-\infty}^{\infty} \frac{n}{R} \bar{\psi}_n \psi_n + \left(m \sum_{n=-\infty}^{\infty} \bar{\psi}_n L + h.c. \right) = \bar{\Psi} M \Psi. \quad (8)$$

If auxiliary field σ acquires a non-vanishing vacuum expectation value, we replace σ in m by its vacuum expectation value $\langle \sigma \rangle$, i. e. $m = Ng \langle \sigma \rangle$. The eigenvalues of matrix M determine the masses of 4-dimensional fermions. The eigenvalue equation is given by [7]

$$\begin{aligned} & \det(M - \lambda I) \\ &= \left[\prod_{j=1}^{\infty} \left(\lambda^2 - \left(\frac{j}{R} \right)^2 \right) \right] \left[\lambda^2 - |m|^2 - 2|m|^2 \lambda^2 \sum_{l=1}^{\infty} \frac{1}{\lambda^2 - (l/R)^2} \right] = 0. \end{aligned} \quad (9)$$

It should be noted that the solutions $\lambda = j/R$ obtained by setting the first factor in the middle of Eq. (9) to vanish is not the eigenvalues of Eq. (9) since they are canceled by the same factor in the denominator in the second factor. The real eigenvalues are obtained from the second factor in the middle of Eq. (9). The summation in the third term of the second factor in Eq. (9) can be performed as it is the Fourier series expansion of the cotangent function and thus the eigenvalue equation reduces to the simpler form [7],

$$\lambda R = \pi |mR|^2 \cot(\pi \lambda R). \quad (10)$$

We would like to see whether we have a possibility of getting the light solution in the eigenvalue equation (10). We confine ourselves to the case $|m| \ll 1/R$. By expanding the solution of Eq. (10) in powers of $|mR|$ we obtain [7]

$$\lambda_{\pm 0} R = \pm |mR| \left(1 - \frac{\pi^2}{6} |mR|^2 + \mathcal{O}(|mR|^4) \right), \quad (11)$$

$$\lambda_{\pm n} R = \pm n \left(1 + \frac{|mR|^2}{n^2} + \mathcal{O}(|mR|^4) \right) \quad (n \neq 0). \quad (12)$$

Obviously we find from Eqs. (11) and (12) that the lightest eigenvalue is given by

$$\lambda_{\pm 0} = \pm |m| \quad \text{for} \quad |m| \ll 1/R. \quad (13)$$

Thus we conclude that within our scheme there is a possibility of having the light fermion masses which is much smaller than the mass of the Kaluza-Klein modes of the bulk fermion. It should be noted here that in Eq. (13) we have two kinds of fermions with the positive and negative mass. These fermions are, however, indistinguishable since they have exactly the same properties. The next step that we have to proceed is to show that this light fermion mass is obtained as a result of the dynamical mass generation mechanism and can really be small.

IV. EFFECTIVE POTENTIAL FOR COMPOSITE FIELDS

We are now interested in the actual value of the lightest fermion mass in the 4-dimensional brane as is given in Eq. (13). Thus we have to study the dynamical mechanism of generating the non-vanishing vacuum expectation value of composite field σ which is involved in the expression $m = Ng \langle \sigma \rangle$. For this purpose we would like to calculate the effective potential for composite field σ .

Our 4-dimensional Lagrangian (5) is rewritten with the matrix representation introduced in the last section as follows,

$$\mathcal{L}^{(4)} = \bar{\Psi}(M + Ii\cancel{\partial})\Psi - |\sigma|^2. \quad (14)$$

The generating functional Z for our system is given by

$$Z = \int [\mathcal{D}\bar{\Psi}][\mathcal{D}\Psi][\mathcal{D}\sigma][\mathcal{D}\sigma^*] e^{i \int d^4x \mathcal{L}^{(4)}}. \quad (15)$$

By performing the path-integration for fermion field Ψ in Eq. (15) we find in the leading order of the $1/N_f$ expansion,

$$Z = \int [\mathcal{D}\sigma][\mathcal{D}\sigma^*] e^{-i \int d^4x V(\sigma)}, \quad (16)$$

$$V(\sigma) \equiv |\sigma|^2 - \int \frac{d^4k}{i(2\pi)^4} \ln \det(M + I\cancel{k}), \quad (17)$$

where V is the effective potential for the auxiliary field σ . By performing the Wick rotation in the momentum integration in Eq. (17) we rewrite the effective potential such that

$$V(\sigma) = |\sigma|^2 - \frac{1}{8\pi^2} \int_0^{\Lambda^2} d(k_E^2) k_E^2 \ln \det(M^2 + I k_E^2), \quad (18)$$

where k_E stands for the Euclidean momentum and Λ is the momentum cut-off. Note that in addition to the divergence in the momentum integration the divergence in the Kaluza-Klein mode sum shows up in general. In our 5-dimensional model this divergence is not present while, if we start from the space-time dimensions higher than six, the divergence does exist and we have to rely on the regularization method developed recently in Ref. [9]. The same argument applies to the case where we derived Eq. (10).

After some calculations we find that

$$V(\sigma) = |\sigma|^2 - \frac{1}{2\pi^2} \int_0^\Lambda dx x^3 \ln \left[x^2 + |m|^2 (\pi x R) \coth(\pi x R) \right] - \frac{1}{2\pi^2} \sum_{j=1}^{\infty} \int_0^\Lambda dx x^3 \ln \left[x^2 + \left(\frac{j}{R} \right)^2 \right]. \quad (19)$$

The gap equation to determine the vacuum expectation value $\langle |\sigma| \rangle$ of $|\sigma|$ reads

$$\frac{\partial V(\sigma)}{\partial |\sigma|} = 2|\sigma| \left\{ 1 - \frac{g^2}{2\pi^2} \int_0^\Lambda dx \frac{x^3}{2x \tanh(\pi x R) + g^2 |\sigma|^2} \right\} = 0. \quad (20)$$

By numerical observation of Eq. (20) we find that there exists a non-trivial solution for $|\sigma|$ for a suitable range of parameters g and R and the solution corresponds to the true minimum of the effective potential. Accordingly the fermion mass is generated dynamically. Here the auxiliary field σ (or the composite field $\bar{L}\psi$) acquires a vacuum expectation value. The continuous symmetry that is broken by this dynamical process is the U(1) symmetry existed in the original Lagrangian (1):

$$\psi \rightarrow e^{i\beta} \psi, \quad L \rightarrow e^{-i\beta} L. \quad (21)$$

Moreover the phase transition associated with this symmetry breaking is of second order as is seen in Fig. 1.

In the case of the second order phase transition the critical radius may be obtained by solving the equation derived from Eq. (20) by setting $|\sigma| = 0$,

$$1 - \frac{g^2}{4\pi^2} \int_0^\Lambda dx \frac{x^2}{\tanh(\pi x R)} = 0. \quad (22)$$

Eq. (22) determines the relation between R and g with the suitable choice of the cut-off parameter Λ .

It is interesting to note here that, if we started from 3 dimensions instead of 5 dimensions and regarded the 2-dimensional world as a physical world, we would have obtained an equation for the critical radius corresponding to Eq. (22) as follows,

$$1 - \frac{g^2}{2\pi} \int_\lambda^\Lambda dx \frac{1}{\tanh(\pi x R)} = 0, \quad (23)$$

where parameter λ is the cut-off of the low momentum integration which is needed to remove an infrared divergence created by letting $\sigma \rightarrow 0$. The integration in Eq. (23) is easily performed and we find that

$$\frac{g^2}{2\pi^2} = R / \ln \frac{\sinh(\pi \Lambda R)}{\sinh(\pi \lambda R)}. \quad (24)$$

Eq. (24), if it is inverted, gives us a formula for the critical radius as a function of the four-fermion coupling constant. Similarly Eq. (22) gives us a relation between R and g if we solve the equation numerically.

V. GENERATION OF DYNAMICAL FERMION MASSES

As was shown in Eq. (13) the lowest fermion mass on the 4-dimensional brane is $m = Ng < |\sigma| >$ where $< |\sigma| >$ is determined by solving Eq. (20). By using the numerical estimation we calculate the fermion mass m as a function of radius R for fixed coupling constant g . The result is shown in Fig. 2 where m , R and g are normalized by the cut-off parameter Λ . It is immediately recognized that the fermion mass generation takes place below the critical radius and so is kept small near the critical radius. The critical curve which represents the critical radius as a function of the coupling constant is shown in Fig. 3 as the curve for $m = 0$. It is easy to see that $g^2\Lambda^3 = 12\pi^2$ as $R \rightarrow \infty$ on the critical curve.

In our model the bulk fermions reside in the 5-dimensional space-time and their Kaluza-Klein modes show up as fermions with masses n/R in the 4-dimensional space-time. Since those fermions are not observed in the present experimental situation except for the zero mode which mixes with the fermion on the brane, their masses have to be very high and hence $1/R$ should be much higher than several TeV. Thus the compactification scale R for the fifth dimension is considered to be very small. On the other hand the bulk graviton lives in the higher dimensional space-time and the compactification scale $1/R_G$ for the extra-dimensional space associated with the bulk graviton could be smaller than $1/R$ as is suggested in Ref. 2. Our cut-off scale Λ is introduced to suppress the divergence appearing in the integration in the effective potential. Hence it is to be determined by the region of the validity of our effective four-fermion theory. We suppose that $\Lambda > 1/R$ since the fundamental theory which derives our effective theory is realized at much higher scale than the compactification scale for the bulk fermions. In Fig. 3 we recognize that, if $g^2\Lambda^3 \sim 12\pi^2$, the fermion mass is kept small for a wide range of radius R except for the small R region.

VI. CONCLUSIONS

We have found within our model that in spite of the presence of the large mass scale $1/R$ in the theory the fermion masses on the 4-dimensional brane can be made small as a consequence of the interaction among the bulk and brane fermions: the mixing of the brane fermions with the bulk fermions does not lead to the lightest fermion masses of order $1/R$ and also the dynamically generated fermion masses are not of order $1/R$. This result is obtained because the dynamical fermion masses generated under the second-order phase transition are small irrespectively of $1/R$ near the critical radius. In our model the possibility of having low mass fermions resulted from the dynamical origin. This mechanism is quite different from the ones in other approaches in which low mass fermions are expected to show up as a result of the kinematical origins [6–8,10,11].

It is tempting to make a final comment on the conjecture that the nature chooses the compactification scale near the critical radius so as to have low mass fermions. In this connection it may be interesting to suppose that the existence of the critical radius is a result of the physical process that the compactification of the fifth dimensional space is driven by the force unknown to us and this force is balanced by the pressure coming from the fermionic Casimir energy [12,13].

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FIGURES

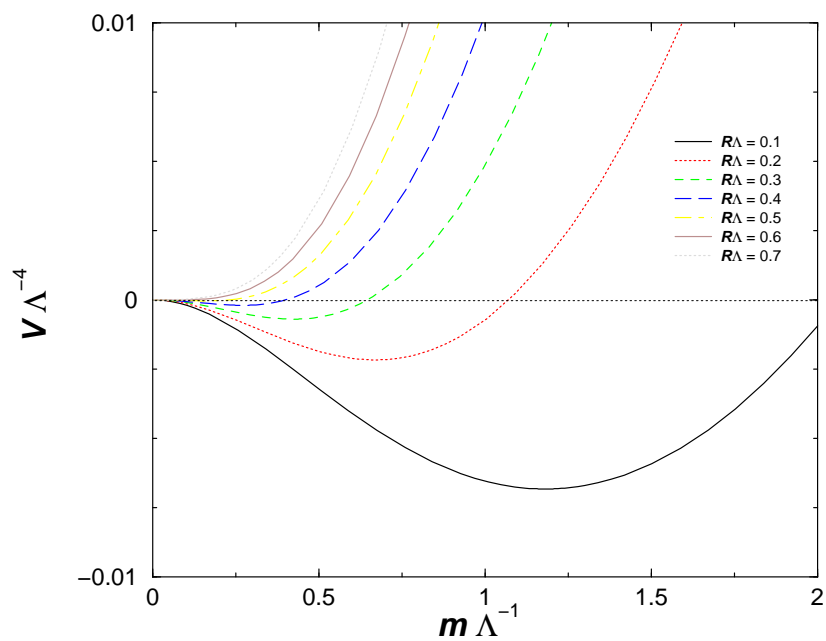


FIG. 1. Typical behavior of the effective potential

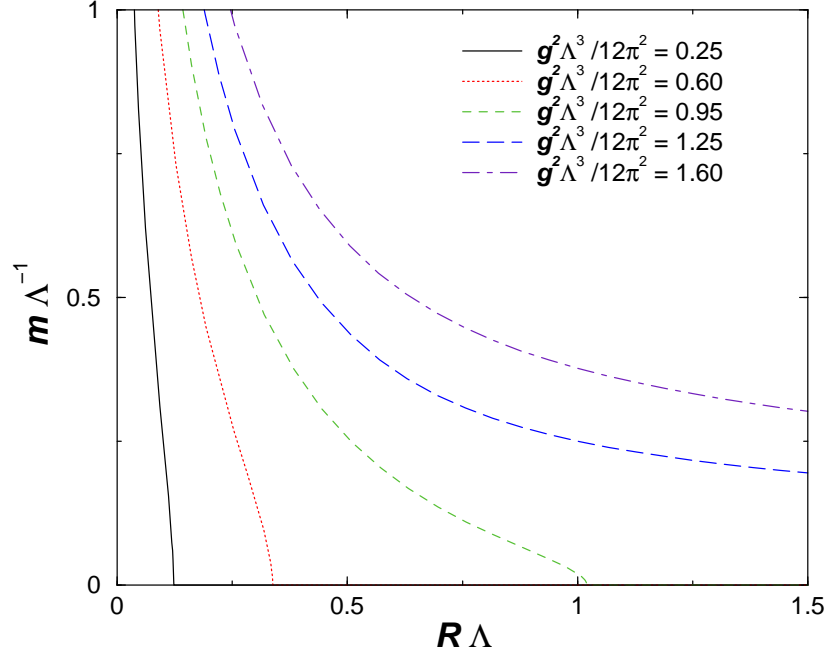


FIG. 2. Dynamical fermion mass as a function of R with g fixed.

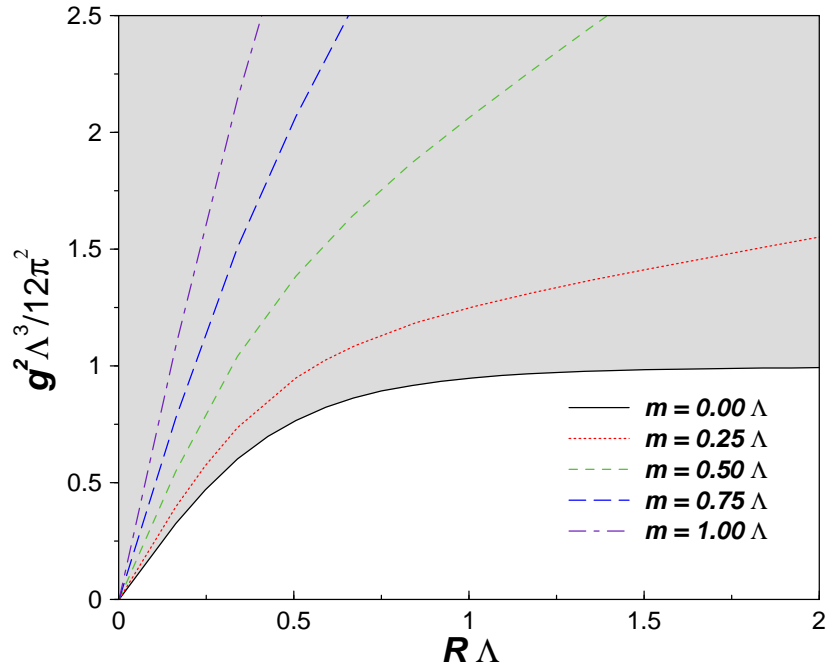


FIG. 3. Critical radius as a function of g .