A Study on Model-Based Performance Evaluation of Dependable Computer Systems
（ディペンダブルコンピュータシステムのモデルベース性能評価に関する研究）

Dissertation submitted in partial fulfillment for the degree of Doctor of Engineering

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Abstract

Model-based performance evaluation is an analytical approach for quantifying system performance indices by modeling the dynamic behavior of system as stochastic processes. Queueing analysis and Markov modeling are typical methods for the model-based performance evaluation. Compared to simulation approach, the model-based approach can provide highly-accurate assessments of performance measures. Therefore, this is commonly used to evaluate performance criteria of the system in the presence of rare events such as system failures.

On the other hand, as software system is widely used in our daily lives, it becomes more important to prevent fatal system failures in computer system design. In other words, several dependability measures such as system reliability, system availability and data integrity, should be estimated only from the design of system architecture before starting the implementation. This approach is called the model-based software performance analysis, which is one of the most attractive topics even in software engineering.

In this thesis, we consider model-based performance evaluation of three kinds of computer system; distributed database system, open-source software and virtualized system, respectively.

Firstly, we focus on the performance evaluation of distributed database systems with conventional snapshot isolation (CSI) and prefix-consistent snapshot isolation (PCSI) by considering the occurrence of communication failures between a master and replicas. We also revisit our probabilistic models for CSI and PCSI with the restart scheme for the communication failure. We investigate the effect of update interval of snapshot and restart timing for the communication with respect to the abort probability and system throughput.

Secondly, we turn our attention to the maintenance scheduling for open source software products. Applying a patch is one of effective fault-tolerant techniques. We consider an optimized patch management model from the perspective of users by applying an NHPP to the bug-discovery process. Also, with analyzing the characteristic of open source software by applying software reliability models, we predictively propose an optimal maintenance schedule for user according to numerical illustrations.

Finally, we dedicate our interest to the performance evaluation of virtualized
system with software rejuvenation. Concretely, we evaluate the virtualized system by using the criterion of resiliency, which is an attitude for measuring the deviation of system when changes happen. We present MRSPNs (Markov regenerative stochastic Petri nets) for the virtualized system with cold and warm software rejuvenation and employ the technique of transient analysis through PH (phase-type) expansion. This technique can reduce MRSPNs to a CTMC approximately. After applying PH expansion to MRSPNs for the virtualized system with software rejuvenation, we present the quantitative measure to evaluate the system resiliency based on CTMC analysis.
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Chapter 1

Introduction

Model-based performance evaluation is an analytical approach for quantifying system performance indices by modeling the dynamic behavior of system as stochastic processes. Queueing analysis and Markov modeling are typical methods for the model-based performance evaluation. Compared to simulation approach, the model-based approach can provide highly-accurate assesses of performance measures. Therefore, this is commonly used to evaluate performance criteria of the system in the presence of rare events such as system failures.

On the other hand, as software system is widely used in our daily lives, it becomes more important to prevent fatal system failures in computer system design. In other words, several dependability measures such as system reliability, system availability and data integrity, should be estimated only from the design of system architecture before starting the implementation. This approach is called the model-based software performance analysis, which is one of the most attractive topics even in software engineering.

In this thesis, we consider model-based performance evaluation of three kinds of computer system; distributed database system, open-source software and virtualized system, respectively.

1.1 Optimization of Prefix-consistent Snapshot Isolation

In database system including distributed database system, there are four properties; (atomicity, consistency, isolation, durability), to guarantee the data integrity of database.[28] Atomicity property guarantees each transaction handled
entirely instead of partially. Consistency property ensures that any transaction will bring the database from one valid state to another. Isolation property is defined as invisibility of operations of a transaction to the other transactions. Durability property guarantees that transactions that have committed will survive permanently. Since the isolation property critically affects throughput of transactions in database system, we should take account of the trade-off between isolation level and system performance. When attempting to maintain the highest level of isolation, a DBMS usually acquires locks on data which may result in a loss of concurrency. The most basic implementation of the isolation is based on a lock of data when it is read or written. However, it is known that the lock-based isolation adversely affects the performance of database system. Recent DBMSs adopt a more effective method using snapshot, so-called snapshot isolation.\cite{9} In the snapshot isolation, each transaction makes a copy (snapshot) of the data at the beginning of a transaction, and updates the data on the snapshot, instead of the original data. After the update operation, each transaction sends a request for updating the original data, the DBMS processes the requests according to the first-committer-wins rule. It has an advantage of the abort probability of a transaction to the lock-based isolation. As a novel isolation scheme, snapshot isolation has received much attention in various fields. Ramesh et al. have implemented snapshot isolation for achieving atomicity in multi-row transactions using RDBMS.\cite{56} Also, snapshot isolation has been utilized for management of scalable transactions on cloud computing platforms.\cite{52}

On the other hand, since the snapshot isolation causes an overhead for getting snapshot, the performance seriously degrades in the case of the system with large latency for getting snapshot such as distributed database system. To overcome this problem, Elnikety et al. proposed generalized snapshot isolation to mitigate the overhead for getting snapshot.\cite{18} Concretely, they presented an implementation of generalized snapshot isolation called a prefix-consistent snapshot isolation (PCSI), which periodically updates snapshot for a fixed time interval, and examined the effectiveness of PCSI through the simulation study. Bernabé-Gisbert and Zuikivicūtė also discussed the performance of PCSI based on a model.\cite{10} They examined that PCSI improved response time for a transaction. On the other hand, PCSI essentially degrades system throughput and
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abort probability of a transaction unless the time interval for updating snapshot is optimized. Neither of [18] and [10] discussed such the optimization of the design parameters of PCSI.

1.2 Maintenance Strategy of Open Source Software

Nowadays, as well as the growing scale of software, the development cost is rapidly increasing. Enterprises and individuals incline to update software instead of purchasing new products. Therefore, the life cycle of software gets longer and the burdens of maintenance and modification become significant. At the same time, open source software becomes popular because of the price and the modifiability. Many communities have been comprised for better communications between developers and users, and it also leads to swift feedbacks when bugs or vulnerabilities are discovered. Combining these two aspects, we are interested in the maintenance strategy of open source software.

The development of open source software is different from traditional software. Traditional (closed source) software is usually developed by firms (vendors) who are responsible for all the phases of software life cycle. That means the maintenances and updates are provided by vendors unilaterally, even after releasing. In the other hand, the quality enhancements of open source software are promoted by both users and developers. Users have rights and interests of reporting an experienced software failure to developers who are in charge of modifying the potential bug related to the failure. Thus, the maintenance of open source software can be thought as lots of bug fixes and several version-ups. It is essentially the same as the maintenance of closed source software, but the life cycle of open source software is much longer. The difference is, bugs are reported by users not testers. However, from the point of view of bugs features, there is few distinctions between open source and closed source software.

To be a user of open source software, besides reporting bugs to developers after experiencing failures, they also have to keep their own duplication up-to-date. The maintenance of open source software, however, are considered as traditional means, i.e., patches and updates. A patch is a part of program designed to fix bugs with, update a computer program or the supporting data,
and improve the usability or performance. One patch is always released by
vendor/developer with updates against one or several reported bugs or security
vulnerabilities. That is saying the patch releasing is strongly related with the
discoveries of bugs or vulnerabilities.

Generally, for the sake of reliability and security of software, patches should
be developed immediately users discover bugs. However, the development of
patches incur time and labor expenses for developers/vendors. For the closed
source software, vendors plan to release patches at a specified time interval,
and the patches fix all the bugs which have been discovered until the release
time. Due to the limitation of number of developers, the patches of open source
software cannot be developed as soon as the bugs discovered neither. That is
the motivation of discussion to the management of patches/updates.

Cavusoglu et al. have proposed a formulation for the cost of development and
distribution of patches, and also discussed the optimized periodic patch release
from the vendor perspective based on the cost function with respect to a steady-
state criterion.[14, 13] As previous research, Okamura et al. have discussed a
patch release strategy from the perspective of vendors by cost criterion.[47] Their
model assumed a NHPP as the number of discovered bugs and formulated the
expected total cost by considering the damage of exploit bugs before and after
a patch release. However, these papers have just discussed about what is the
best timing for releasing patches from the vendors’ angle. By investigating the
behavior of users, we can realize that not all patches are applied immediately
they are released. For convenience, users may occasionally apply patches even
it will bring much more risk of bug occurring. Also, less patch applying means
less cost of software update.

1.2.1 Related Works

In the past decades, software reliability models (SRMs), which describe and
analyze the software bug-discovery phenomenon, have been discussed in litera-
tures extensively.[36, 43] The classical and most important SRMs may be the
non-homogeneous Poisson process (NHPP) models developed by Goel and Oku-
moto, which describe the stochastic behavior of detecting potential bugs in the
test phase and predict the software reliability easily.[22] After that, Goel[23],
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Yamada et al.[70] and others[36, 42, 1] have proposed the different NHPP-based SRMs. These SRMs are based on different debugging scenarios, and can qualitatively catch the software reliability growth phenomenon in test phases of software products.

On the other hand, because the development of available patches are governed by bug-discovery and bug-correction processes, the consideration of bug-correction process is essential for analyzing maintenance strategy of open source software, particularly from users’ perspective. However, almost all the NHPP-based SRMs assume the bug-correction process instantaneously, and only focus on the bug-discovery phenomenon. Schneidewind[59, 60] and Gokhale et al.[24, 25] have discussed some concepts of software bug-correction phenomenon, and explained the dependency between the bug-discovery/correction profiles with analytical models and real data. Shibata et al.[61] have developed a novel approach to analyze the software bug-discovery/correction processes simultaneously. Their idea to describe the bug-discovery/correction processes is to use an $M_t/M/\infty$ queueing model, where a fault is discovered and corrected by exponentially distributed random detection and repair times, respectively. Based on queueing theory[11], they have assumed that the number of corrected bugs can be modeled as an output process of an infinite server queue.

In addition, several mathematical models of patch management have been proposed. Arora et al.[5] have considered the optimized plan for vulnerability exposures over a software life cycle. Beattie et al.[7] have focused on the risk that a patch re-injects a failure, and considered the optimal timing to apply patches maximizing system availability. However, they have not dealt with the properties of the discovery process. Cavusoglu et al. [14, 13] proposed the patch release and management models from both vendor and user perspectives. In addition, they discussed an equilibrium point of patch release times based on a game theory between a vendor and a user. As an extension of Cavusoglu et al.[14, 13], Okamura et al. have described a patch management model by using non-homogeneous bugs-discovery process.[47] They have discussed an optimal algorithm for minimizing the damage cost only from vendor perspective. Apart from mathematical models, Brykczynski and Small reported practices of security patch management, and also emphasized an importance of economic security
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patch management as a part of information asset management.[12]

Moreover, we talk about the optimizing of maintenance scheduling. Marseguerra et al. have presented an optimization approach based on the combination of a Genetic Algorithms maximization procedure with a Monte Carlo simulation, and a stochastic model of plant operation is developed from the standpoint of its reliability/availability behavior.[39] Xia et al. have discussed the maintenance scheduling with consideration of multi-attribute model (MAM). They applied advance-postpone balancing (APB) and Maintenance time window (MTW) methods to utilize maintenance opportunities.[67, 68] Also, towards the distributed energy system, they have developed a model-iteration algorithm to optimize the maintenance schedule cycle by cycle.[69] Although these works were done for hardware system, the methodologies for optimization of maintenance schedule is worth to be referred.

1.3 Resiliency of Virtualized Systems

Virtualized system is one of the most flexible architectures to manage many servers. In recent years, the virtualized system is used to build not only the enterprise cloud system but also the private cloud which provides services for its own staff in a small office. The virtualized system has the two-tier managements for virtual machines (VMs) and virtual machine monitor (VMM). The VM is an approach of virtualization which behaves as a real computer. It is to create software components by emulating behavior of hardware units, and to control them in a software platform. VM can make multiple OS environments without redundant hardware cost. And a running virtual machine can be moved between different physical machines without disconnecting the client. On the other hand, VMM is the monitoring system to check behavior of VMs running on VMM. In general, since administrators check VMs through VMM, the cost for management can be reduced. In addition, VMM is generally installed on a physical machine, and VMs shares the resources of physical machines such as CPU, memory and network. Thus the virtual machine is also said to be good for the efficiency as well as the management cost. However, although the maintenance for VMs is easy in the virtualized system, it is not easy to make a maintenance plan for VMM because VMM should continuously run while VMs provide their services.
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That is, the aging phenomenon of VMM is serious problem in the virtualized system.

In [37, 38], Machida et al. discussed software rejuvenation policies in a virtualized system. The software rejuvenation is a proactive maintenance to avoid the system failure caused by aging-related bugs.[29] The aging-related bugs are defined as the bugs that cause the system performance degradation by long time usage of the system. Typical examples of aging-related bugs are memory leak and fragmentation. They are also reported on the virtualized platform such as Xen. In general, it is difficult to find the root cause of aging-related bugs, and we empirically know that the proactive maintenance such as reboot and restarting processes is effective to prevent the system failure by aging-related bugs. Such maintenance is generally called software rejuvenation.[31] The software rejuvenation operation is often managed by a time-triggered policy, i.e., the system automatically execute the rejuvenation operation under scheduled time. Machida et al.[37, 38] discussed the steady-state measures under three kinds of rejuvenation policies called cold-VM, warm-VM and migrate-VM rejuvenations in the virtualized system. Also, Okamura et al.[49] evaluated the transient measures for the virtualized system with software rejuvenation.

The software aging is caused by the resource exhaustion with long-term operation. On the other hand, it is also important to evaluate the system performance when a suddenly change occurs in practice. The change includes changes of system configuration and environments of system such as rate of arrivals. The extreme example is a disaster like flood, fire and earthquake. To avoid the worst scenario, it is useful to predict what happens in the system if the system suffers such disasters. The resiliency is known as one of the attributes of system to be evaluated for such changes. In brief, the resiliency is a resistance of system for a change. In the research field of dependability computing, many definitions of resiliency were proposed to characterize this concept. Laprie[33] and Simoncini[62] defined the resiliency as the resistance of service delivery when facing changes. In addition, based on this definition, Gohsh et al.[21] tried to quantify the system resiliency in IaaS cloud through the modeling of system with continuous-time Markov chains (CTMCs).
1.4 Organization of Dissertation

This thesis is organized as follows:

Firstly, in Chapter 2, we focus on the performance evaluation of conventional snapshot isolation (CSI) and PCSI based on probabilistic models and discuss the optimization of the design parameter of PCSI by using the probabilistic models for the distributed database system. In [35], the model-based performance evaluation of CSI and PSCI with the optimization has been discussed. However, [35] assumed that any failure did not occur during the communication between a master and replicas. Generally, in distributed environment, communication failures are non-negligible factors that negatively affect the performance of systems in the distributed environment. Thus, one of the purpose of this chapter is to reveal the impact of communication failures to the system performance. Concretely, this chapter revisits our probabilistic models for CSI and PCSI with the restart scheme for the communication failure which discussed by van Moorsel and Wolter.[40] We investigate the effect of update interval of snapshot and restart timing for the communication with respect to the abort probability and system throughput.

In Chapter 3, we consider a patch management model from the perspective of users by applying an NHPP to the bug-discovery process. We focus on another part of the patch management game by Cavusoglu et al. from user perspective.[14, 13] We expand the time-based patch release policy proposed by Okamura et al. into a number-based policy and formulate the total cost from user perspective.[47] Also, with analyzing the characteristic of open source software by applying software reliability models, we predictively propose an optimal updating schedule for user according to numerical illustrations.

In Chapter 4, we focus on the quantification of resiliency of the virtualized system with software rejuvenation. Concretely, according to the manner of [21], we evaluate the resiliency of the virtualized system. However, since Machida et al.[37, 38] originally presented MRSPNs (Markov regenerative stochastic Petri nets) for the virtualized system with software rejuvenation, we could not apply the same approach as [21] to the virtualized system model directly. Then we employ the technique in [49], i.e., the transient analysis through PH (phase-type)
expansion. This technique can reduce MRSPNs to a CTMC approximately. After applying PH expansion to MRSPNs for the virtualized system with software rejuvenation, we present the quantitative measure to evaluate the system resiliency based on CTMC analysis.

Finally, the thesis is concluded with some remarks and future directions in Chapter 5.
Chapter 2

Performance Evaluation of Snapshot Isolation in Distributed Database System

Database systems are widely used in many fields. As the scale of database systems increases, more firms decide to deploy the database system in a distributed environment for the sake of data integrity fault tolerance. For guaranteeing the consistency of data, database management systems adopt isolation policies. This chapter discusses probabilistic models for snapshot isolation of database management system. Snapshot isolation is an effective method to enhance the consistency of database system. Although, it degrades the system performance where the system has large network latency such as distributed database system. Also, under the failure-prone environment, a restart scheme is considered as one countermeasure. This chapter proposes probabilistic models for the dynamics of snapshot isolation of database system and exhibits the optimization of system performance with respect to updating interval of snapshot isolation within the failure-prone environment from the analytical point of view. Numerical experiments are conducted to validate the effectiveness of analytical results by using real traffic data.


2.1 Database Management System

Database management system (DBMS) is a fundamental software to control transactions that arrive at the database system. In DBMS, there are four significant properties abbreviated as ACID (atomicity, consistency, isolation, durability). The atomicity is the property ensuring that either all or none of the operations have been executed when a transaction is completed. This corresponds to the commit/abort scheme. The consistency is to ensure that a transaction does not change the database to any of inconsistent states whenever the state of database is consistent at the beginning of the transaction. The isolation is to make all the operations of a transaction invisible to other transactions. The durability is to ensure that all the successful transactions must persist even though a system failure occurs. In particular, since the isolation affects the performance of database, four isolation levels are defined in ANSI/ISO SQL standard; read uncommitted, read committed, repeatable reads and serializable. The serializable is the highest level of isolation. This level requests starting and ending times of all the transactions to be serial in the execution history of transactions. In this level, we prevent any of inconsistency such as dirty reads, non-repeatable reads and phantom reads [9].

This chapter focuses on the serializable level of isolation. There are two implementation schemes to achieve the serializable level. One method is the lock-based control. In the lock-based control, DBMS acquires locks on the data during the transaction is executed. The implementation of lock-based control is simple. However, when a transaction spends long processing time, the performance of database such as system throughput is drastically degraded, since all the transactions that arrive in the processing time are canceled or aborted. Another implementation is based on the version of database, called the version-based control or the snapshot isolation (SI). In the SI, DBMS does not acquire a lock on the data. Instead of locks, DBMS makes a snapshot of database before starting a transaction, and checks write conflicts on the data by using the snapshot at which each transaction is completed. That is, the transaction that has no conflict with other transactions is only committed at the termination (first-committer-wins rule [9]). Compared with the lock-based control, the SI is expected to provide high system throughput.
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On the other hand, the SI poses a problem with response time overhead to get the last snapshot. In the conventional snapshot isolation (CSI), since DBMS should get the latest snapshot before processing a transaction, it can be equated to the fact that the processing time of a transaction under the SI is longer than that under the lock-based control. Specifically, in the case where the SI is applied to distributed database system with a master server, the network latency to get the latest snapshot should be considered as a part of response time. To overcome this problem, Elnikety et al. [18] proposed the generalized snapshot isolation (GSI) which allows us to use the ‘potentially’ latest snapshot. Under the first-committee-wins rule, the first transaction committed on a snapshot is a winner on the snapshot. Thus the probability of commit generally depends on the age of snapshot at which the transaction starts. The probability of commit with ‘young’ snapshot is higher than that with ‘older’ snapshot, because the ‘older’ snapshot is likely to be changed by other transactions. In other words, if we could maintain a certain level of the probability of commit, it is not necessary to use the latest snapshot. Elnikety et al. [18] focused on such the property of SI, and proposed an implementation of GSI called prefix-consistent snapshot isolation (PCSI) to enhance the response time. In the PCSI, the process to get the latest snapshot is executed at every prefixed time interval. Figures 2.1 and 2.2 illustrate communication between the master and the replica in conventional SI (CSI) and PCSI.

However, one of the transactions on the same snapshot may be committed under the first-committee-wins rule. In the PCSI, a snapshot is used for all the transactions that arrive during the prefixed time interval for updating snapshot, namely, the number of transactions using the same snapshot in the PCSI is larger than that in the CSI. It indicates that the system throughput of PCSI is lower than that of CSI. This motivates us to consider the optimal time interval for updating snapshot in the PCSI.

2.2 Latency Distribution in Failure-Prone Environment

As shown in the previous section, Elnikety et al. [18] have extended the snapshot isolation scheme to the distributed database system with innegligible latency of
communication between replicas and master. Their scheme implicitly assumed that a replica can get a response from the master with probability one. However, when we focus on the network/transmission layer in the network model, this assumption does not hold in any network environment. M. Tamer Özsu et al. have discussed fundamental principles of distributed data management and data management in different network architectures [51]. Though TCP (transmission control protocol) equips the retransmission scheme to ensure the end-to-end communication, the latency of communication strongly depends on the communication environment. Richard L. Graham et al. have detailed the features of an end-to-end network failure-tolerant message-passing system [27]. In particular, in the case of failure-prone environment, it is useful to design both high and low layers by considering the effect of retransmission scheme in
low layer to the latency distribution in high layer. Hsieh et al. have studied the maximum latency guarantee in networks [30]. In addition, the performance model by Luo et al. [35] also assumed that the first moment of latency is finite. If the probability that the communication failure occurs is not zero, the first moment of latency becomes infinite. Thus we should consider the retransmission scheme and snapshot isolation simultaneously in the model.

van Moorsel and Wolter [40] discussed the completion time under the general restart scheme that can directly be applied to the retransmission scheme of the Internet. According to the restart scheme [40], we first discuss the latency distribution in the failure-prone environment.

Define the following random variables:

- \( L (> 0) \): the latency of communication from a sender to a receiver through the network. This is a random variable having a cumulative distribution function (c.d.f.) \( F_L(t) \).

Since the communication failure occurs in the failure-prone environment, the c.d.f. of \( F_L(t) \) is allowed to be defective, i.e., \( F(\infty) < 1 \). Then the probability of communication failure corresponds to \( 1 - F(\infty) \). Also, the first moment can be defined by

\[
E[L] = \int_{0}^{\infty} tdF_L(t) = \int_{0}^{\infty} \mathcal{P}_L(t) dt, \tag{2.1}
\]

where, in general, \( \mathcal{P}(\cdot) = 1 - F(\cdot) \). Hence, when the probability of communication failure is not zero, the first moment of \( L \), \( E[L] \) is not finite.

van Moorsel and Wolter [41] considered a general restart scheme. In their
scheme, the sender waits for an acknowledgement from the receiver by the limit $\tau$ ($>0$). That is, if the sender does not get an acknowledgement from the receiver until time unit $\tau$, the sender resends a request to the receiver with the restart overhead time $c$ ($>0$). The sender repeatedly sends requests until obtaining an acknowledgement from the receiver. Figure 2.3 illustrates the possible sample path under the restart scheme by van Moorsel and Wolter [41]. For the sake of simplicity, the sender is assumed to be cancel the previous request before resending a new request. Let $C$ be a completion time under such a restart scheme. Then the c.d.f. of the completion time is given by

$$F_C(t) = P(C \leq t) = \begin{cases} 1 - \frac{1}{F_{L+L}(\tau)} F_{L+L}(t - k(\tau + c)), & k(\tau + c) \leq t < k(\tau + c) + \tau \\ 1 - \frac{1}{F_{L+L}(\tau)} F_{L+L}(k(\tau + c) + \tau), & k(\tau + c) + \tau \leq t < (k + 1)(\tau + c). \end{cases}$$

(2.2)

In the above equation, $F_{L+L}$ indicates the c.d.f. of a round-trip time without communication failure that is a 2-fold convolution of $L$;

$$F_{L+L}(t) = \int_0^t F_L(t - x)dF_L(x).$$

(2.3)

The expected time of the completion time holds the following equation:

$$E[C] = \int_0^\tau E[C|C = x]dF_{L+L}(x) + \int_\tau^\infty (\tau + c + E[C])dF_{L+L}(x)$$

$$= \int_0^\tau xdF_{L+L}(x) + (\tau + c + E[C])F_{L+L}(\tau).$$

(2.4)

Then we have

$$E[C] = \frac{\int_0^\tau F_{L+L}(x)dx + \tau F_{L+L}(\tau)}{F_{L+L}(\tau)}.$$  (2.5)

From the above equation, we find that $E[C]$ is always finite even if $F_L(\infty) < 1$. Also the Laplace-Stieltjes (LS) transform of $C$ can be derived from

$$E[e^{-sC}] = \int_0^\tau E[e^{-sC}|C = x]dF_{L+L}(x) + \int_\tau^\infty e^{-s(\tau + c)}E[e^{-sC}]dF_{L+L}(x)$$

$$= \int_0^\tau e^{-sx}dF_{L+L}(x) + e^{-s(\tau + c)}E[e^{-sC}]F_{L+L}(\tau),$$

(2.6)

and

$$E[e^{-sC}] = \frac{\int_0^\tau e^{-sx}dF_{L+L}(x)}{1 - e^{-s(\tau + c)}F_{L+L}(\tau)}.\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 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2.3. CONVENTIONAL SNAPSHOT ISOLATION (CSI) MODEL

The completion time under the restart scheme can be represented by

\[ C = (\tau + c)N_F + L_1 + L_2, \]

where \( N_F \) is the number of restarts, \( L_1 \) and \( L_2 \) are latencies
for a request and an acknowledgement, respectively, provided that \( L_1 + L_2 \leq \tau \). Letting \( P_1 = (\tau + c)N_F + L_1 \) and \( P_2 = L_2 \) provided that \( L_1 + L_2 \leq \tau \), we have

\[ E[P_1] + E[P_2] = E[P_1 + P_2] = E[C]. \] (2.8)

Also, the LS transforms of \( P_1 \) and \( P_2 \) are

\[ E[e^{-sP_1}] = E[e^{-s(\tau + c)N_F}] \times E[e^{-sL_1}|L_1 + L_2 \leq \tau] \]

\[ = \sum_{k=0}^{\infty} e^{-s(\tau + c)k} \frac{F_{L_1+L_2}(\tau)}{1 - e^{-s(\tau + c)}F_{L_1+L_2}(\tau)} \times E[e^{-sL_1}|L_1 + L_2 \leq \tau] \]

\[ = \int_0^{\tau} \int_0^t e^{-sx} f_L(t-x) dF_L(x) dt \]

\[ = \int_0^{\tau} e^{-st} F_L(t) dF_L(t), \] (2.9)

and

\[ E[e^{-sP_2}] = E[L_2|L_1 + L_2 \leq \tau] \]

\[ = \int_0^{\tau} \int_0^t e^{-sx} f_L(t-x) dF_L(x) dt \]

\[ = \int_0^{\tau} e^{-st} F_L(t) dF_L(t). \] (2.10)

It should be noted that \( E[e^{-sC}] \neq E[e^{-sP_1}]E[e^{-sP_2}] \) since \( P_1 \) and \( P_2 \) are not mutually independent random variables.

2.3 Conventional snapshot isolation (CSI) model

Consider a distributed database system having a central server (master) and several replicas. We suppose that all the replicas have a copy of the database in the master. Thus, in the practical situation, the master and replicas should execute the replication procedure such as 2-phase commit. However, since this chapter focuses only on the performance of SI between the master and a replica, we ignore the effect of a request message from the master to replicas. In addition, we consider only the arrival stream of update transactions, because read transactions are never aborted in the scheme of SI and there is no effect of read transactions to the performance.

Define the following random variables:
CHAPTER 2. PERFORMANCE EVALUATION OF SNAPSHOT ISOLATION

- $C (> 0)$: the latency of communication between the master and a replica with the restart scheme described in Section 3. Similar to Section 3, we decomposes the total latency $C$ as $C = C_1 + C_2$, where $C_1$ is the time for the request arrives at the master and $C_2$ is the time required from the master to the replica. Note that the time $C_1$ includes canceled requests by restarts.

- $T (> 0)$: the time spent by a transaction (random variable having a c.d.f. $F_T(t)$).

Suppose that the arrival stream of update transactions to a replica is a Poisson process with rate $\lambda_R$, i.e., the inter-arrival time of update transactions is given by an exponential distribution with mean $1/\lambda_R$. When a new transaction arrives at the replica, the replica requests the latest snapshot to the master. The total time for executing all the operations of a transaction is given by $T$ having the c.d.f. $F_T(t)$. After completing all the operations, the replica sends a message of request for commit to the master. Similar to the request for snapshot, the latency for the commit request is also given as the round-trip time $C$.

On the other hand, the master receives requests for commit from the other replicas in accordance with a Poisson process with rate $\lambda_A$. According to the first-committer-wins rule, the master decides a request for commit is committed or aborted. Concretely, in the CSI scheme, if no commit occurs during the latencies and processing time of a transaction, the transaction is committed. Otherwise, if one or more transactions are committed during the latencies and the processing time, the transaction is aborted. Figure 2.4 depicts possible sample paths when transactions are aborted and committed.

Next we derive the performance measures in CSI model. This chapter considers system throughput, abort probability and response time as criteria of performance on the distributed database system. The system throughput is defined as the number of committed transactions per unit time, i.e.,

$$ST = \lim_{t \to \infty} \frac{E}{t} \left( \text{the number of committed transactions during } [0, t] \right). \quad (2.11)$$

The system with high throughput is better than the system with low throughput.
2.3. CONVENTIONAL SNAPSHOT ISOLATION (CSI) MODEL

To derive the system throughput of CSI, we focus on the age of snapshot. The age of snapshot is renewed at which a request for snapshot arrives at the master. Then we define the time interval between successive renewal points of the age of snapshot as one cycle. From the renewal reward theorem [58], the system throughput can be rewritten as

\[
ST_{CSI} = \frac{P_r \left\{ \text{a transaction that arrives during a cycle is committed} \right\}}{E[\text{time length of one cycle}]}.
\]  

(2.12)

The numerator comes from the fact that one snapshot allows only one transaction to be committed (the first-committee-wins rule). Let \( P_r \) denote the numerator in Eq. (2.12). In the case of CPI, \( P_c \) is equivalent to the probability

Figure 2.4: Possible sample paths of CSI.
that the first transaction is committed, i.e., we consider the probability that
\( C_2 + T + C_1 \) is faster than the arrival time of the request for commit from other
replicas. Here, in general, the probability that a random variable \( X \) having a
c.d.f. \( F(t) \) is lower than an exponentially distributed random variable \( Y \) with
mean \( 1/\lambda \) is given by
\[
\int_0^\infty \lambda e^{-\lambda t} P(X \leq t) dt = \lambda \int_0^\infty e^{-\lambda t} F(t) dt = F^*(\lambda),
\] (2.13)
where \( F^*(s) \) is the LS transform of \( F(t) \), namely,
\[
F^*(s) = \int_0^\infty e^{-\lambda s} dF(t).
\] (2.14)
From the LS transforms of \( C_2 \) and \( C_1 \), we have
\[
P_c = \int_0^\infty \lambda_A e^{-\lambda_A t} P(C_2 \leq t) dt \int_0^\infty \lambda_A e^{-\lambda_A t} P(T \leq t) dt
\times \int_0^\infty \lambda_A e^{-\lambda_A t} P(C_1 \leq t) dt
= F_{C_1}^*(\lambda_A) F_{C_2}^*(\lambda_A) F_T^*(\lambda_A),
\] (2.15)
where \( F_{C_1}^*(s) = E[e^{-sP_1}] \) and \( F_{C_2}^*(s) = E[e^{-sP_2}] \) are defined as Eqs. (2.9) and
(2.10). Also, the expected time length of one cycle is given by \( E[C_2 + T + C_1 +
C_2 + X_A + C_1] \), where \( X_A \) is a random variable representing the time when a
new transaction arrives at the replica after receiving the acknowledgement of a
commit request. From the memoryless property of exponential distribution, we
get \( E[X_A] = 1/\lambda_R \). Then the system throughput of CSI can be obtained by
\[
ST_{CSI} = \frac{F_{C_1}^*(\lambda_A) F_{C_2}^*(\lambda_A) F_T^*(\lambda_A)}{2E[C] + E[T] + 1/\lambda_R}.
\] (2.16)
Since the arrival rate of transaction \( \lambda_R \) is defined as the number of transac-
tions per unit time, the abort probability of a transaction is obtained by using
the system throughput, i.e.,
\[
AP_{CSI} = 1 - \frac{ST_{CSI}}{\lambda_R}.
\] (2.17)
Also, the response time is defined by the expected time until receiving the
acknowledgement for a commit request from starting operations of a transaction.
Then we have
\[
RT_{CSI} = E[C_1 + C_2 + T + C_1 + C_2]
= 2E[C] + E[T].
\] (2.18)
2.4. PREFIX-CONSISTENT SNAPSHOT ISOLATION (PCSI)

Remark 4.1 (Non-homogeneous Poisson process): Non-homogeneous Poisson process (NHPP) is also considerable for the arrivals in master. NHPP is widely used for describing numerous random phenomena in many fields. However, it augments the difficulty of derivation of formulations and the computation complexity. Thus, we consider Poisson process in this chapter.

2.4 Prefix-consistent snapshot isolation (PCSI)

Similar to the model of CSI, we consider the database with a master and replicas. The update transactions that arrive at a replica are according to a Poisson process with rate \( \lambda_R \), and the requests for commit to the master follows a Poisson process with mean \( \lambda_A \). Moreover, we define the random variables that the latency of communication (round-trip time) \( C \) and the processing time of a transaction \( T \) under the failure-prone environment. The c.d.f.s of \( C \) and \( T \) are also given by \( F_C(t) \) and \( F_T(t) \), respectively. According to the restart scheme of van Moorsel and Wolter [41], the c.d.f. of \( C \) can be presented in Eq. (2.2) with the restart timing and overhead \( \tau \) and \( c \). Also the latency is divided into \( C_1 \) and \( C_2 \) which are latency from a replica to the server and from the server to a replica under the restart scheme.

In the PCSI, the snapshot in the replica is updated when the age of snapshot exceeds \( U \). The age of snapshot is larger when the time interval of updating snapshot is longer. On the other hand, the short update interval causes the overhead to obtain the snapshot from the master. Furthermore, we assume that, if the time until receiving an acknowledgment exceeds update interval \( U \), the updating snapshot is delayed to the time until receiving the acknowledgement. Figure 2.5 illustrates sample paths of our PCSI model.

Consider the performance measures in the PCSI. From the renewal reward theory, the system throughput of PCSI can also be defined as Eq. (2.11). Define a cycle as time interval between two successive renewal points of the age of snapshot. Then the system throughput under the PCSI is given by

\[
ST_{PCSI} = \frac{\Pr \left\{ \begin{array}{c}
\text{a transaction that arrives} \\
during a cycle and it is committed
\end{array} \right\}}{E[\text{time length of one cycle}]}.
\] (2.19)
First we consider the probability that a transaction arriving in a cycle is committed. Let $P_p(U)$ be the numerator of Eq. (2.19) because the probability is a function of time interval for updating snapshot $U$. Since the inter-arrival time
of transactions is given by an exponential random variable, we obtain
\[
P_p(U) = \int_0^U \lambda Re^{-\lambda rt} e^{-\lambda At} dt \\
\times \int_0^\infty \lambda Ae^{-\lambda At} P(C_1 \leq t) dt \\
\times \int_0^\infty \lambda Ae^{-\lambda At} P(C_2 \leq t) dt \\
\times \int_0^\infty \lambda Ae^{-\lambda At} P(T \leq t) dt
\]
\[
= \frac{\lambda R}{\lambda R + \lambda A} \left( 1 - e^{-(\lambda R + \lambda A)U} \right) F^*_{C_1}(\lambda A) F^*_{C_2}(\lambda A) F^*_T(\lambda A). \tag{2.20}
\]

Next we derive the expected time of one cycle. Let \( C_p(U) \) denote the expected time of one cycle. For the notational simplification, \( G(t) \) is the c.d.f. of the random variable \( T + C_1 + C_2 = T + C \) since \( C_1 \) and \( C_2 \) belong to the same round trip. In other words, \( G(t) \) can be obtained by the inverse LS transform of the following LS transform:
\[
G^*(s) = F^*_T(s) F^*_C(s) = F^*_T(s), \tag{2.21}
\]
where
\[
F^*_C(s) = E[e^{-sC}] = \frac{\int_0^T e^{-sx} dF_{L+L}(x)}{1-e^{-s(\tau+c)F_{L+L}(\tau)}}, \tag{2.22}
\]
As mentioned before, system automatically renews the snapshot in \( U \) time in case transactions terminated before time interval \( U \). However, it is possible that transactions are being processed until \( U \) time. For this situation, we assume system does not update the snapshot until the accomplishment of transactions, i.e., one cycle is considered as the period from prior snapshot to the end of delayed transactions. Then the expected time of one cycle is given by
\[
C_p(U) = E[C_2] \\
+ \int_0^U \left( UG(U-t) + \int_0^\infty sdG(s) \right) \lambda Re^{-\lambda nt} dt \\
+ Ue^{-(\lambda R+\lambda A)U} + E[C_1] \\
= U + E[C] + (E[T] + E[C])(1 - e^{-(\lambda R)U}) \\
- \int_0^U \overline{G}(s)(1 - e^{-\lambda R(U-s)}) ds, \tag{2.23}
\]
where \( \overline{G}(s) = 1 - G(s) \). Then the system throughput becomes
\[
ST_{PCSI}(U) = \frac{P_p(U)}{C_p(U)}. \tag{2.24}
\]
Also, the abort probability of a transaction and the response time can be obtained as follows.

\[
AP_{PCS}(U) = 1 - \frac{ST_{PCS}(U)}{\lambda_R},
\]

\[
RT_{PCS} = E[T + C_1 + C_2] = E[C] + E[T].
\]

Note that the response time of PCSI does not depend on the time interval for updating the snapshot \( U \).

As mentioned before, the system throughput of the PCSI is always less than that of CSI. In addition, the system throughput changes with the time interval for updating snapshot \( U \). Therefore, in the design of PCSI, it is important to find the optimal update interval maximizing the system throughput. This chapter presents an analytical result on the optimal update timing in the PCSI scheme.

Consider the first derivatives of \( P_p(U) \) and \( C_p(U) \) with respect to \( U \). Then we have

\[
\frac{d}{dU} P_p(U) = \lambda_R e^{-(\lambda_R + \lambda_A)U} F_{C_1}^*(\lambda_A) F_{C_2}^*(\lambda_A) F_{T}^*(\lambda_A)
\]

and

\[
\frac{d}{dU} C_p(U) = 1 + (E[T] + E[C])\lambda_R e^{-\lambda_R U}
- \int_0^U G(s)\lambda_R e^{-\lambda_R(U-s)} ds.
\]

Here we define the function \( q(U) \) which is the numerator of the first derivative of \( ST_{PCS}(U) \):

\[
q(U) = C_p(U) \frac{d}{dU} P_p(U) - P_p(U) \frac{d}{dU} C_p(U).
\]

Then we have the following result on the optimal update timing:

**Theorem:** There is a unique and finite solution of \( U^* \) maximizing the system throughput such that \( q(U^*) = 0 \) in the PCSI scheme under the restart scheme.

The maximum throughput is given by

\[
ST_{PCS}(U^*) = \frac{e^{-(\lambda_R + \lambda_A)U^*} F_{C_1}^*(\lambda_A) F_{C_2}^*(\lambda_A) F_{T}^*(\lambda_A)}{(1/\lambda_R)(1 - P_e(U^*)) + (E[C] + E[T])e^{-\lambda_R U^*}},
\]

\[
(2.30)
\]
where $P_e(U)$ is the probability that a transaction arrives during $U$ and the time receiving the acknowledgement for a commit request exceeds the time for updating snapshot $U$; $P_e(U) = \int_0^U C(U - s) \lambda R e^{-\lambda s} ds$.

**Proof:** Consider the function $\tilde{q}(U) = e^{-(\lambda_R + \lambda_A)U}q(u)$, which has the same sign of the first derivative of $ST_{PCS1}(U)$ with respect to $U$. Then we get

$$
\frac{d}{dU} \tilde{q}(U) = -\lambda_R \lambda_A e^{(\lambda_R + \lambda_A)U} \left(\frac{1}{\lambda_R} (1 - P_e(U))
+ \frac{1}{\lambda_A} G(U) + (E[C] + E[T])e^{-\lambda_R U})\right) P_p(U).
$$

(2.31)

It is straightforward to find $d\tilde{q}(U)/dU < 0$ for any $U$. Hence the function $q(U)$ is a monotonically decreasing function of $U$. From Eqs. (2.19)–(2.29), we have

$$
q(0) = \lambda_R F_{C_1}^*(\lambda_A) F_{C_2}^*(\lambda_A) F_T^*(\lambda_A) C_p(0) > 0
$$

(2.32)

and

$$
q(\infty) = -P_p(\infty)
= -\frac{\lambda_R}{\lambda_R + \lambda_A} F_{C_1}^*(\lambda_A) F_{C_2}^*(\lambda_A) F_T^*(\lambda_A) < 0.
$$

(2.33)

The sign of $q(U)$ is equal to the sign of the first derivative of $ST_{PCS1}(U)$, and henceforth, $ST_{PCS1}(U)$ is a concave function having a global maximum point $U^*$ such that $q(U^*) = 0$. Also, from $q(U^*) = 0$, the maximum throughput can be obtained:

$$
ST_{PCS1}(U^*) = \frac{P_p(U^*)}{C_p(U^*)} = \frac{d}{dU} P_p(U) \bigg|_{U = U^*} = \frac{d}{dU} C_p(U) \bigg|_{U = U^*}.
$$

(2.34)

From the above theorem, we find that there exists a unique optimal update timing $U^*$ which minimizes the abort probability, and the optimal update timings in terms of system throughput and abort probability are identical.

### 2.5 Numerical Experiments

In this section, we present numerical examples for the system throughput of CSI and PCSI under the failure-prone environment. We examine how much the system throughput of PCSI decreases compared to CSI. As mentioned before,
although the response time of PCSI is improved, the system throughput of PCSI is less than that of CSI. Thus we investigate the effectiveness of determining the optimal time interval for updating snapshots in PCSI.

2.5.1 Distribution Assumption

Suppose that the latency of communication between a replica and the master $L$ is the following defective gamma distribution:

$$F_L(t) = p_s \int_0^t \beta_L^{\alpha_L} \frac{s^{\alpha_L-1} e^{-\beta_L s}}{\Gamma(\alpha_L)} ds, \quad 0 \leq t < \infty,$$

where $\alpha_L$ and $\beta_L$ are shape and scale parameters and $p_s$ represents the success probability of communication. Although the mean time of $L$ is infinite, the mean time provided that $L < \infty$ can be obtained as $E[L|L < \infty] = \frac{\alpha_L}{\beta_L}$.

Also, the LS transform of $L$ can be given by

$$F_L^*(s) = p_s \left( \frac{\beta_L}{s + \beta_L} \right)^{\alpha_L}.$$

(2.36)

Similarly, the processing time of a transaction follows the gamma distribution:

$$F_T(t) = \int_0^t \beta_T^{\alpha_T} \frac{s^{\alpha_T-1} e^{-\beta_T s}}{\Gamma(\alpha_T)} ds, \quad 0 \leq t < \infty,$$

where $\alpha_T$ and $\beta_T$ are shape and scale parameters. The shape parameter decides the age property of gamma distribution. If the shape parameter is less than 1, the gamma distribution has the decreasing failure rate (DFR) property. On the other hand, when the shape parameter is greater than 1, the gamma distribution has the increasing failure rate (IFR) property. The mean time of processing becomes $E[T] = \frac{\alpha_T}{\beta_T}$. The LS transform of $T$ is

$$F_T^*(s) = \left( \frac{\beta_T}{s + \beta_T} \right)^{\alpha_T}.$$

(2.38)

Then the LS transform of $G(t)$ is given by

$$G^*(s) = \left( \frac{\beta_T}{s + \beta_T} \right)^{\alpha_T} \int_0^t \frac{e^{-s x} dF_{L+L}(x)}{1 - e^{-s(\tau + \omega)}F_{L+L}(\tau)},$$

where the c.d.f. $F_{L+L}$ can be derived as the 2-fold convolution of $F_L(t)$, namely,

$$F_{L+L}(t) = p_s^2 \int_0^t \frac{\beta_L^{2\alpha_L} s^{2\alpha_L-1} e^{-\beta_L s}}{\Gamma(2\alpha_L)} ds, \quad 0 \leq t < \infty.$$

(2.40)

The c.d.f. $G(t)$ can be computed by the numerical inverse Laplace transform technique such as Gaver’s method (see Appendix).
2.5.2 Response Time

We first investigate the impacts to response time of CSI and PCSI under failure-prone environment. In the experiments, we fix the arrival rate of transaction in the replica $\lambda_R = 1$, i.e., the inter-arrival time of transaction becomes 1. We also set the arrival rate of transaction in the master $\lambda_A = 1$. Moreover, for investigating the effect of restart scheme, we set the restart time limit in three cases, $\tau = 0.5, 1, 2$ and a fixed restart overhead time $c = 0.5$ constantly. Table 2.1 presents the response times of CSI and PCSI where $\lambda_A = 1$, $\alpha_T = \alpha_L = 1$, $E[T] = 0.01$ and $\tau = 0.5$. Note that the response times can be calculated by Eq. (2.18) and (2.26). As same as the conclusion showed in [10, 18], it can be found from Table 2.1 that the PCSI is effective to reducing the response time. Particularly in the case when latency is large, i.e., $E[L] = 1$, PCSI provides about half the response time of CSI. Therefore, in order to examine the impact caused by communication failures, we show the response times of CSI and PCSI with fixing the mean time of communication latency $E[L] = 1$ and $\alpha_T = \alpha_L = 1$ in Table 2.2. From this table, it can be observed that response times under both isolation policies reduce as the restart time limit $\tau$ gets enlarged. In contrast, Table 2.3 shows the response times when latency is comparatively short, $E[L] = 0.001$. It is inferred that restart scheme reduces system performance significantly when communication latency is too large to be ignored. With considering the network environment, the usage of request restart scheme is supposed be implemented in the database systems with small communication delay. In addition, we inspect the effects from distribution of latency time. Let $\alpha_L$, the shape parameter of latency time which is following defective gamma distribution, equals 0.5, 1, 2, 5, respectively. Table 2.4 presents the response times in a set of shape parameters of latency time distribution and restart time limits. Notice that the response time of CSI while $\alpha_L = 5$ and $\tau = 0.5$ is much larger than other situations. Since the property of shape parameter in gamma distribution, it can be supposed that the mean time of latency stays steady when $\alpha_L$ is less than 1, and stays variable when $\alpha_L$ is larger than 1. While $\alpha_L = 5$, the communication latency may values in wider range, and it exactly represents different reasons by which the communication delays caused.
## Table 2.1: Response times of CSI and PCSI ($\tau = 0.5, \alpha_T = \alpha_L = 1$)

<table>
<thead>
<tr>
<th>$E[T]$</th>
<th>$E[L]$</th>
<th>CSI</th>
<th>PCSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.0120</td>
<td>0.0110</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>0.0300</td>
<td>0.0200</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2168</td>
<td>0.1134</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.5515</td>
<td>1.7808</td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td>0.1020</td>
<td>0.1010</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.1200</td>
<td>0.1100</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.3068</td>
<td>0.2034</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.6415</td>
<td>1.8708</td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td>0.5020</td>
<td>0.5010</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.5200</td>
<td>0.5100</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.7068</td>
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<tr>
<td>1</td>
<td>4.0415</td>
<td>2.2708</td>
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</tr>
<tr>
<td>0.001</td>
<td>1.0020</td>
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<tr>
<td>1</td>
<td>1.0200</td>
<td>1.0100</td>
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</tr>
<tr>
<td>0.1</td>
<td>1.2068</td>
<td>1.1034</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.5415</td>
<td>2.7708</td>
<td></td>
</tr>
</tbody>
</table>
2.5. NUMERICAL EXPERIMENTS

<table>
<thead>
<tr>
<th>$E[T]$</th>
<th>$\tau$</th>
<th>CSI</th>
<th>PCSI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>3.5515</td>
<td>1.7808</td>
</tr>
<tr>
<td>0.01</td>
<td>1</td>
<td>2.5920</td>
<td>1.3010</td>
</tr>
<tr>
<td>2</td>
<td>2.1665</td>
<td>1.0883</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>3.6415</td>
<td>1.8708</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>2.6820</td>
<td>1.3910</td>
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<tr>
<td>2</td>
<td>2.2565</td>
<td>1.1783</td>
<td></td>
</tr>
<tr>
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<td>0.5</td>
<td>4.0415</td>
<td>2.2708</td>
</tr>
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<td>0.5</td>
<td>1</td>
<td>3.0820</td>
<td>1.7910</td>
</tr>
<tr>
<td>2</td>
<td>2.6565</td>
<td>1.5783</td>
<td></td>
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<tr>
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<td>0.5</td>
<td>4.5415</td>
<td>2.7708</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3.5820</td>
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</tr>
<tr>
<td>2</td>
<td>3.1565</td>
<td>2.0783</td>
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</table>

<table>
<thead>
<tr>
<th>$E[T]$</th>
<th>$\tau$</th>
<th>CSI</th>
<th>PCSI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>0.012</td>
<td>0.011</td>
</tr>
<tr>
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<td>1</td>
<td>0.012</td>
<td>0.011</td>
</tr>
<tr>
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<td>0.012</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.102</td>
<td>0.101</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0.102</td>
<td>0.101</td>
</tr>
<tr>
<td>2</td>
<td>0.102</td>
<td>0.101</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>0.502</td>
<td>0.501</td>
</tr>
<tr>
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<td>0.502</td>
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<tr>
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</tr>
<tr>
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<td>1</td>
<td>1.002</td>
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<tr>
<td>2</td>
<td>1.002</td>
<td>1.001</td>
<td></td>
</tr>
</tbody>
</table>
2.5.3 System Throughput

Next we discuss the system performances of CSI and PCSI in terms of throughput. In Tables 2.5–2.7, where we set the restart time $\tau = 0.5, 1, 2$ respectively, the system throughputs of PCSI with optimized time interval show a decrease comparing with that in CSI. In addition, PCSI with restart scheme performs about 90% of CSI in term of throughput, when latency is contrastively small. And the ratio between throughputs of PCSI and CSI, when $E[L] = 1.0$, is reduced to about 55%. It implies that if the network environment is crowded, system has to pay more expense for ensuring successful communications. In contrast, restart scheme is more effective under unblocked network condition. On the other hand, the performances are almost equal for each restart time $\tau$, except the high latency situation, i.e., $E[L] = 1.0$. It can be implied that the restart time limit will affect the system performance more significantly when latency is large. Thus, the trade-off of the system performance and the guarantee of success communication should be considered especially in lagging network.

For inspecting the sensitivity of throughput to the distribution parameters, we fix parameters as $\tau = 1$, $E[L] = 0.001$, $E[T] = 0.01$. Table 2.8 shows
2.5. NUMERICAL EXPERIMENTS

Table 2.5: System throughput of CSI and PCSI ($\tau = 0.5, \alpha_T = \alpha_L = 1$).

<table>
<thead>
<tr>
<th>E[L]</th>
<th>CSI</th>
<th>PCSI</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.9745</td>
<td>0.9025</td>
<td>0.0453</td>
</tr>
<tr>
<td>0.010</td>
<td>0.9244</td>
<td>0.7383</td>
<td>0.1370</td>
</tr>
<tr>
<td>0.100</td>
<td>0.5803</td>
<td>0.3565</td>
<td>0.4197</td>
</tr>
<tr>
<td>1.000</td>
<td>0.0306</td>
<td>0.0166</td>
<td>0.9868</td>
</tr>
</tbody>
</table>

Table 2.6: System throughput of CSI and PCSI ($\tau = 1, \alpha_T = \alpha_L = 1$).

<table>
<thead>
<tr>
<th>E[L]</th>
<th>CSI</th>
<th>PCSI</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.9745</td>
<td>0.9025</td>
<td>0.0453</td>
</tr>
<tr>
<td>0.010</td>
<td>0.9244</td>
<td>0.7383</td>
<td>0.1370</td>
</tr>
<tr>
<td>0.100</td>
<td>0.5803</td>
<td>0.3565</td>
<td>0.4197</td>
</tr>
<tr>
<td>1.000</td>
<td>0.0401</td>
<td>0.0219</td>
<td>0.9950</td>
</tr>
</tbody>
</table>

the throughputs with fixing $\alpha_L = 2$. The throughputs of CSI and PCSI are not much differences while the shape parameter of processing time changes. It can be inferred that the system throughput strongly depends on the mean time of latency instead of shape parameter. Conversely, Table 2.9 presents the throughputs where $\alpha_L$ changes from 0.5 to 1. And it tells an identical relationship between throughputs and shape parameter of latency.

2.5.4 Simulations

In this section, we conduct a simulation experiment based on group data of real traffic measured in Hiroshima University, Japan. The targeted host is installed

Table 2.7: System throughput of CSI and PCSI ($\tau = 2, \alpha_T = \alpha_L = 1$).

<table>
<thead>
<tr>
<th>E[L]</th>
<th>CSI</th>
<th>PCSI</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.9745</td>
<td>0.9025</td>
<td>0.0453</td>
</tr>
<tr>
<td>0.010</td>
<td>0.9244</td>
<td>0.7383</td>
<td>0.1370</td>
</tr>
<tr>
<td>0.100</td>
<td>0.5803</td>
<td>0.3616</td>
<td>0.4143</td>
</tr>
<tr>
<td>1.000</td>
<td>0.0465</td>
<td>0.0256</td>
<td>0.9928</td>
</tr>
</tbody>
</table>
in the Department of Information Engineering in Hiroshima university. The accessing records are collected for one day and the number of total records is 16529. We further aggregate the group data at 1 hour’s time interval so as to summary the characteristic of arrival stream into parameters in Figure 2.6. Also, by observing the traffic packets data in Figure 2.7, we can empirically gain the density of traffic which can be applied as the parameters of processing time. Then we set Monte Carlo simulations with the these system parameters and survey the system throughputs and response times.

Next, we numerically calculate the throughput and response time by using our formulations. The mean time between arrivals $1/\lambda_A$ is 0.087 min and the mean processing time $E[T]$ is 0.018 min. We assume the latency time $E[L] = 0.01$, $\alpha_T = \alpha_L = 1$, $c = 0.5$ and $\tau = 0.5$. Table 2.10 shows the comparison of simulation and analysis results with CSI policy. The response times are almost the same and the analytical throughput are close to that of simulation. The correctness of our formulations can be validated. On the other hand, we generate requests under PCSI policy by same parameters. Since the time interval of snapshot is difficult to be determined by observed data, we firstly use the optimal value obtained by proposed equations, $U^* = 0.12$. Then,
we set different intervals of snapshot to investigate the effectiveness of proposed optimization of snapshot interval. Table 2.11 shows the simulation results with optimal and constant intervals, i.e., $U=0.12$, 1 and 0.05. The throughput under optimal interval is larger than others. It proves the necessity of optimizing the snapshot interval of PCSI. Meanwhile, comparing Table 2.10 and Table 2.11, optimized throughput in PCSI is close to that in CSI. It indicates that PCSI policy is effective unless the time interval is optimized. According to the simulation experiments, our conclusions are proved usable for evaluating and optimizing the performance of CSI and PCSI.

Table 2.10: Comparison of simulation and analysis in CSI

<table>
<thead>
<tr>
<th></th>
<th>Simulation</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Throughputs</td>
<td>0.6749</td>
<td>0.6870</td>
</tr>
<tr>
<td>Response times</td>
<td>0.0299</td>
<td>0.0300</td>
</tr>
</tbody>
</table>
### Table 2.11: Comparison of simulation and analysis in PCSI

<table>
<thead>
<tr>
<th></th>
<th>Simulation</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimal</td>
<td>U=1</td>
</tr>
<tr>
<td>System Throughputs</td>
<td>0.6358</td>
<td>0.0935</td>
</tr>
<tr>
<td>Response times</td>
<td>0.0278</td>
<td>0.0281</td>
</tr>
</tbody>
</table>
Chapter 3

Optimal Planning for Open Source Software Updates

Optimal Planning for Open Source Software Updates

Open source software is widely deployed for both academic and commercial proposes. However, failures and attacks against open source software are greatly reported. Being different from traditional software, failures of open source software are reported by users and fixed by developers. With the overheads and costs of updates, users would not maintain the software immediately the latest update is released. We are attracted by the optimal maintenance problem for open source software from users’ perspective. In this chapter, we proposes periodic and aperiodic maintenance management models with non-homogeneous bug-discovery/correction processes in terms of total expense. Also, according to a dynamic programming algorithm, we numerically derive the optimal maintenance schedule under aperiodic policy. In numerical examples, we investigate the efficiency of proposed policies by mathematical experiments. And, based on bug reports of Hadoop MapReduce, we predictively illustrate the optimal maintenance scheduling for a real open source software product.

3.1 Bug-Correction Process

We focus on the bug-discovery/correction process in the open source project. In general, the open source project has no clear distinction between implement and testing phases. If users discover yet-undetected bugs in using the software, they report details of the bugs to the community of open source project and
developers start to fix the bugs. Thus the bug-discovery process seems to be evolutionary, and it is different from the traditional software development process such as waterfall model. However, if we focus on particular versions of the software, it is known that the number of discovered bugs is able to be represented by the traditional software reliability growth model (SRGM).

In this chapter, we generally apply the NHPP-based SRGM for characterizing the bug-discovery phenomenon due to two reasons: 1) The effectiveness of NHPP-based SRGMs are affirmed for describing the stochastic behavior of the number of detected faults, because of their tractability and goodness-of-fit performance. Also, dozens of NHPP-based SRGMs are proposed and improved for fitting different software projects. [36, 53] 2) According to these models, several software reliability assessment tools are developed for evaluating software reliability and predicting the residual bugs. [48, 55] These tools make it possible to find the best matching model for particular software product.

Also, as mentioned by Shibata, [61] the repair time for one bug is considered as an exponential random variable. Thus, the generation of available patches is governed as the output of an $M_t/M/\infty$ queueing model. According to queueing theory, the output stream is following a NHPP model if the arrival, i.e., the discovery process, is also a NHPP.

In the software reliability engineering, the SRGM has been used to present the behavior of the number of faults detected in testing phase. Generally, the model is constructed under the following assumptions:

1. The software involves a fixed and finite number of faults before testing and it is a Poisson random variable with mean $\omega$.

2. The times to correct a fault are stochastic and mutually independent random variables having the cumulative distribution function $F(t)$.

According to the above assumption, the cumulative number of faults corrected
before time $t$ has the following probability mass function (p.m.f.):

$$P(B(t) = n) = \sum_{m=n}^{\infty} P(B(t) = n|B(0) = m)P(B(0) = m) = \sum_{m=n}^{\infty} \binom{m}{n} F(t)^n(1 - F(t))^{m-n} \frac{\omega^m}{m!} \exp(-\omega) = \frac{(\omega F(t))^n}{n!} \exp(-\omega F(t)).$$ (3.1)

The above p.m.f. is equivalent to the non-homogeneous Poisson process (NHPP) with mean value function $E[B(t)] = \omega F(t)$.

When we focus on the particular versions of software, the above two assumptions are also plausible even in the open source software. Thus in this chapter, we use the traditional SRGM to represent the bug-discovery process in the open source project, i.e., the NHPP $B(t)$ means the number of discovered bugs in the particular versions of open source software. In fact, when $F(t)$ is a truncated logistic distribution, the corresponding NHPP model is called the inflection S-shaped model whose mean value function draws a logistic curve. [44, 45] It has been reported that the logistic curve fitted to the actual bug (vulnerability) discovery process of open source software. [4, 3, 66, 2]

### 3.2 Maintenance Cost Model

In this chapter, we consider the maintenance model for the system whose infrastructure is constructed by open source software; for example, the virtual environment is provided by Xen\(^1\) as infrastructure, some large-scale file systems involve MapReduce\(^2\) as an important module, and even a simple Linux-based system also relies on Linux kernel\(^3\). In such system, from the reliability and security points of view, the infrastructure software should be kept to the latest and stable version. Even if the infrastructure has a security hole, the system may suffer malicious attacks and eventually happens the system or security failure such as system down, intrusion and falsification. On the other hand, during the update of infrastructure, the system should be stopped. That is, there is a trade-off relationship between failure and maintenance.

---

\(^{1}\)http://www.xenproject.org/
\(^{2}\)http://hadoop.apache.org/
\(^{3}\)https://www.kernel.org/
Suppose that the open source software is continuously maintained in the project. That is, if one reports a bug, developers immediately try to fix the bug. Then the reliability growth of the particular versions of open source software is given by an NHPP model described in the previous section. Let \( B(t) \) be the cumulative number of corrected bugs and its p.m.f. is written by
\[
P(B(t) = n) = \frac{(\omega F(t))^n}{n!} \exp(-\omega F(t)), \quad n = 0, 1, \ldots
\] (3.2)

Also we assume that the particular versions of open source software will be used as infrastructure in the calendar time period \([t_s, t_e]\). Note that the calendar time starts at the time when the development of particular versions of software begins. Without loss of generality, \( m \) updates are applied in the period \([t_s, t_e]\), i.e., we define the time sequence of maintenance as \( t_s = t_0 < t_1 < t_2 < \cdots < t_m < t_{m+1} = t_e \). For notational convenience, \( t_0 \) and \( t_{m+1} \) are equivalent to \( t_s \) and \( t_e \) respectively.

Here we focus on the number of residual bugs in the software. When the number of corrected bugs \( B(t) \) is known, the number of residual bugs \( R(t) \) is predicted by
\[
R(t) = B(\infty) - B(t);
\]
\[
P(R(t) = n) = \frac{(\omega F(t))^n}{n!} \exp(-\omega F(t)), \quad n = 0, 1, \ldots
\] (3.3)

where \( F(t) = 1 - F(t) \). On the other hand, the number of residual bugs in user environment can be decreased only at the time when the system is updated. Figure 3.1 illustrates possible sample paths of the number of residual bugs in the latest version of repository and in user environment. As shown in the figure, the bugs corrected during \([t_{i-1}, t_i]\) are fixed only at \( t_i \) in user environment, although the number of residual bugs in the latest version continuously decreases.

To build the cost model for maintenance, we define the following two cost parameters:

- \( c_m \): The maintenance cost per software update.
- \( c_f \): The penalty/recovery cost per failure.

As mentioned before, the maintenance incurs some penalty cost while the system is unavailable and the work cost for applying the update. The cost \( c_m \) indicates the total cost required for the system to undergo maintenance. When the system
is failed by the residual bugs, the cost \( c_f \) is required to recovery the system down, intrusion and falsification. For the sake of simplification, even if the system failure is happened, the software update is not executed and the system is recovered as the same version of software. Also, maintenance and recovery time durations are negligible since time intervals of updates are relatively larger than maintenance and recovery time durations.

Here we assume that the frequency of failure occurrences is proportional to the number of residual bugs, i.e., the failure rate of software at time \( t \) is given by \( \lambda R(t) \), where \( \lambda \) is the parameter meaning the failure rate per bug. In particular, it should be noted that the failure rate is piecewise constant at the period \([t_{i-1}, t_i] \) for each \( i = 1, \ldots, m \) in the user environment. Thus the expected number of failures \( X_i \) at the \( i \)-th period \([t_{i-1}, t_i] \) in the user environment is given by

\[
E[X_i] = \sum_{n=0}^{\infty} E[X_i | R(t_{i-1}) = n] P(R(t_{i-1}) = n)
= (t_i - t_{i-1}) \lambda E[R(t_{i-1})]
= (t_i - t_{i-1}) \lambda \omega F(t_{i-1}). \tag{3.4}
\]

Thus the expected total maintenance cost in the period \([t_0, t_{m+1}] \) becomes

\[
C(t_1, \ldots, t_m; t_0, t_{m+1}) = c_m m + \sum_{i=1}^{m+1} c_f E[X_i]
= c_m m + c_f \lambda \omega \sum_{i=1}^{m+1} (t_i - t_{i-1}) F(t_{i-1}). \tag{3.5}
\]

### 3.3 Optimal Update Scheduling

Based on the cost model, the problem is to find the optimal schedule for updates minimizing the expected total maintenance cost;

\[
(t_1^*, \ldots, t_m^*) = \arg\min_{(t_1,\ldots,t_m)} C(t_1,\ldots,t_m; t_0, t_{m+1}). \tag{3.6}
\]

Note that \( t_0 \) and \( t_{m+1} \) are fixed. In the chapter, we discuss two different schedules under respective constraints.
3.3.1 Periodic Policy

The first policy is the periodic policy under which the time intervals of updates are identical. In fact, the maintenance is performed every week, month or year in many pragmatic systems. Let $s$ be the time interval of updates. The expected total cost can be rewritten by

$$C(s; t_s, t_e) = c_m h + c_f \lambda \omega \sum_{i=0}^{h-1} F(t_s + is) + c_f \lambda \omega [(t_e - t_s) - hs] F(t_s + hs),$$

(3.7)

where $h$ is the number of updates that is given by $h = \max\{n \text{ is a non-negative integer}; t_e - t_s > ns\}$. The last term corresponds to the penalty/recovery cost at the period $[hs, te]$ since the time interval of update is $(t_e - t_s) - hs$. According to the above, we find some properties of the optimal maintenance schedule. For example, if the number of residual bugs does not decrease, namely $F(t) = \text{constant}$, in the time period $t \in [t_s, t_e]$, then the optimal maintenance policy becomes the policy under which the system update is never applied. The cost in this case is given by

$$C(s; t_s, t_e) = c_f (t_e - t_s) \lambda \omega F(t_s), \quad \text{for } s \geq t_e - t_s.$$  

(3.8)
3.3. OPTIMAL UPDATE SCHEDULING

Since \( \lim_{s \to 0} C(s; t_s, t_e) = +\infty \), Eq. (3.8) also gives the upper bound of the minimum cost. On the other hand, the penalty/recovery cost depends on the expected area of the number of residual bugs; \( \mathbb{E} \left[ \int_{t_s}^{t_e} R(t) dt \right] \). Then the lower bound of minimum cost is given by

\[
C(s; t_s, t_e) \geq c_f \lambda \omega \int_{t_s}^{t_e} \overline{F}(t) dt, \quad \text{for any } s.
\]

(3.9)

From these results, we have

\[
c_f \lambda \omega \int_{t_s}^{t_e} \overline{F}(t) dt \leq C(s^*; t_s, t_e) \leq c_f (t_e - t_s) \lambda \omega \overline{F}(t_s),
\]

(3.10)

where \( s^* \) the optimal time interval minimizing Eq. (3.7).

3.3.2 Aperiodic Policy

The second is the aperiodic policy under which the time intervals of updates are allowed to be different. If \( \overline{F}(t) \) is constant, the system update is meaningless to reduce the maintenance cost. On contrary, while the number of residual bugs rapidly decreases, we intuitively knows that it is better to perform the system updates frequently. That is, the appropriate time intervals of updates depend on behavior of the number of residual bugs and are not constant. From this insight, we consider the aperiodic policy to determine the optimal maintenance schedule minimizing the maintenance cost.

First we consider the optimization problem when the number of updates in \([t_s, t_e]\) is fixed as \( m \). In Eq. (3.5), we find that the original problem can be reduced to the optimization of the following equation:

\[
\min_{t_1, \ldots, t_m} \sum_{i=1}^{m+1} (t_i - t_{i-1}) \overline{F}(t_{i-1}).
\]

(3.11)

This implies that cost parameters do not affect the optimal maintenance schedule when the number of updates is fixed. By taking the first derivatives of Eq. (3.11) with respect to \( t_1, \ldots, t_m \), we obtain the optimality condition;

\[
t_{i+1}^* - t_i^* = \frac{\overline{F}(t_{i-1}^*) - \overline{F}(t_i^*)}{f(t_{i+1}^*)}, \quad \text{for } i = 1, \ldots, m,
\]

(3.12)

where \( t_0^* = t_s \) and \( t_{m+1}^* = t_e \) are fixed. Ideally, by solving the above non-linear equations, we get the optimal maintenance schedule under the fixed number of updates. However, it is not easy to compute it from the computational point of view.
In this chapter, we consider the computation of the optimal maintenance schedule based on the dynamic programing (DP) instead of solving Eq. (3.12) directly. [8] The DP provides the optimization algorithm by dividing the original problems to several sub-problems. Let $t_1^*, \ldots, t_m^*$ be the optimal maintenance schedule. According to the optimality principle, we have the following equations:

\begin{align*}
V_i^* &= \min_t V_i(t_{i-1}^*, t), \quad i = 1, \ldots, m, \\
V_{m+1}^* &= V_{m+1}(t_m^*, t_{m+1}^*), \\
V_i(t_{i-1}, t) &= (t - t_{i-1})F(t_{i-1}) + V_{i+1}^*, \quad i = 1, \ldots, m, \\
V_{m+1}(t_m, t) &= (t - t_m^*)F(t_m^*).
\end{align*}

In general, the optimality equation can be solved by the policy iteration. [54] However, $V_i(t_{i-1}, t)$ is a linear function with respect to $t$. Then, to solve the above equation, we define

\begin{align*}
\tilde{V}_i(t) &= (t - t_i^*)F(t_i^*) + \tilde{V}_{i+2}(t), \quad i = 1, \ldots, m - 1, \\
\tilde{V}_m(t) &= (t - t_{m-1}^*)F(t_{m-1}^*) + \tilde{V}_{m+1}(t).
\end{align*}

Dissimilar to Eqs. (3.13) through (3.16), the function $\tilde{V}_i(t)$ has the minimum value in $t \in [t_{i-1}^*, t_{i+1}]$. Thus the policy iteration can be applied to solve them. Concretely, the proposed policy iteration algorithm can be described as follows.

- **Step 1:** Determine initial values
  \begin{align*}
  u &:= 0, \\
  t_0 &= t_0^0 < t_1 < \cdots < t_m < t_{m+1} = t_e,
  \end{align*}

- **Step 2:** Solve the following optimization problems and update the optimal maintenance schedule:
  \begin{align*}
  t_0^{(u+1)} &:= t_s, \\
  t_i^{(u+1)} &:= \arg\min_{t_i^{(u)}} \tilde{V}_i(t), \quad \text{for } i = 1, \ldots, m, \\
  t_{m+1}^{(u+1)} &:= t_e.
  \end{align*}
3.4. NUMERICAL EXAMPLES

- **Step 3:** For all $i = 1, \ldots, m$, if $|t_i^{(u+1)} - t_i^{(u)}| < \delta$, stop the algorithm, where $\delta$ is an error tolerance level. Otherwise, let $u := u + 1$ and go to Step 2.

In Step 2, an arbitrary optimization technique should be applied. Since $\tilde{V}_i(t)$ has the minimum value in the range $[t_{i-1}, t_{i+1}]$, it is not difficult to find it. For instance, the golden section method would be effective to find the solution. [26]

Next we consider the optimal number of updates $m^*$. The problem can be formulated as follows.

$$
\min_m \left( c_m m + c_f \omega S^*(m) \right), \tag{3.22}
$$

where

$$
S^*(m) = \min_{t_1, \ldots, t_m} \sum_{i=1}^{m+1} (t_i - t_{i-1}) \mathcal{P}(t_{i-1}). \tag{3.23}
$$

Note that $S^*(m)$ depends on the optimal maintenance schedule.

Suppose that the cost for update is positive, i.e., $c_m > 0$. Then the maintenance cost infinitely increases as $m$ increases, since the expected total failure cost has the lower bound shown in the previous section. Thus it is easy to see that the optimal number of updates exists in finite domain; $0 \leq m^* < \infty$. Moreover, Eq. (3.23) is a monotonically decreasing function of $m$ and $c_m m$ is linearly increasing with respect to $m$. Then we find the optimal number of updates satisfying

$$
m^* = \min\{ m \geq 0; c_m + c_f \omega (S^*(m) - S^*(m-1)) > 0 \}. \tag{3.24}
$$

3.4 Numerical Examples

3.4.1 Characteristic Analysis

We first investigate the characteristics of periodic and aperiodic maintenance policies by numerical experiments. As mentioned before, the cost parameters have no effect to the optimal maintenance schedule under a fixed number of $m$. Without considering the cost parameters, we first define two mean value functions for a bug-correction process:

$$
R(t) = \omega(1 - e^{-at}) \tag{3.25}
$$
and

\[ R(t) = \frac{\omega(1 - e^{-at})}{1 + be^{-at}}. \] (3.26)

The former assume that a bug-correction time follows an exponential distribution, and is the same as traditional NHPP model proposed by Goel and Okumoto. [22] The latter is based on the assumption that a bug-correction time is a truncated logistic random variable, [44, 45] i.e., the related model is an inflection S-shaped model and its mean value function is almost as the same as the ones proposed by concerning literatures. [4, 3, 66, 2] We call the models ‘Exp’ and ‘Log’ in this section.

To derive the optimal maintenance times for these two models, we set the number of total potential bugs is \( \omega = 100 \) and distributions’ parameters are \( a = 0.2, b = 20.0 \). Then the mean value functions are plotted in Figure 3.2. For simulating the scenario that users are using open source software, we set \( t_s = 12 \) (months) and the software life cycle is \( t_e - t_s = 24 \) (months). That is, user starts using the software released one year ago and continues for two years.

![Figure 3.2: Mean value functions of Exp and Log.](image)

Next, we compare the costs of periodic and aperiodic policies. Aperiodic policy suggests user maintaining in optimized schedule. On the contrary, periodic policy make updates in fixed time interval. For the convenience of calculation, we set the interval is determined by \( (t_e - t_s)/(m + 1) \). The cost parameters are set as \( c_m = 1, c_f = 1 \), i.e., maintenance activity is considered as the same as the
countermeasure against a failure. Table 3.1 shows the costs of Exp model under aperiodic and periodic policies. It is clear to realize that one or two maintenances are insufficient for reducing the risk of potential bugs in terms of damage cost. However, as the frequency increases, the tendency of total cost reverses into increasing. The non-monotonicity of total cost implies the existence of $m^*$, the best maintenance frequency. In Table 3.1, we can gain the lowest cost under aperiodic and periodic policies when $m^* = 9$ and $m^* = 11$ respectively. Besides, we can observe the same phenomenon in Table 3.2 for Log model, although the reduction of cost is not significant.

Table 3.1: Costs of aperiodic and periodic policies for Exp.

<table>
<thead>
<tr>
<th>$N$</th>
<th>Aperiodic Cost</th>
<th>Periodic Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150.097</td>
<td>170.942</td>
</tr>
<tr>
<td>2</td>
<td>122.646</td>
<td>137.600</td>
</tr>
<tr>
<td>3</td>
<td>111.843</td>
<td>123.042</td>
</tr>
<tr>
<td>4</td>
<td>106.440</td>
<td>115.300</td>
</tr>
<tr>
<td>5</td>
<td>103.433</td>
<td>110.733</td>
</tr>
<tr>
<td>6</td>
<td>101.695</td>
<td>107.890</td>
</tr>
<tr>
<td>7</td>
<td>100.706</td>
<td>106.081</td>
</tr>
<tr>
<td>8</td>
<td>100.196</td>
<td>104.940</td>
</tr>
<tr>
<td>9</td>
<td>100.012</td>
<td>104.256</td>
</tr>
<tr>
<td>10</td>
<td>100.058</td>
<td>103.896</td>
</tr>
<tr>
<td>11</td>
<td>100.273</td>
<td>103.776</td>
</tr>
<tr>
<td>12</td>
<td>100.617</td>
<td>103.837</td>
</tr>
</tbody>
</table>

3.4.2 Real Data Analysis

We present an optimal maintenance schedule based on the bug reports of a real open source software, Apache Hadoop MapReduce. Figure 3.3 describes the cumulative number of bugs for Hadoop MapReduce Version 2.0 reported on Apache bug-tracking system\(^4\) from Oct.2011 to Sept.2013, and estimated

\(^4\)https://issues.apache.org/
Table 3.2: Costs of aperiodic and periodic policies for Log.

<table>
<thead>
<tr>
<th>(N)</th>
<th>Aperiodic Cost</th>
<th>Periodic Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>183.952</td>
<td>187.768</td>
</tr>
<tr>
<td>2</td>
<td>158.500</td>
<td>162.044</td>
</tr>
<tr>
<td>3</td>
<td>147.566</td>
<td>150.629</td>
</tr>
<tr>
<td>4</td>
<td>141.792</td>
<td>144.433</td>
</tr>
<tr>
<td>5</td>
<td>138.433</td>
<td>140.736</td>
</tr>
<tr>
<td>6</td>
<td>136.394</td>
<td>138.431</td>
</tr>
<tr>
<td>7</td>
<td>135.155</td>
<td>136.978</td>
</tr>
<tr>
<td>8</td>
<td>134.438</td>
<td>136.086</td>
</tr>
<tr>
<td>9</td>
<td>134.079</td>
<td>135.583</td>
</tr>
<tr>
<td>10</td>
<td>133.978</td>
<td>135.359</td>
</tr>
<tr>
<td>11</td>
<td>134.066</td>
<td>135.343</td>
</tr>
<tr>
<td>12</td>
<td>134.153</td>
<td>135.476</td>
</tr>
</tbody>
</table>

mean value functions of bug-discovery models. Here we applied an estimation method proposed by Okamura et al. to figure out the most fitting NHPP-based SRM. [48] The bug-discovery model is approximately estimated as a truncated extreme-value maximum (TEVM) distribution, whose mean value function is given by

\[
R(t) = \omega \frac{\Lambda(t) - \Lambda(0)}{1 - \Lambda(0)},
\]

(3.27)

\[
\Lambda(t) = \exp(-\exp(-\frac{t - b}{a}))
\]

(3.28)

where \(a\) and \(b\) are parameters of TEVM distribution respectively. The parameter estimation for all the models involved by Okamura et al. is based on maximum likelihood estimation (MLE). [48] The estimated parameters for TEVM model are described in Table 3.3. This table also presents the corresponding maximum logarithmic likelihood (MLL) and Akaike’s information criterion (AIC). Commonly the AIC is used as an information criterion to assess the goodness-of-fit between data and model. The model with a smaller AIC is better fitted to data. From this argument, it can be indicated that TEVM model is better fitted to
the bug data for Hadoop MapReduce than the others. In addition, the mean value function in Fig. 3.3 predictively shows number of found bugs from Oct. 2013 for about one more year.

![Figure 3.3: Hadoop: bug data and estimated mean value functions.](image)

Table 3.3: Estimation results for bug data.

<table>
<thead>
<tr>
<th>Model</th>
<th>MLL</th>
<th>AIC</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truncated Extreme-Value Maximum</td>
<td>-268.66</td>
<td>543.32</td>
<td>( \omega = 389.02, )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \alpha = 193.92, )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \beta = 382.67. )</td>
</tr>
</tbody>
</table>

Next we perform the proposed maintenance policies with the estimated mean value function of TEVM model which is supposed as the best fitting model in terms of AIC in the next year. We initially investigate the time intervals between two updates under periodic and aperiodic policies. Assume user start using Hadoop MapReduce after two years, \( t_s = 730 \) and the life period of the
software is one year, i.e. $t_e - t_s = 365$, and the first application of Hadoop MapReduce starts at Oct. 2013. Then the total number of updates during the software life period is given by $m = 12$. We use the estimated parameters of TEVM model to predict the residual bugs of software. Figure 3.4 illustrates the time intervals of updates under two policies when $m = 12$. The intervals of aperiodic policy are not constant strictly, and the intervals at the ending part become longer than the ones at the beginning.

![Figure 3.4: Time intervals of updating times ($m = 12$).](image)

We also exam the determination of total update times, $m^*$. By setting the cost parameters as $c_m = 1$, $c_f = 1$, and the constant bug rate $\lambda = 0.004$, we obtain Figure 3.5. This figure numerically shows the existence of $m^*$ under both aperiodic and periodic polices. From the parameters setting, we may tell a scenario; if the costs of repairing a failure and applying a update are identical, user should update the software 6 times for gaining the least cost.

With the above parameters, we further investigate the effectiveness of optimization of $m$. The optimized cost under aperiodic policy and the costs with different $m$ are shown in Table 3.4. Under periodic policy, user update patches
in fixed interval, which are set as half a year, two months, one month and two weeks. Since the life period is given by a constant, the total numbers of update are fixed as 1, 5, 11 and 25. Comparing these values with the cost when the number of updates is optimized, we can see that the costs of periodic policy are much higher than the aperiodic one, unless \( m = 5 \). It can be imply that our proposal is useful to reduce the total cost. Also, by deriving \( m^* \), redundant updates can be averted. In practice, the available updates are not released as frequent as one per two weeks. The result shows a practical significance for maintenance scheduling.

Based on these numerical results, we can imply that aperiodic policy provides more efficient maintenance plan while users update the software frequently or infrequently. However, the periodic policy performs almost identically when users apply the patches the same times as aperiodic policy. Thus, we can suggest for the users of MapReduce module that the total number of patches should be determined first. After that, the detailed patch timing can be decided equally during the time span of usage. On the other hand, considering the characteristic of software, we can see that the MapReduce module is used after sufficient testing. That means residual bugs are few. In this situation, the periodic policy is suitable for maintenance scheduling. If users begin to use a software product without enough testing, the aperiodic policy should be strongly recommended from the economic point of view.

Table 3.4: Total costs for aperiodic and periodic maintenance scheduling.

<table>
<thead>
<tr>
<th>( m^* )</th>
<th>Aperiodic</th>
<th>( m )</th>
<th>Periodic</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>54.0655(60-days)</td>
<td>1</td>
<td>66.897(180-days)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>55.6854(60-days)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>58.2553(30-days)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25</td>
<td>70.4708(14-days)</td>
</tr>
</tbody>
</table>
Figure 3.5: Optimized number of update times $m$. 
Chapter 4

Quantifying Resiliency of Virtualized System with Software Rejuvenation

This chapter considers how to evaluate the resiliency for virtualized system with software rejuvenation. The software rejuvenation is a proactive technique to prevent the failure caused by aging phenomenon such as resource exhaustion. In particular, according to Gohsh et al. (2010), we compute a quantitative criterion to evaluate resiliency of system by using continuous-time Markov chains (CTMC). In addition, in order to convert general state-based models to CTMCs, we employ PH (phase-type) expansion technique. In numerical examples, we investigate the resiliency of virtualized system with software rejuvenation under two different rejuvenation policies.

4.1 Virtualized System with Software Rejuvenation

Consider the virtualized system with software rejuvenation proposed in [37, 38]. The system consists of virtual machines (VMs) and a virtual machine monitor (VMM). The virtual machines provide services for the end users. In general, VMs are used for Internet servers such as Web, mail and DB. Thus the availability of VMs is important for quality of services. On the other hand, VMM manages VMs in a physical machine. If VMM was stopped, all the VMs on VMM should be stopped. Therefore it is important to decide optimal timings
for triggering software rejuvenations for VM and VMM.

Machida et al. [37, 38] discussed three kinds of software rejuvenation policies in such virtualized system; cold-VM, warm-VM and migrate-VM rejuvenations. The cold-VM rejuvenation is the simplest rejuvenation. In the cold-VM rejuvenation, all the VMs running on the VMM are shut down before the rejuvenation for VMM is performed. That is, although both VM and VMM are rejuvenated by this operation, the time to shut down all the VMs is required. In addition, the jobs running on VMs should be stopped or canceled before all the VMs are shut down. The warm-VM rejuvenation is also called as the fast rejuvenation [32]. In the warm-VM rejuvenation, instead of shutting down all the VM hosts, they are resumed after the VMM rejuvenation and suspended before it. Since the jobs running on the VMs are stored on memory, jobs are not canceled by the rejuvenation though the job completion time becomes longer. Also, the downtime is shorter than the cold-VM rejuvenation, because the time to wake up from suspend mode is faster than boot time generally. The third rejuvenation is the migrate-VM rejuvenation. This scheme requires another VMM which has the same environment as the original VMM. The live migration is a well-known and convenient function of virtualized system, which can move running VMs from one VMM to another VMM without downtime of service. The migrate-VM rejuvenation utilizes the live migration of virtualized system. That is, running VMs on one VMM are moved onto another VMM before the VMM rejuvenation. After completion of the VMM rejuvenation, the migrated VMs are back to the rejuvenated VMM. Although the jobs running on VMs are never canceled by the warm-VM and migrate-VM rejuvenations, the software aging of VMs is not rejuvenated by both the warm-VM and migrate-VM rejuvenations. In this chapter, we consider the situation where a single VMM is available such as private clouds. Since the migrate-VM rejuvenation requires two or more VMMs as stand-by system. The chapter focuses on only cold-VM and warm-VM rejuvenations, which are easily implemented even in a private cloud environment.
4.2 MRSPN-based Analysis

4.2.1 MRSPN Modeling

Markov regenerative stochastic Petri nets (MRSPNs) are one of the versatile tools for model-based performance evaluation [16, 20], which is extensions of generalized stochastic Petri nets (GSPNs). MRSPN can deal with GEN transitions whose firing time follows a general distribution, while GSPN consists of only immediate transitions and EXP transitions whose firing time follows an exponential distribution. Thus MRSPNs have high representation ability for stochastic modeling, and in fact, many application models arising in computer communication systems can be described by MRSPNs [17, 34].

(i) MRSPN for Cold-VM Rejuvenation:

Figure 4.1 and 4.2 illustrate MRSPN representation for the cold-VM rejuve-
nation presented in [37, 38]. The model is divided into four layered MRSPNs. The MRSPN (a) indicates the behavior of VMM under the cold-VM rejuvenation. Each places represent the state of VMM;

- $P_{hup}$: VMM is in the normal state.
- $P_{hfp}$: VMM is in the failure-probable state, namely, VMM is degraded by aging-related bugs.
- $P_{hfail}$: VMM has been failed (failure state).
- $P_{hdet}$: The failure has been detected and the repair operation is started (repair state).
- $P_{hrej}$: The rejuvenation operation is executed (rejuvenation state).

Also, each transition presents the event that occurs in VMM;

- $T_{hfp}$: VMM becomes the failure-probable state.
- $T_{hrej}, T_{hfprej}$: The rejuvenation operation for VMM is started.
- $T_{hrej}$: The rejuvenation operation is completed.
- $T_{hfail}$: The failure occurs in VMM.
- $T_{hdet}$: The failure has been detected and the repair operation is started.
- $T_{hrepair}$: The repair operation is completed.

The MRSPN (b) indicates the behavior of VMM rejuvenation policy. It is managed by time-triggered policy, namely, the rejuvenation is triggered by a periodic time which is represented by a GEN transition (black bar). $T_{ghinterval}$ is enabled when a token is deposited in $P_{hup}$ or $P_{hfp}$. The places $P_{hpolicy}$ means that VMM is ready to execute rejuvenation operation. When the guards $ghpolicy$ is satisfied, the tokens move to $P_{htrigger}$ and start the rejuvenation. When the VMM rejuvenation finishes, the immediate transition $T_{hreset}$ is enabled and a token is moved to $P_{hclock}$. In MRSPN (a), the transitions $T_{hrej}$ and $T_{hfprej}$ fire when tokens are moved into $P_{htrigger}$. Once the rejuvenation operation is started, the time counter of rejuvenation resets.
4.2. MRSPN-BASED ANALYSIS

Similar to the MRSPN (a) and (b), MRSPN (c) and (d) model the behaviors of VM and VM rejuvenation policy under the cold-VM rejuvenation. The places $P_{vup}$, $P_{vfp}$, $P_{vfail}$, $P_{vdet}$, $P_{vrej}$ correspond to the state; normal, failure-probable, failure, repair and rejuvenation states of VM. Also $P_{vsd}$ and $P_{vfpsd}$ represent that the VM is shut down due to the VMM rejuvenation. The place $P_{vstop}$ means that the VM is stopped by a failure of VMM or VMM rejuvenation. The transitions $T_{vfp}$, $T_{vrejt}$, $T_{vfprejt}$, $T_{vfail}$, $T_{vdet}$ and $T_{vrepair}$ are similar to the events in VMM. On the other hand, $T_{vpre}$ and $T_{vfppre}$ indicate the preventive shutdown for VM. The transition $T_{vdw}$ is the event where VM is stopped due to the VMM failure. Also, $T_{vsd}$ and $T_{vfpsd}$ are the completion of shutdown and $T_{vrestart}$ means the completion of restart of VM.

Cold-VM rejuvenation makes the hosted VM shut down before triggering VMM rejuvenation. This policy is represented in the guard functions for the immediate transitions $T_{vpre}$ and $T_{vfppre}$. Either $T_{vpre}$ or $T_{vfppre}$ are enabled when a token is deposited in $P_{hpolicy}$ in the VMM clock model. Then a token is moved into $P_{vsd}$ or $P_{vfpsd}$ in the VM model. When a token is deposited in $P_{vstop}$ by firing $T_{vsd}$ or $T_{vfpsd}$, the immediate transition $T_{hpolicy}$ is enabled by $[ghpolicy]$ and a VMM rejuvenation process starts. This ensures that VMM rejuvenation does not start until the hosted VM is shut down properly. As in the definition of $[ghpolicy]$, we assume that VMM rejuvenation can start only when a token is deposited in $P_{vstop}$, $P_{vfail}$, $P_{vdet}$ or $P_{vrej}$.

(ii) MRSPN for Warm-VM Rejuvenation:

Figure 4.3 presents MRSPN for the behavior of VM under the warm-VM rejuvenation [37, 38]. In the warm-VM rejuvenation, VM goes to suspend mode before executing the VMM rejuvenation. Thus only the behavior of VM in the warm-VM rejuvenation is different from the behavior of VM in the cold-VM rejuvenation. The guard function $[gvinterval]$ is modified to enable $T_{vinterval}$ even while a VM is in suspended state (a token is deposited in $P_{vsusd}$ or $P_{vfpsusd}$). The clock for VM rejuvenation is not affected by VM suspension. The guard function $[ghpolicy]$ enables $T_{hpolicy}$ only when a token is deposited in $P_{vsusd}$, $P_{vfpsusd}$, $P_{vfail}$, $P_{vdet}$ or $P_{vrej}$.

Warm-VM rejuvenation uses the suspend operation to stop the execution of VM at VMM rejuvenation. This policy is represented in the guard functions
for the immediate transitions $T_{vpcre}$ and $T_{vfppre}$. One of the two immediate transitions $T_{vpcre}$ or $T_{vfppre}$ is enabled when a token is deposited in $P_{hpolicy}$ in the VMM clock model. When a token is deposited in $P_{vsusd}$ or $P_{vfpsusd}$ by firing $T_{vsus}$ or $T_{vfpsus}$, the VMM rejuvenation can start. One of the immediate transitions $T_{vpost}$ or $T_{vfppost}$ is enabled, when the VMM rejuvenation is completed and a token is deposited in $P_{hup}$ in the VMM model. The hosted VM resumes the execution when a token is deposited in $P_{vup}$ or $P_{vf}$ by the transitions $T_{vresm}$ or $T_{vfpsusm}$. As defined in [ghpolicy], the VMM rejuvenation can start only when a token is deposited in $P_{vsusd}$, $P_{vfpsusd}$, $P_{vfail}$, $P_{vdet}$ or $P_{vrej}$.

4.2.2 Transient Analysis

In [37, 38], Machida et al. derived the steady-state availability of the virtualized system under the cold-VM and warm-VM rejuvenation based on the MRSPN models. On the other hand, Okamura et al. [49] performed the transient analysis for the virtualized system with cold-VM and warm-VM rejuvenation. Since the resiliency analysis requires the transient analysis, we describe the transient analysis of MRSPN.

The analysis begins with building MRGP (Markov regenerative process) from the MRSPN. MRGP is a stochastic point process which has both regenerative and non-regenerative time points. Generally, we consider a stochastic process $\{M(t); t \geq 0\}$ with discrete state space. If $M(t)$ has time points at which the process probabilistically restarts itself, the process is called regenerative, and the time points are called regeneration points. Otherwise, the time points when $M(t)$ does not restart are called non-regeneration points. Specifically, when state transition at the regeneration points is governed by a discrete Markov chain (DTMC), the process $M(t)$ is an MRGP. There are several methods for the transient analysis of MRGP such as the supplementary variable (SV) method and discretization approach [20, 64]. In this chapter, we apply the phase-type (PH) expansion to analyze MRGP.

The fundamental idea of PH expansion is to replace general distributions in MRGP with approximate PH distributions, and it can reduce the original MRGP to an approximate CTMC. The PH distribution is defined by the distri-
4.2. MRSPN-BASED ANALYSIS

but for an absorbing time in a CTMC, and it is known that PH distribution can approximate any distribution with high precision [6]. Concretely, the probability density function (p.d.f.) of PH distribution is given by

\[ f_{PH}(t) = \alpha \exp(Tt)\xi, \]  

(4.1)

where \( T \) is an infinitesimal generator on transient states in an absorbing CTMC. The column vector \( \xi \) is the transition rates from transient states to the absorbing state. When \( 1 \) is a column vector whose entries are 1, we have \( \xi = -T1 \). Also \( \alpha \) is an initial probability vector over the transient states.

To apply the PH expansion, we classify all the states of MRGP based on what GEN transitions are enabled. In both cold-VM and warm-VM rejuvenations, there are two GEN transitions: \( T_{hinterval} \) and \( T_{vinterval} \). Then we can divide the state space of MRGP into three subspaces \( S_E, S_H \) and \( S_{HV} \). The subspace \( S_E \) consists of the states where any GEN transition is not enabled. The subspace \( S_H \) is a set of the states where only \( T_{hinterval} \) is enabled. The subspace \( S_{HV} \) is a set of the states where both \( T_{hinterval} \) and \( T_{vinterval} \) are enabled. For these subspaces, we define \( Q_{x,y}, x, y \in \{ E, H, HV \} \) as the infinitesimal generators of non-regenerative transitions from \( S_x \) to \( S_y \). Moreover, \( A^H_{x,y} \) and \( A^V_{x,y} \) are transition probability matrices from \( S_x \) to \( S_y \) by firings of \( T_{hinterval} \) and \( T_{vinterval} \), respectively. Then the approximate CTMC infinitesimal generator with PH expansion \( Q^{PH} \) can be written by

\[ Q^{PH} = Q_{0}^{PH} + Q_{1}^{PH}, \]  

(4.2)

\[
Q_{0}^{PH} = \begin{pmatrix}
Q_{E,E} & Q_{E,H} \otimes \alpha_H & O \\
Q_{H,E} \otimes 1 & Q_{H,H} \otimes T_H & Q_{H,HV} \otimes I \otimes \alpha_V \\
Q_{HV,E} \otimes 1 \otimes 1 & Q_{HV,H} \otimes I \otimes 1 & Q_{HV,HV} \otimes T_H \otimes T_V
\end{pmatrix},
\]  

(4.3)

\[
Q_{1}^{PH} = \begin{pmatrix}
O & O & O \\
A^H_{H,E} \otimes \xi_H & O & O \\
A^H_{HV,E} \otimes \xi_H \otimes 1 & A^V_{HV,H} \otimes I \otimes \xi_V & O
\end{pmatrix},
\]  

(4.4)

where \( \otimes \) and \( \oplus \) are Kronecker product and sum and \( I \) and \( O \) are an identity matrix and zero matrix, respectively. In the above, general distributions of \( T_{hinterval} \)
and $T_{\text{interval}}$ are approximately given by the following PH distributions:

\[ f_{\text{interval}}(t) \approx \alpha_H \exp(T_H t) \xi_H, \quad (4.5) \]
\[ f_{\text{interval}}(t) \approx \alpha_V \exp(T_V t) \xi_V. \quad (4.6) \]

Then the approximate transient probabilities can be computed by the ordinary transient analysis of CTMC with the approximate infinitesimal generator $Q^{\text{PH}}$.

Furthermore, there are several techniques to obtain the PH distribution which approximates a general distribution. In this chapter, we use the EM-based maximum likelihood estimation (see Appendix).

### 4.3 Quantification of System Resiliency

In this section, we introduce the quantitative measure for evaluating system resiliency based on CTMC model. As mentioned before, the resiliency is one of the effects of system indices when a change happens. According to [21], we evaluate the resiliency of virtualized system with rejuvenation based on the CTMC model in the following steps:

- **Step 1**: Compute the state probability vector and availability in the steady-state.
- **Step 2**: Generate the infinitesimal generator when the change is applied.
- **Step 3**: Compute the transient behavior of availability after the change by using the infinitesimal generator of Step 2 where the initial probability vector is the steady state probability vector in Step 1.

In [21], settling time, peak overshoot or undershoot, peak time and $\gamma$-percentile time are computed from the transient behavior. This chapter computes the amount of deviation of measure which is defined by the area of difference between the steady-state measure and the deviation of measure by the change. Concretely, let $Q_N$ and $Q_C$ be infinitesimal generators of system behavior during normal and change conditions. Without loss of generality, the change is applied in the period from $t = 0$ to $t = T_c$. Also it is assumed that the measure of interest is given by a dot product of the state probability vector $\pi$ and a reward vector (column vector) $r$, namely, $\pi r$. In the chapter, since we consider
the availability as a measure of interest, the elements of reward vector are 0 or 1. If the system is available at a state, the reward of the state is 1. Otherwise, if the system is unavailable at a state, the reward is 0.

The steady-state probability vector $\pi_s$ in the normal condition can be obtained from the following equations:

$$\pi_s Q_N = 0, \quad \pi_s 1 = 1. \quad (4.7)$$

By considering the state probability vector at any time can be represented by the matrix exponential of infinitesimal generator, the amount of deviation of measure (AD) is given by

$$AD = \int_0^{T_e} |\pi_s r - \pi_s \exp(Q_C t)r| dt + \int_0^\infty |\pi_s r - \pi_s \exp(Q_C T_e) \exp(Q_N t)r| dt. \quad (4.8)$$

Figure 4.4 illustrates an example of the amount of deviation of measure. The area in the figure corresponds to the amount of deviation of measure. If the following equations hold for any $t \geq 0$:

$$\pi_s r > \pi_s \exp(Q_C t)r, \quad (4.9)$$

$$\pi_s r > \pi_s \exp(Q_C T_e) \exp(Q_N t)r, \quad (4.10)$$

then Eq. (4.8) can be simplified to

$$AD = \pi_s r T_e - \pi_s \int_0^{T_e} \exp(Q_C t)dt r + (\pi_s - \pi_s \exp(Q_C T_e)) \int_0^\infty \exp(Q_N t) dt r, \quad (4.11)$$

where we use $\pi_s \exp(Q_N t) = \pi_s$. In the above, we can apply the technique of cumulative measures [57] to the integrals of matrix exponentials.

4.4 Experiment

In the experiment, we investigate the amount of deviation of system availability when a change is applied. The model parameters are set as Table 4.1 which are also used in [37, 38]. Also VM and VMM rejuvenations are triggered by random variables. Due to the operating situation, VM and VMM rejuvenations may be practically varied. VM rejuvenation is periodically performed at time
Table 4.1: Model parameters in the experiment.

<table>
<thead>
<tr>
<th>Transition</th>
<th>Mean Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thfp</td>
<td>1 month</td>
</tr>
<tr>
<td>Thfail</td>
<td>1 week</td>
</tr>
<tr>
<td>Thdet</td>
<td>5 minutes</td>
</tr>
<tr>
<td>Threpair</td>
<td>1 hour</td>
</tr>
<tr>
<td>Threju</td>
<td>2 minutes</td>
</tr>
<tr>
<td>Tvpf</td>
<td>1 week</td>
</tr>
<tr>
<td>Tvfail</td>
<td>3 days</td>
</tr>
<tr>
<td>Vdet</td>
<td>5 minutes</td>
</tr>
<tr>
<td>Vrepair</td>
<td>30 minutes</td>
</tr>
<tr>
<td>Vreju</td>
<td>1 minute</td>
</tr>
<tr>
<td>Vad, Vfspd</td>
<td>30 seconds</td>
</tr>
<tr>
<td>Vrestart</td>
<td>30 seconds</td>
</tr>
<tr>
<td>Vaus, Vfpsus</td>
<td>0.08 seconds</td>
</tr>
<tr>
<td>Vreasm, Vfpreasm</td>
<td>0.8 seconds</td>
</tr>
</tbody>
</table>

interval which follows the normal distribution with mean 0.5, 1, 2 or 7 days, and their coefficients of variation are fixed as 0.5. Similarly, VMM rejuvenation is periodically performed at time interval which follows the normal distribution with mean 7 (days) and the coefficient of variation 0.1. These normal distributions are approximated by PH distributions with 50 phases by applying EM algorithm [46] (see Appendix). Furthermore, we consider the following change condition:

- The mean time of VM failure after aging is changed to 1/60 (hrs).

In this case, the change causes the degradation of availability. Then we can use the formula of Eq. (4.11). Also we set the time duration of change as $T_e = 5, 30, 60$ minutes.

First we investigate the steady-state availabilities when the change continues, i.e., $T_e \rightarrow \infty$. Table 4.2 shows the steady-state availabilities when the rejuvenation periods in cold-Vm and warm-VM rejuvenations are 0.5, 1, 2 and 7 days. The columns ‘Normal’ mean the steady-state availabilities under the
Table 4.2: The steady-state availabilities in normal and change conditions.

<table>
<thead>
<tr>
<th>Rejuvenation interval</th>
<th>Cold-VM Normal</th>
<th>Cold-VM Change</th>
<th>Warm-VM Normal</th>
<th>Warm-VM Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 days</td>
<td>0.9948</td>
<td>0.9917</td>
<td>0.9976</td>
<td>0.9946</td>
</tr>
<tr>
<td>1 day</td>
<td>0.9970</td>
<td>0.9940</td>
<td>0.9980</td>
<td>0.9952</td>
</tr>
<tr>
<td>2 days</td>
<td>0.9978</td>
<td>0.9951</td>
<td>0.9979</td>
<td>0.9956</td>
</tr>
<tr>
<td>7 days</td>
<td>0.9975</td>
<td>0.9956</td>
<td>0.9972</td>
<td>0.9958</td>
</tr>
</tbody>
</table>

normal condition, and the columns ‘Change’ indicate the steady-state availabilities under the change condition. From this table, we can see that there is no remarkable difference between normal and change conditions in terms of steady-state availabilities even though the failure rate in failure-probable state drastically increases in the change condition. This is because the aging phenomenon rarely occurs because of the software rejuvenation. In the table, the change does not affect the steady-state availability strongly, and thus the effect of change condition is not clear in terms of steady-state availability.

Next we investigate the system resiliency against the change. Figures 4.5, 4.6 and 4.7 illustrate the behavior of availabilities after the change is applied. In the figures, the arrow indicates the time duration of change. Also cold(0.5), cold(1), cold(2) and cold(7) mean the availabilities under the cold-VM rejuvenation. The number in the bracket indicates the time interval of rejuvenation for VM, i.e., 0.5, 1, 2 and 7 days time intervals of rejuvenation. Similarly, warm(0.5), warm(1), warm(2) and warm(7) are the availabilities under the warm-VM rejuvenation. For all the cases, the availability decreases by applying the change. However, the speeds to go back to the steady state depend on the rejuvenation policies. As the rejuvenation interval is small, the speed to go back to the steady state becomes faster. Also by comparing the cold-VM and warm-VM rejuvenations, we find the effect of change in the cold-VM rejuvenation is smaller than that in the warm-VM rejuvenation.

Tables 4.3 and 4.4 present the amount of deviation of availability under the cold-VM and warm-VM rejuvenations, respectively. From these tables, we conclude that the time duration of change is insensitive to the system availability,
and that the virtualized system with cold-VM rejuvenation is robust for the change compared to the system with warm-VM rejuvenation.

The main difference between cold-VM and warm-VM rejuvenations is whether the VMs are shut down or not at the VMM rejuvenation. Thus the VMs under the cold-VM rejuvenation have more opportunities to execute rejuvenation, compared to the warm-VM rejuvenation, and the probability of aging of VM under the cold-VM rejuvenation is relatively low. This implies that the probability that VMs are failed under the warm-VM rejuvenation is higher than that under the cold-VM rejuvenation when the change occurs. This is the main reason that the cold-VM rejuvenation is more robust than the warm-VM rejuvenation. From this insight, we also find that it is important to maintain the aging of VMs against a suddenly change of system condition.

Table 4.3: The amount of deviation of availability for the virtualized system with software rejuvenation (cold-VM rejuvenation).

<table>
<thead>
<tr>
<th>Time duration for the change (minutes)</th>
<th>5</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rejuvenation interval 0.5 days</td>
<td>0.0156</td>
<td>0.0170</td>
<td>0.0186</td>
</tr>
<tr>
<td>1 days</td>
<td>0.0275</td>
<td>0.0289</td>
<td>0.0304</td>
</tr>
<tr>
<td>2 days</td>
<td>0.0437</td>
<td>0.0450</td>
<td>0.0463</td>
</tr>
<tr>
<td>7 days</td>
<td>0.0682</td>
<td>0.0694</td>
<td>0.0704</td>
</tr>
</tbody>
</table>

Table 4.4: The amount of deviation of availability for the virtualized system with software rejuvenation (warm-VM rejuvenation).

<table>
<thead>
<tr>
<th>Time duration for the change (minutes)</th>
<th>5</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rejuvenation interval 0.5 days</td>
<td>0.0219</td>
<td>0.0233</td>
<td>0.0249</td>
</tr>
<tr>
<td>1 days</td>
<td>0.0377</td>
<td>0.0390</td>
<td>0.0404</td>
</tr>
<tr>
<td>2 days</td>
<td>0.0563</td>
<td>0.0576</td>
<td>0.0587</td>
</tr>
<tr>
<td>7 days</td>
<td>0.0765</td>
<td>0.0776</td>
<td>0.0785</td>
</tr>
</tbody>
</table>
4.4. EXPERIMENT

Guard functions

Figure 4.2: Part 2 of MRSPNs for the cold-VM rejuvenation [37, 38].
Figure 4.3: MRSPNs for the warm-VM rejuvenation [37, 38].

Guard functions

gpolicy: (#Pvsusd=1) || (#Pvfsusd=1) || (#Pvfail=1) || (#Pvde=1) || (#Pvrej=1)
gvinterval: (#Pvup=1) || (#Pvf=1) || (#Pvusd=1) || (#Pvfsusd=1)

Figure 4.4: A schematic illustration of the amount of deviation of measure.
4.4. EXPERIMENT

Figure 4.5: The behavior of availabilities after the change ($T_e = 5$ minutes)

Figure 4.6: The behavior of availabilities after the change ($T_e = 30$ minutes)
Figure 4.7: The behavior of availabilities after the change ($T_e = 60$ minutes)
Chapter 5

Conclusions

5.1 Summary and Remarks

In Chapter 2, we have dealt with the performance analysis of snapshot isolation in the database system by using probabilistic models. We have developed the models for CSI and PCSI and have derived the optimal time interval for updating snapshot in PCSI with restart scheme under the failure-prone communication environment. In the numerical examples, we have examined the effectiveness of the optimization of update timing. As a result, although the PCSI is effective to reduce the response time, the system throughput of PCSI degrades, compared to CSI. The presence of communication failure discussed in this paper is necessary and fundamental in the application of PCSI in distributed database system which has large network latency. With considering the network environment, the usage of request restart scheme is supposed be implemented in the database systems with small communication delay. In this paper, we have revealed that the effect of communication failure on the performance of distributed database system analytically. Furthermore, Monte Carlo simulations have been conducted for investigating the effectiveness of our proposal. The results of simulations indicated the effectiveness of our proposal. As a conclusion of the comparison of CSI and PCSI, it can be implied that the CSI is better at keeping the data integrity, and PCSI is better at improving the communication effectiveness in distributed environment, especially when network is crowed.

In Chapter 3, we have proposed an update management model for open
source software from the customer’s point of view based on a non-homogeneous Poisson process. We have also simply modeled user’s behavior on maintenance activity based on non-homogeneous bug-discovery/correction processes by applying an unified software reliability model. As an extension of previous study, [47] we have proposed a novel maintenance model in terms of the update cost and recovery cost for users. By surmising users’ behavior, two maintenance policies (periodic and aperiodic) have been discussed theoretically. For aperiodic policy, we derive the optimized maintenance schedule by DP algorithm. In the numerical examples, we have examined the characteristics of two policies by assuming two kinds of bug-correction processes, exponential and logistic. The existences of optimized number of update times have been proven under both policies. Moreover, based on the real bug data for Hadoop MapReduce, we have shown that the optimized aperiodic maintenance policy is more effective than the periodic one. However, we have just considered the patches for fixing common bugs, i.e., fatal bugs or security holes are not taken into account. The counteraction for emergent accidents must be the next topic of our research. On the other hand, the best policy with consideration for both users and vendors could be discussed in future.

In Chapter 4, we have evaluated the resiliency of the virtualized system with software rejuvenation. In particular, we have defined one of the quantitative criteria for evaluating the resiliency as the amount of deviation of measure, and have presented how to compute it in CTMC models. In the experiment, we focused on the amount of deviation of system availability for the virtualized system with software rejuvenation under two different rejuvenation policies. Since the original model was described by MRSPNs, the model has been approximated by a CTMC model through the PH expansion. Based on the scenario; the failure rate of virtual machine increases, we have evaluated the resiliency of system. As a result, we find that the time duration of change is insensitive to the system availability, the system with cold-VM rejuvenation is more robust for the change and the system with rejuvenation has the high ability of self-recovery. Besides, we must notice that, the resiliency in this chapter is related as the deviation of availability. If we change the performance index, the analysis of resiliency might show us different results.
5.2 Future Works

Concerning PCSI, the parameters should be estimated from empirical data collected in working database servers, and different servers have different values of parameters. Thus we will discuss autonomic algorithms for deciding the optimal time interval from empirical data based on the presented models in future.

For Chapter 4, we will investigate the resiliency of system under the migrate-VM rejuvenation, and further analyze the system resiliency of a variety of system such as checkpoint system and backup system in the future. Additionally, we try to combine the resiliency analysis and fault-tree models and consider the resiliency to disasters in the life-line system.
Appendix A

EM-based PH fitting

This paper uses the PH fitting based on maximum likelihood (ML) estimation. Briefly speaking, ML estimation is to determine the parameters minimizing the Kullback-Leibler (KL) divergence. Given an arbitrary probability density function \( f(t) \), the KL divergence \( KL(f, g) \) between \( f(t) \) and any probability density function \( g(t) \) is defined by

\[
KL(f, g) = \int_0^\infty f(t) \log \frac{f(t)}{g(t)} dt = \int_0^\infty f(t) \log f(t) dt - \int_0^\infty f(t) \log g(t) dt.
\] (A.1)

The problem is to find \( g(t) \) maximizing \( \int_0^\infty f(t) \log g(t) dt \). Applying a suitable numerical integration technique, we have

\[
\int_0^\infty f(t) \log g(t) dt \approx \sum_{i=1}^K w_i \log g(t_i),
\] (A.2)

where \( w_i \), including \( f(t_i) \), is a weight. The discretized points and their associated weights are computed by any numerical quadrature. Eq. (A.2) implies that the parameter of \( g(t) \) are determined by the ordinary ML estimation with the weighted samples \((t_1, w_1), \ldots, (t_K, w_K)\). In this paper, the weighted samples are obtained from the numerical integration with the double exponential formula [63].

On the other hand, this paper applies the EM (expectation-maximization) algorithm to derive the ML estimations for PH distribution from weighted samples [46]. Fig. A.1 summarizes one EM-step of the EM algorithm for PH distributions with weighted samples \((t_1, w_1), \ldots, (t_K, w_K)\). In Fig. A.1, the symbols \([\cdot]_i\) and \([\cdot]_{i,j}\) are the \(i\)-th entry of vector and \((i, j)\)-entry of matrix, respectively.
Also, $\Delta t_k = t_k - t_{k-1}$ is a time interval, $q > \max_i |[T]_{i,i}|$ is the maximum event rate, the matrix $P$ equals $I + T/q$ with the identity matrix $I$, and $R_k$ is a right truncation point with tolerance error $\varepsilon$;

$$
\sum_{u=0}^{R_k} e^{-q\Delta t_k} \frac{(q\Delta t_k)^u}{u!} \geq 1 - \varepsilon. \quad \text{(A.3)}
$$

In Fig. A.1, the time complexity of the EM-step is $O(\eta K)$ where $\eta$ is the number of non-zero elements in $T$. This PH fitting algorithm in the paper can be utilized by *mapfit* [50] as a package of statistical analysis tool R.
EM-step for PH fitting

1: Global step:

1.1 Forward: Compute $\hat{\xi}_k$ and $\hat{\alpha}_k$ for $k = 1, \ldots, K$;

$$\hat{\alpha}_1 \leftarrow \alpha, \quad \hat{\xi}_1 \leftarrow 1$$

$$\hat{\alpha}_k \leftarrow \hat{\alpha}_{k-1} \exp(T \Delta t_k), \quad \hat{\xi}_k \leftarrow \exp(T \Delta t_k \hat{\xi}_{k-1}), \quad \hat{\xi}_0 \leftarrow 1$$

1.2 Backward: Compute $\hat{\alpha}_k$ for $k = K - 1, \ldots, 1$;

$$\hat{\alpha}_1 \leftarrow \hat{\alpha}_{K+1} \exp(T \Delta t_{K+1}) + w_k \alpha, \quad \hat{\alpha}_K \leftarrow \frac{w_K \alpha}{\alpha \hat{\xi}_K}$$

2: Local step: For each time interval $\Delta t_k$, $k = 1, \ldots, K$;

2.1 Forward: Compute $b_u$ for $u = 1, \ldots, R_k$;

$$b_u \leftarrow P b_{u-1}, \quad b_0 \leftarrow \hat{\xi}_{k-1}$$

2.2 Backward: Compute $c_u$ for $u = R_k - 1, \ldots, 0$;

$$c_u \leftarrow c_{u+1} P + e^{-\theta \Delta t_k} \frac{(\theta \Delta t_k)^{u+1}}{(u+1)!} \alpha_k, \quad c_{R_k} \leftarrow e^{-\theta \Delta t_k} \frac{(\theta \Delta t_k)^{R_k+1}}{(R_k + 1)!} \alpha_k$$

2.3 Aggregation: Compute $H_k \leftarrow \frac{1}{q} \sum_{u=0}^{R_k} b_u c_u$.

3: Aggregation: Compute $H \leftarrow \sum_{k=1}^{K} H_k$.

4: Update:

$$[\alpha]_{i} \leftarrow \frac{\sum_{k=1}^{K} w_{k}(\alpha) \xi_{k}^{i} / \sum_{k=1}^{K} w_{k}}{\alpha \hat{\xi}_K}, \quad [\xi]_{i} \leftarrow \frac{\sum_{k=1}^{K} w_{k}(\alpha) \xi_{k}^{i} / [H]_{i,i}}{[H]_{i,i}}, \quad [T]_{i,j} \leftarrow \frac{[T]_{i,j} / [H]_{i,i}}{[H]_{i,i}}, \quad i \neq j.$$

Figure A.1: EM-based PH fitting with weighted samples [46].
Bibliography


Publication List of the Author


