MARKET EFFICIENCY AND STOCK PRICES VOLATILITY

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JOSÉ MIGUEL DUARTE LEITE PINTO DOS SANTOS
株式市場の効率性と株価変動

広島大学大学院社会科学研究科経済学専攻

JOSE MIGUEL DUARTE LEITE PINTO DOS SANTOS
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CHAPTER 1

INTRODUCTION

As is well known, Thomas Kuhn (1970) asserts that scientists attempt to resolve anomalies observed in the course of normal science, and scientific revolutions are the consequences of those attempts. Kuhn also emphasizes that a scientific progress is not made only with scientific revolutions but also with small incremental improvements inside the existing theoretical framework. This is because most anomalies can be explained soon or later within the theoretical framework from which they originate. Kuhn (1970) gives an example of such a case:

How, then, to return to the initial question, do scientists respond to the awareness of an anomaly in the fit between theory and nature? What has just been said indicates that even a discrepancy unaccountably larger than that experienced in other applications of the theory need not draw any very profound response. There are always some discrepancies. Even the most stubborn ones usually respond at last to normal practice. Very often scientists are willing to wait, particularly if there are many problems available in other parts of the field. We have already noted, for example, that during the sixty years after Newton's original computation, the predicted motion of the moon's perigee remained only half of that observed. As Europe's best mathematical physicists continued to wrestle unsuccessfully with the well-known discrepancy, there were occasional proposals for a modification of Newton's inverse law. But no one took these proposals very seriously, and in practice this patience with a major anomaly proved justified. (p.81)

Every theory in any field always seems to leave some phenomena unexplained. The usual work of the scientist, both in natural and social sciences, is to make small improvements and adjustments either in the theoretical framework or in its interpretation so that the unexplained phenomena can be brought inside the existing framework.

In this thesis, we examine critically the evidence concerning some stock market anomalies that seem to undermine two important existing frameworks in financial economics: the efficient markets hypothesis and the rational expectations hypothesis. Our main objective is to attempt to explain stock prices behaviour, especially the apparently anomalous one, within the efficient markets and rational
expectations paradigms.

This thesis is divided into two parts: in Part I theoretical considerations pertaining to the efficiency of stock markets are presented; in Part II empirical evidence against the efficiency of stock markets is presented and reexamined.

After presenting in this chapter an introduction to the contents of the thesis, we begin Part I with the presentation of the main ideas relating to efficient markets in Chapter 2, *Stock Markets and the Efficient Markets Hypothesis*. We examine first the concept of "market" and define it: a process by which buyers and sellers interact freely to determine the price and quantities transacted (Cournot (1838), Marshal (1890), Samuelson and Nordhaus (1985)). Then, by examining the main role that stock markets perform (Copeland and Weston (1988), Mishkin (1986)) we define what we understand by stock markets and exchanges (Michie (1992)). Although we deal with the secondary market and its efficiency, the role of stock markets, i.e., the transfer of funds between savers and investors will always stay in the forefront of this work. A short presentation of the historical development of stock markets (Galbraith (1954), Friedman and Schwartz (1963), Malkiel (1985), Neal (1992)) serves as a prelude to the descriptive account of the evolution of the concept of efficient markets (Smith (1776), Bachelier (1900), Cowles (1933), Keynes (1936), Samuelson ((1965), (1973)), Fama (1970), Rubinstein (1975), LeRoy (1989)). We end this chapter by distinguishing some traits associated with efficiency. We argue that since the contribution to the allocative efficiency in an economy is the ultimate goal of stock markets, operational and informational efficiency are important in the measure that they contribute to that end (Baumol (1965), Copeland and Weston (1988), Friedman and Laibson (1989)).

In Chapter 3, *Theory of Fundamentals and Bubbles*, we present the behavioural specification for the stock market model which is based on an arbitrage equation between the return of a share and the interest paid by a riskless asset (Blanchard and Fisher (1989)). We see that under certain restrictions there is a solution to this equation, called fundamentals, that explains stock prices as the discounted expected flow of future dividends. Next we consider how this fundamental model and the efficient markets hypothesis explain the movements in stock prices (Hayek (1945), Hirshleifer and Riley (1979)). Based on Ross, Westfield and Jaffe (1990) and Elton and Gruber (1987), we propose a classification of market informational efficiency related to the speed with which stock prices react to new information.

We also propose the distinction between channeling efficiency and reaction
efficiency with the analysis of how the occurrence of an event leads to changes in stock prices. The purpose of this proposal is to distinguish the time spans between the occurrence of an event and its perception by the relevant market agents, and between this perception and the moment that prices fully reflect the occurrence of the event.

Then we proceed to examine another possible solution to the arbitrage equation when one of the restrictions imposed to derive the fundamental model is lifted: bubbles (Blanchard and Watson (1982), Tirole (1982), (1985)). After defining bubbles we examine the possible causes of their formation and bursting and then see under what conditions they cannot exist (Diba and Grossman (1985)).

Part II begins Chapter 4 dedicated to the description of some Empirical Tests of the Efficient Markets Hypothesis. First we define random walks, martingales and fair games (Copeland and Weston (1988), Pindyck and Rubinfeld (1976)) and see the relation between the degree of risk aversion and these models (LeRoy (1982)). Then we explore the relationship between martingales and the fundamental model (Samuelson (1965), (1973) and LeRoy (1989)). We also make brief considerations concerning statistical tests of weak form efficiency (Fama (1965), Granger and Morgenstern (1963), Fama and Blume (1966)) and weak form efficiency anomalies (Thaler (1987), French (1980), French and Roll (1986), Ariel (1987), and others). We then examine a major anomaly, e.g., empirical observation that the variance bounds inequality is reversed (Shiller (1981a), LeRoy and Porter (1981)) and that the orthogonality equality is not satisfied (Cochrane (1991)).

In Chapter 5, Testing for Fundamentals and Bubbles in Japanese Stock Prices, we present the results of some empirical tests including weak form efficiency tests, a variance bounds test, tests for fundamentals under both the hypothesis that the expected real interest rate is constant and the hypothesis that it is variable. We end this chapter by also presenting some tests for bubbles.

In Chapter 6 Bounded Time Horizons and Efficient Markets we present a possible solution for the excess volatility anomaly. Shiller's (1981a) assumption of a linear trend for stock prices and dividends is replaced by a basic structural model (a type of stochastic trend model) for dividends (Harvey and Todd (1983), Clark (1987)), that can capture not only temporary shocks but also permanent shocks (Nelson and Plosser (1982)). We contend this basic structural model conforms better to reality especially when data with long time span is considered. Technically, the use of this model allows the easy estimation of the variable (stochastic) trend through the use of smoothing estimation of the state space form of this model. However, the adoption of
the basic structural model, by assuming that the trend of dividends is an integrated process of order two, implies that the corresponding fundamental stock prices are too volatile, sometimes negative. In order to resolve this problem we note another assumption made in Shiller (1981a) that investors form expectations about future dividends for an infinite time horizon. This assumption is no more reasonable under the basic structural model though it is so under the linear trend model. Thus assuming that the time horizon of investors is limited, we empirically estimate the span of this horizon and verify that there is a stable relationship between actual stock prices and fundamental stock prices.
PART I
THEORY
CHAPTER 2

STOCK MARKETS AND THE EFFICIENT MARKETS HYPOTHESIS

1. Definition of market

For centuries a market has been commonly defined as a place where things are bought and sold. Alfred Marshall, in his *Principles of Economics* (1890), quoted a footnote in A. Cournot (1838) in giving some definitions of markets:

Economists understand by the term Market, not only a particular market place in which things are bought and sold, but the whole of any region in which buyers and sellers are in such free intercourse with one another that the prices of the same goods tend to equality easily and quickly. (p. 270)

This view of market seems to represent a progress of the concept from a view of a pre-modern age of economics into that of a modern age. Here the emphasis is no more in the "place" but in the free intercourse that makes the prices of the same goods tend to equality.

Cournot's definition in the version given by Marshall does not restrain market to a certain place, but allows it to be the whole region where there is free intercourse between different economic agents. It seems also to be able to accommodate the (more modern) idea that the actual transaction of those things that can be bought and sold is not required, but that the free intercourse of market participants is enough to make the prices of same goods equal.

In another way this definition is also strikingly modern. Already we sense in it elements of the efficient markets concept: arguably the "free intercourse" of people is propitious to the generation and the free flow of information (a condition necessary to market efficiency), and the "[tendency of prices] to equality easily and quickly [due to the free intercourse]" brings to mind the definition that requires prices to fully reflect all available information. This sense is reinforced when Marshall adds to the above definition the following explanation:

The more nearly perfect a market is, the stronger is the tendency for the same price to be paid for the same thing at the same time in all parts
of the market.

Marshall still linked market to place, although with a much broader meaning that includes the "whole of any region in which [there is] free intercourse". More recent views of market do not refer to it as a place but as a system, process or method. Samuelson and Nordhaus (1985) give the following definition:

A market is as a process by which the buyers and sellers of a good interact to determine its price and quantity.

This definition is based on five main concepts. To make them more clear we restate it as a process (e.g., a series of actions or operations designed to achieve an end) involving interaction (e.g., to influence or to act upon each other) between buyers and sellers to determine (e.g., to decide or settle; this is the purpose or end of the process) the price and quantity of a good (meaning here something that has economic utility or satisfies an economic want).

Here we have almost all the components of Marshall’s definition plus one more. Interaction corresponds to Marshall’s "intercourse", and determination is equivalent to his "equality" (of the prices of the same goods). The new element is that market is not viewed as a place anymore, but as a process.

However, in this definition there is something missing that is present in Marshall’s: freedom. For although interaction signifies mutual influence of agents, this by itself does not imply that the mutual influence is free. An example of a process of interaction to determine the prices and quantities of goods that lacks freedom is central planning.

Bringing together Marshall’s and Samuelson and Nordhaus’ views we define market as a process by which buyers and sellers of goods interact freely to determine its price and quantity.

In our world many markets exist (commodities markets like those for corn, wheat, copper, etc, manufacture markets like those for textiles, cars, airplanes, etc, and services markets like those for baby-sitting, accounting, teaching, etc) but in what follows we will deal exclusively with financial markets and more specifically with stock markets.
2. The role of stock markets

Stock markets share with the other financial markets one of its main purposes. The main purpose of financial markets is to transfer funds between lenders (savers) and borrowers (producers/investors) efficiently. The more efficient are financial markets in transferring funds from those who have them in excess to those who are short of them, the larger is their positive effect on welfare.

Other institutions compete with financial markets in the transfer of funds between savers and producers. These are financial intermediaries such as depository institutions (commercial banks, etc), contractual savings institutions (insurance companies, etc) and others.

Also, several financial markets compete between them and with financial intermediaries to perform this transfer of funds the more efficiently as possible. Financial markets can be divided into money and capital markets according to the maturity of the securities (e.g., claims of the savers on the producers) traded in that market. Thus money markets are the financial markets where short-term debt securities are traded, and capital markets are markets where medium and long-term debt and equity securities are transacted. Capital markets can be divided into debt and equity markets according to the nature of the securities exchanged. Debt markets are where the trading of bonds (e.g., claims of the lenders to a specified interest payment at (usually) regular intervals until an established date when the principal is repaid) takes place, and equity markets are where the trading of shares of stock (e.g., claims in the net income and the assets of business firm) occurs.

Both the debt and the equity markets can be divided into primary and secondary markets according to whether new issues or previously issued securities are sold and bought. Secondary markets of both debt and equity can take the form of exchanges or of over-the-counter markets. Both exchanges and over-the-counter markets are systems (or processes) of trading. Their main difference (in our age a minor one) is that trading in the former is centralized in one location whereas in the latter it takes place in "the whole of any region in which buyers and sellers are in (...) intercourse." Major disparities exist between different exchanges in the way they are organized (some with continuous trading others with discontinuous trading, some where dealers are responsible to maintain a liquid and orderly market in the stocks which they are specialists, others where brokers just match buy and sell orders, etc) and unlikenesses exist also between over-the-counter markets in different countries in
the way they are regulated.

From the arguments of the previous section we define stock market to be a process by which buyers and sellers of shares of stock interact freely to determine their prices and the quantities exchanged. This definition comprises both primary and secondary equity markets, and also exchanges and over-the-counter markets. In spite of what follows being referred mainly to the secondary equity markets that take place in exchanges it could as well refer also to what happens in the over-the-counter market. And although most of the issues analysed are specific to secondary markets, the larger problem addressed, that of efficiency and rationality, is also relevant in primary markets, and the more so to the extent that those markets influence each other.

We follow closely Michie (1992) in defining stock exchange as a market where specialized intermediaries trade shares of stock under a common set of rules and regulations through a closed system dedicated to that purpose. Thus a stock market can be composed by several exchanges, the over-the-counter market, and other non-organized trading. We will use the term stock exchange only to indicate what takes place in organized exchanges, but will use stock market both in the sense of the definition given above that includes both exchanges and the over-the-counter market, secondary and primary markets, and also in the sense of organized exchanges.

We will not deal here in any detail with why the transfer of funds made possible and performed by stock and other financial markets and financial intermediaries increases welfare, but will just assume that it does. Our main objective is to inquire whether stock markets seem to be acting efficiently in performing that transfer or not. We will do this not directly but indirectly, by observing how stock prices behave and comparing that behaviour with the behaviour we would expect if stock markets were performing their role efficiently.

We will assume that for stock markets to perform their function of transferring funds efficiently they must behave subject to a number of restrictions. The fundamental restriction is that they must be determined according to a rational model of valuation that takes account of all available information relevant for that evaluation. This is informational efficiency and will be used as a proxy of the degree of efficiency in the transfer of funds performed by the stock market.

Thus we will not need to deal with the interesting subject of how the stock market performs this role of transferring funds between savers and producers. We will
assume that it does so through the price mechanism: changes in preferences, technologies, etc., are reflected in the prices of goods, which in turn affect the wages and profitability of their producers; changes in profitability of firms affect their stock prices which affect their ability to raise funds either through financial intermediaries or through the debt or equity markets; this ability to raise funds is then reflected in the ability to expand or necessity to contract the production of those goods for which the demand changed due to changes in preferences or for which supply changed due to changes in technologies (or other causes). The relevance of what follows depends on how well these price mechanisms work, that is to say, how well prices convey information inside and among different markets. As Morck, Shleifer and Vishny (1990) pointed out, if the stock market does not affect real economic activity, the debates over market informational efficiency, though exciting, are not important. The same can also be said if the stock market is not affected by the real economic activity.

The view that stock markets are socially inefficient institutions for the allocation of funds does not always imply the rejection of the market economy. However, it puts those who hold that the market organization of the economy is efficient but that stock markets are not efficient, in the difficult necessity of finding alternative institutions that can perform the function of allocation of funds in the economy better by providing "a more sober and hence more accurate measure of value" (Cochrane (1991)), and that do it according to market laws.

Two functions performed specifically by stock markets and subordinated to the one stated above are providing liquidity to the holders of shares of stock (the owners of the instruments of production, e.g., capital), and providing a pricing mechanism for those shares (and the capital they represent).

As a ready market for securities, stock markets ensure their liquidity, and thus allows lenders and borrowers to easily transfer funds among them. This can be understood in two ways or at two levels, as referring to the secondary market or relating to the primary market. When referring to the secondary market, lenders are those market agents who adjust their portfolios by buying shares and borrowers are those who, for some reason, want to raise money and thus sell them. When relating to the primary market, those referred as savers-lenders are the ones who buy new (first hand) shares, those referred as producers-borrowers are the ones who issue them. Although logically the secondary market cannot exist without the existence of a primary market, primary markets difficulty emerge and develop without secondary markets. The existence of secondary markets that offer liquidity encourages the channelling of savings to corporate investment, and it is in this way that stock markets
perform their role of making possible the transfer of funds between savers (lenders) and borrowers (producers). This channelling can be done with different degrees of efficiency. As will be seen later (in section 5) this degree of efficiency depends on several factors that may be present in varying degrees in different stock markets.

Stock markets also have the function of providing a pricing or a valuation mechanism of the capital represented by the shares of firms. Through this pricing mechanism, stock markets allocate capital among different firms by determining stock prices that reflect the fundamental value of a company's investment (equal to the present value of the stream of expected income accruing to the firm).

If the economic mechanisms that link secondary to primary equity markets, and both these markets to the real economy does not work properly then stock markets do not perform any economically useful function and may perform an (arguably) socially harmful one. To use the words of Morck, Shleifer and Vishny (1990) market efficiency "would not be important if the stock market did not affect real economic activity. If the stock market were a sideshow, market inefficiencies would merely redistribute wealth between smart investors and noise traders." The economic mechanisms that are usually thought to link stock markets to the real economy are two. One is the above mentioned price mechanism that may affect the real economy through its effect on investment by its influence on the costs of funds and conditions for external financing. Another possible mechanism is through the effect on the wealth of market participants.

It is a well established fact that stock returns are highly correlated with real investment (Bosworth (1975), Fama (1981), Barro (1990), Blanchard, Rhee, and Summers (1993)), although there is some disagreement concerning the cause-effect relationship.

The relationship between stock returns and consumer demand through a possible wealth effect seems to be more dubious. The effect of variations in stock prices on consumer spending does not seem strong nor significant or only marginally significant for U.S. data (Arena (1964), (1965), Bhatia (1972), Bosworth (1975)). A reason usually given for so weak effect is that share holdings are highly concentrated among the rich (Blume, Crockett and Friend (1974)). Moreover it is dubious that capital gains or losses have much influence in the consumption decisions of such individuals because capital gains and losses from shares are highly transitory and only a small proportion are actually realized (Bosworth (1975)).

If the link between equity markets to the real economy does indeed work, but those markets are in some way inefficient then their effect on the real economy can
be an harmful one. Inefficiency can arise due to the lack of some desirable characteristic on the part of the market, for example, operational or informational efficiency. Inefficiency can also arise when market participants do not behave in a fully rational way but let their decisions to be influenced by social factors such as fashions and fads (Shiller (1984) and (1989, Chapter 2)).

In this case, when stock markets are not efficient, prices will not convey any useful information from stock markets to the real economy but only misleading noise. If for example, stock markets exhibit excess volatility (Shiller (1981a), LeRoy and Porter (1981)) or any other cyclical, seasonal or other pattern not linked to the real economic activity (such as calendar-based anomalies (Thaler (1987), French (1980), French and Roll (1986), Ariel (1987)), performance related anomalies (Dreman (1982), De Bond and Thaler (1985), (1987)), and weather related anomalies (Saunders (1993))) then the effect of stock markets on the real economy will not be a beneficial effect.

3. The development of stock markets

3.1 The origins of securities markets

Stock markets and exchanges grew out slowly from the experience with other trading activities. Traders of agricultural and other products in the fairs of the Middle Ages besides using foreign coins and bullion found it convenient to start using credit, which required the supporting documents of drafts, notes and bills of exchange.

The Italians can trace their stock exchanges as far back as the 13th and 14th centuries. The French place the origin of their "Bourse" in the 12th century, when trading started in commercial bills of exchange, or at latest in the beginning of the 14th century, when the profession of courrier de change (the forerunner of the modern french stockbroker) was created. However, Michie (1992) argues that the sporadic and limited trading, in those few securities that existed, was "perfectly well handled by private negotiation or as one of the many items bought and sold in the mercantile exchanges" and thus no real stock exchange appeared until much later. Thus, he adds, "it was only from the 17th century onwards, with the appearance of both negotiable instruments representing national indebtedness and transferable stocks issued by corporate enterprise, that the volume of business generated by securities was such as to justify the beginnings of professional intermediation and organized markets".
Thus, although it seems that markets for bonds and stocks were already in existence in some countries since about the 13th century (since Roman times according to Michie (1992)), it is doubtful that stock exchanges saw light before the 17th century.

The term "bourse" began to mean stock exchange when in Bruges (in what is now Belgium), at about the start of the 14th century, merchants began gathering in front of the house of the Van der Buerse family to trade securities. From similar origins in trade and commerce, the institutional beginnings of stock exchanges began to appear during the 16th and 17th centuries in other countries of Western and Central Europe.

As intercontinental trade grew and governments' need to finance more and more expensive wars increased, the need for banks and insurance companies appeared. Intermittent capital shortages due to economic fluctuations and wars induced governments, banks and insurance companies, and the large trading companies to issue securities. From the existing exchanges for commercial notes and bills, it was an easy transition to the establishment of stock exchanges for transferable securities.

The first bond issue on the record was probably that of the city state of Florence, in the middle of the 14th century, which could be traded on a secondary market. In the middle of the 16th century the government of Charles V in the Netherlands introduced perpetual (but redeemable) annuities that were heritable, transferable and therefore fit for resale, but whose secondary market seems to have remained with modest dimensions. And in the late 16th century, in Antwerp, another financial innovation occurred, this time in the payments system, with the development of the foreign bill of exchange with serial endorsements. Because multiple endorsements increased the security of this negotiable instrument they also increased their liquidity. This innovation was carried over first to Amsterdam and then to London after the Portuguese Jews were expelled from Antwerp.

### 3.2 The first stock exchanges

The fusion of the characteristics of both perpetual annuity and the foreign bill of exchange with serial endorsements were in the origin of the modern shares of stock first issued by the Dutch East India Company (Neal (1992)). These shares were both transferable and safe long-term investments as the perpetual annuity, and safe,
liquid and internationally accepted as the foreign bill of exchange. Stock holders in the Dutch East India Company were not entitled to redeem their capital from the company but could freely sell them to another investor, and also were not entitled to fixed interest payments but only to whatever dividends might be distributed each year.

By early 17th century, shares of the Dutch East India Company started being traded in Amsterdam, the principal centre for securities trading in the world until the Napoleonic wars. In Amsterdam not only domestic but also foreign stocks were traded in the behalf of nationals and foreigners. It was in Amsterdam that many modern techniques of stock exchange dealing, like price lists and broker's dealing for their own account first appeared.

In the next hundred years, several companies, especially in France and England, issued shares that were traded in non-organized stock markets. The more famous were large trading companies like the English East India Company and the South Sea Company in England and the Mississipi Company in France.

It was following the fiasco of the Mississipi Company that the first organized stock exchange saw light, in France, in 1724. This exchange was created and tightly regulated by the government. This created the tradition of control and regulation of stock exchanges by the government that still exists in continental Europe. An example of such tradition can be found also in the stock exchange of Lisbon in Portugal created forty year after the French exchange.

The French revolution and the wars that followed it destroyed temporarily the stock markets in France and Holland. The market that would dominate securities trading for the more than one hundred years that followed, was London, whose stock exchange combined the trading techniques of Amsterdam with the organizational control (although an internal one) of Paris. The rise to dominance of the London stock market was due primarily to the rapid expansion of the British industry during the eighteen hundreds. During the remaining of the 19th century most countries in Western and Central Europe saw the appearance of stock exchanges.

The first stock exchange in the United States was established in 1791 in Philadelphia, and a second one followed in 1792, in New York, where 24 merchants and brokers decided to charge fixed commissions by acting as agents for other persons and to give preference to each other in their negotiations, set a fixed minimum commission and hold members-only auctions (White (1992)). The principal cause to the development of securities markets in the United States appears to have been the speculative trading in bonds of the Revolutionary government. The New York stock exchange was organized formally in 1817 as the New York Stock and Exchange
Board, and in 1863 it adopted its present name of New York Stock Exchange. At first
government securities formed the bulk of trading but as the 19th century progressed
shares of banks and insurance companies, railroads, steel and petroleum companies
became more and more important.

By the end of the 19th century, trading of securities in organized exchanges
was common in the industrialized nations of Europe, in the United States and in Japan
(the Tokyo Stock Exchange was formed in 1878) and also in countries under European
cultural influence, for example Argentina, Australia and South Africa. Throughout the
world until World War I, the number of securities issued and traded through stock
exchanges kept rising.

3.3 Stock markets in the 20th century

With World War I many exchanges were closed transitorily and some
permanently, but most recovered the temporary loss of their function of transferring
funds between lenders and borrowers during the ten years after the end of the war.

Stock markets suffered in general another deep crisis following the
worldwide crash of 1929 and during the years after World War II. The crash generated
doubts about their economic role and also doubts about their social and moral
goodness. In many countries, took root the view (sometimes fueled by totalitarian
socialistic and nationalistic ideologies) that stock markets are places of greed
manipulated by a faceless international capital and that could not serve the economic
and social good. Thus the view that it is necessary to closely regulate them, or to
close them, grew in most countries. In the United States, in 1934 the Securities and
Exchange Commission was established to supervise new issues and secondary trading
(Kareken (1992), Kitch (1992)), and in 1933 the Glass-Steagall Act was enacted to
prohibit the direct involvement of commercial banks in the securities markets
(Wheelock (1992)). In the 1930s stock exchanges were closed for long periods in
Germany and Spain (among other countries) and when reopened they were reduced
to minor importance (Michie (1992)) being substituted in their functions, in large part,
by the banking system. The reconstruction years after the war were also a period
when governments rather than markets played a dominant role in finance. Stock
exchanges did not reopen after the war in most countries in Asia and Eastern Europe,
war restrictions were slowly removed in European and other countries, and many
governments in newly independent countries did not encourage their creation.
In Japan, before 1945, because most shares were held by the "financial
cliques" (zaibatsu) and the proportion of publicly available shares for sale on the stock
exchanges was small, trading was thin and prices erratic. In 1945 began a campaign
to spread the stock formerly held by the financial cliques and semigovernment
corporations which resulted in increased public stock ownership. When the stock
exchanges re-opened in 1948, individual investors started playing a major role. This
in turn greatly increased the trading on the Japanese stock exchanges. By 1950
individuals owned 61% of the outstanding shares in Japan. However, from the later
fifties until the late eighties the mutual shareholdings among companies became a
dominant characteristic of the stock market. The rapid rise in stock prices in the
eighties and the steep appreciation of the yen made the Tokyo Stock Exchange,
together with the New York Stock Exchange, the largest in the world (Bronte (1981),
Hamao (1992)).

In the late 1950s and in the 1960s the mood changed again with rising stock
prices and trading volumes in Western Europe, United States and Japan due to rapid
economic growth, increased wealth and the advent of new institutional investors such
as pension funds and mutual funds and the more active participation in the market by
others like insurance companies. Stock markets also started to be opened in some
Third World countries such as Nigeria, Kuwait, Taiwan, etc, where they started to be
viewed as instrumental to economic development. In the late 1980s, for the same
reason, stock exchanges were beginning to be allowed to re-open in socialist countries
(Shanghai in 1990) and ex-socialist ones (Budapest in 1990, Warsaw in 1991). Today
many countries in the world have a stock exchange, but most still remain small and
unsophisticated. However, it is to be expected that stock markets in these countries
will evolve to the stage already reached in some other countries where efficient stock
markets provide an important role in pricing capital and providing liquidity to the owners
of that capital.

The institutional arrangements of trading in each stock exchange have
evolved under the constraints of political, social and economic factors. Reflecting
those constraints trading is done in various ways: it may occur on a continuous auction
basis, it may involve only brokers who buy and sell for other people's account or also
dealers who buy and sell for their own account, or it may be conducted only by
generalists or only by specialists or by a combination of both specialists and
generalists. Some, as in Germany, allowed commercial banks, in many ways their
competitors, to become full members, while others, as the London, New York and
Tokyo exchanges excluded them. In Great Britain and the United States there is a
tradition of internal and voluntary autoregulation by the stock exchanges. In continental Europe and Japan the tradition is one of control by the government and regulation by national laws. The efficiency of stock markets depends on these institutional and organizational arrangements.

4. Evolution of the concept of efficient markets

Many people, perhaps most people, do not think that stock markets are efficient either in their pricing or allocative functions. This view is held by most non-specialists and by some specialists. The idea that stock prices are moved by fads and bubbles, bulls and bears is strongly rooted. This is an idea that is quite at odds with the notion that stock markets might perform a useful economic function and not be just a place where money is made by some at the expense of others. The idea that stock markets are not efficient has its main roots in ideological preconceptions, and in superficial generalizations. There have been, however, some thoughtful doubts and many ingenious empirical tests that need to be reexamined carefully. This is because it is in overcoming theoretical objections and in explaining empirical anomalies that our understanding of the functions and working mechanisms of financial markets is increased. Those who hold that financial markets in general, and stock markets in particular are efficient in performing those functions have responded to their critics with different arguments and evidence through time, and in doing so have refined this concept.

The concept of efficient capital markets did not appear suddenly in the early 1970s. It has slowly evolved from the independent work of both theoretical and empirical economists. These different contributions crystallized in Fama’s (1970) definition. We will now examine briefly this evolution. We start with the period LeRoy (1989) denominated of "prehistory of efficient capital markets". Then we will present the more recent evolution of this concept.

4.1 Origins of the concept of efficiency

The concept of efficiency had its remote origins with some of the classical economists. Among them Adam Smith has an important place. In The Wealth of
Nations he exposes his 'invisible-hand' theory where he argues that the interaction of selfish agents (to employ their capital to their maximum private gain) through markets results in the promotion of the public interest (that is, in the efficient use of limited resources). The idea that free interaction through markets promotes efficient allocation of resources is fundamental to the efficient markets theory. The theme of efficient allocation of resources through markets also occurs in other economists of the 19th century.

The first known empirical work to have a relation with the concept (that was later developed) of efficient capital markets is that of Louis Bachelier who found that the prices of the bonds issued by the French government behaved in a way consistent with the random walk model. However, his work does not seem to have had connection with a theory about the workings and functions of markets, nor to have been instrumental in the development of one such theory. Notwithstanding the importance of his findings, his work remained forgotten for half a century and it does not seem to have left any visible mark on the economic thought until it was rediscovered.

The emergence of the concepts directly related to the efficient capital markets hypothesis can be dated back to the 1930s. In the early 1930s Cowles (1933) found and reported that recommendations by brokerage houses did not outperform the market and, in the following year, Working (1934) suggested that random walks have patterns that look like those found in stock prices. These results hinted that neither fundamental nor technical analyses, then widely used methods in forecasting stock prices, were of any worth.

Against these suggestions that the analysis of brokers (fundamental analysis) is not useful and that stock prices are random, in 1934 Graham and Dodd (in *Security Analysis*) and in 1938 Williams (in *The Theory of Investment Value*) popularized the concept, developed by Fisher, of fundamental or intrinsic value. According to this concept an asset's price should equal the discounted value of the cash-flow arising from it. In another camp, Edwards and Magee (1954) tried to refute the arguments against the validity of the analysis of chartists (technical analysis).

Someone who did not believe that stock prices move in accordance with their fundamental or intrinsic values was John M. Keynes who devoted one whole chapter to the stock market and to the importance of investor expectations in *The General Theory of Employment, Interest and Money*. There he wrote:
most persons are, in fact, largely concerned, not with making superior long-range forecasts of the probable yield of an investment over its whole life, but with forecasting changes in the conventional basis of valuation a short time ahead of the general public. They are concerned, not with what an investment is really worth to a man who buys it "for keeps", but with what the market will value it at, under the influence of mass psychology, three months or a year hence. (...) For it is not sensible to pay 25 for an investment of which you believe the prospective yield to justify a value of 30, if you also believe that the market will value it at 20 three months hence. (pp. 154-155).

Keynes' comparisons of the behaviour of investors in stock markets to the ways of participants in a musical chairs game or in a newspaper beauty contest are also famous. They show also that he attached more importance to the psychology of individuals in crowds than to the existence of any objective and rational formula in determining the behaviour of stock prices.

Although Graham and Dodd's and Williams' arguments won the acceptance of many (especially in the securities industry), other studies continued to find that stock prices were random. In the early 1950s Kendall (1953) in a statistical study found that stock prices follow a random walk and, about ten years later, Granger and Morgenstern (1963) found the same using spectral analysis.

Until about this time the random walk model seemed to contradict the idea that stock prices have a fundamental (rational) value. How could a price that reflects a fundamental value behave in a random fashion? In the late 1950s Roberts (1959) gave a first step in solving the contradiction between the two views. He defended that an idealized market of rational individuals should show instantaneous adjustment of prices to the arrival of new information. Because the arrival of new information is random, prices in reflecting it should also behave randomly. The alternative, a systematic slow adjustment of prices to new information, would imply the existence of profitable trading opportunities that were not being exploited (something incompatible with the existence of profit maximizers.

Although backed by empirical evidence and by the above argument, the random walk hypothesis remained somewhat alien to mainstream economic theory. Several problems remained that needed harmonization. For example, how to make the relation between random walk prices and the framework of demand and supply (with preferences and technology)? What allocative purpose serve random stock prices for?
4.2 Martingale models

The contradiction between the random and the fundamentalist views was solved and the possibility of an economic interpretation was offered by Samuelson ((1965) and (1973)). Samuelson showed that if future dividends are random variables generated according to a martingale process with respect to the information sequence \( \{ \phi_t \} \), then the expected present value of those dividends (that is, the stock price) given information \( \phi_t \), should also behave according to that process.

In this way the difference between the random walk and fundamentalist view was bridged. Stock prices can be determined according to their fundamental value of discounted future dividends and still present their observed random characteristics if dividends also follow a random process. And although prices behave randomly their formation can be explained by economic theory.

After Samuelson's papers, the proposition that stock prices are set rationally as the present value of the expected future cash flows came to be widely accepted, and is now one of the cornerstones of financial theory.

4.3 Fama's definitions

The most widely used definition of capital market efficiency was presented by Fama (1970), following the definition previously given by Harry Roberts in an unpublished paper in 1967:

A market in which prices always "fully reflect" the available information is called efficient.

Depending on what type of information is contained in the information set \( \phi \) three types of efficiency can be distinguished: (1) weak-form efficiency when the information set contains only past prices and returns; (2) semi-strong form efficiency when the information set contains any publicly available information; (3) strong-form efficiency when the information set contains all information, publicly available or not.

With this definition and the assumption that the conditions of market equilibrium can be stated in terms of expected returns, Fama drew out the implication that innovations (the difference between the observed value and the expected value...
that was projected a period before on the basis of the information then available) in stock prices and returns are fair games. A problem with this characterization of market efficiency is that it is tautological (LeRoy (1976),(1989)): innovations are always fair games.

Later, Fama (1976) proposed a different definition of capital market efficiency. A capital market is efficient if: (a) it takes into account all information relevant to determination of securities prices; (b) it has rational expectations. This definition implies that the market not only uses all relevant information to determine securities prices, but also that it uses it correctly. As pointed out by LeRoy (1989) if "the market's" information means that everybody participating in that market has the same information, then of course the market must be informationally efficient.

4.4 The Rubinstein-Latham definition

According to the definitions given by Fama ((1970) and (1976)), if a market is efficient with respect to some information, then security prices should be unaffected by revealing that information to the market participants. Rubinstein (1975) and Latham (1985) proposed that the definition of market efficiency be made according to whether the revelation of information causes portfolio changes or not. For a market to be efficient in the Rubinstein-Latham sense, prices and portfolio composition should reflect all relevant information. If, for example, a piece of information does not cause the price of a security to change but causes transactions to occur (because people disagree about the implications of that piece of information and adjust their portfolios accordingly) then the market, though efficient in Fama's sense, is not efficient in the Rubinstein-Latham sense.

Based on this definition, Rubinstein (1975) shows that prices can almost never "fully reflect information" for two main reasons: (1) if there are no future markets, even when individuals have the same beliefs about future states of the world, individuals will engage necessarily in portfolio revision (e.g., posterior trading); 2) when individuals do not share the same beliefs they will engage in speculative trading and thus necessarily have to close their speculative positions.
4.5 LeRoy's insight

LeRoy (1989) presents the theory of efficient capital markets as just the theory of competitive equilibrium applied to financial markets. Its counterpart in international economics would be the principle of comparative advantage of David Ricardo (1817). According to this view differences of information would confer comparative advantage on those who possess more information. Trading of securities is due to the existence of differences of information and thus that differential information is not fully reflected in securities prices as long as there is any trading of securities. When no difference of information exists then all information is fully reflected in the prices of securities and no trading occurs. In this situation the market is in equilibrium.

According to this view of market efficiency, financial prices are determined in consonance with the conditions of equilibrium in competitive markets by the actions of rational agents.

4.6 Problems facing the efficiency paradigm

Since the mid-1970s, and specially since the early 1980s the idea that capital markets are efficient has come increasingly under attack.

One front of attack has been theoretical. One line of argument runs as follows: if stock markets are efficient then there is no possibility of anyone earning abnormal returns. But if nobody can earn abnormal returns, nobody will invest resources in acquiring information concerning fundamental variables. In such a case it is improbable that stock prices can reflect those fundamental variables. On the other hand, if it is possible to earn abnormal returns by collecting and analysing information (and the existence of the securities analysis industry shows it is) then stock prices do not fully reflect all the relevant information (Grossman and Stiglitz (1980)). However, a model by Cornell and Roll (1981) has shown that there is no necessary incompatibility between information collection and analysis and the efficient markets hypothesis. Their basic argument is that those who spend resources in collecting and analysing information will outperform, in terms of gross returns, other investors who use less information. But once the costs incurred in collecting and analysing information are taken into consideration both kinds of investors earn the same net returns.
Another front of attack on the efficient market hypothesis has been empirical. As will be seen in more detail in Chapter 4, since the late 1960s several anomalous patterns in stock prices incompatible with the efficient market hypothesis have been uncovered. Although some have been explained inside the efficiency paradigm as being caused by some operational inefficiency, explanations have yet to be found to most anomalies.

Of special importance among anomalies has been excess volatility. This anomaly has attracted the attention of numerous economists since the first papers by LeRoy and Porter (1981) and Shiller (1981a) because of several reasons. One is the simplicity and intuitive appeal of the inequality tested. Another is due to the visual impact that Shiller's plots have. Two such plots are reproduced in Figures 8 and 9. Shiller (1984) and Tirole (1985), among others, claim that such plots alone are sufficient evidence that stock prices are inconsistent with the valuation model associated with the efficient markets hypothesis: the present value of expected future dividends model. Another reason for the impact of excess volatility is that it is interpreted as to provide in some way more 'scientific' evidence against the efficient markets hypothesis and also to constitute "a distinct and more striking phenomenon from anything conventional finance researchers document in their pedestrian examination of expected returns" (Cochrane (1991)).

After the first papers by LeRoy and Porter (1981) and Shiller (1981) sprang a vast literature trying to explain excess volatility having recourse either to rational bubbles (Flood and Garber (1980), Blanchard and Watson (1982), etc), fads and fashions (Shiller (1984), etc) and noise trading (Cabrales and Hoshi (1992), etc). A vast literature disputing the validity of excess volatility tests and the conclusions drawn from it also has appeared (Flavin (1983), Kleidon (1986), Marsh and Merton (1986), etc). Already ten years ago Fisher (1984) could write that "the balance of the argument for excess variability is now weaker than it was a few years ago". Also, LeRoy (1984) states that "the burden of the proof is now on those who contend that asset prices are too volatile, rather than on those who view the observed behavior of asset prices as consistent with market efficiency".

5. Characterization of efficiency

Capital markets exist because the transfer of funds that they make possible
has a positive effect on welfare. The more efficient capital markets are in transferring funds from those who have them in excess to those who are short of them the more positive is their effect on welfare. To make the concept of efficiency more clear we will next discuss some related concepts.

5.1 Perfect capital markets

An idealized capital market that would achieve maximum welfare is known as "perfect capital market", and is usually defined as requiring the following characteristics (Copeland and Weston (1988)): (a) perfect competition in all markets including the product market, what means that goods and services are produced at minimum average cost and all market participants are price takers; (b) frictionlessness, that is, there are no transaction costs, taxes or regulations that restrict trading prices or volumes; the trading is instantaneous and all assets should be perfectly divisible; (c) perfect information, that is, information is complete, homogeneous (interpreted in the same way by all), free and received simultaneously by all market participants; (d) utility maximization by all consumers and profit maximization by all firms; and (e) rationality by all market participants in pursuing their objectives.

A distinction can be made between instrumental and behavioural characteristics. Perfect competition, frictionlessness and perfect information are instrumental characteristics, that is, characteristics that the market itself needs to have in its regulatory and organizational features if it is to be a perfect market. Utility maximization by consumers and profit maximization by firms and rationality by all market participants in pursuing their objectives are behavioural characteristics that economic agents need to have if the market is to be perfect.

A perfect capital market is thus a market that is allocationally, operationally and informationally efficient and rational.

5.2 Allocative efficiency

It can be shown that savers, after exhausting all their productive investment opportunities with expected returns greater than a certain lending rate, will be better off if they lend their excess funds at that rate than if they invest them in productive assets with a lower rate of return. Investors, on the other hand, after exhausting all
their funds, still have an excess of investment opportunities with expected returns greater than the borrowing rate. They will be better off if they borrow the necessary funds at the ongoing borrowing rate to undertake those investment opportunities. Savers, those with an amount of funds larger than investment opportunities, can become lenders with the existence of financial markets. Likewise, investors, those with a larger amount of investment opportunities than funds, can become borrowers. In this way, both savers and investors become better off with the existence of capital markets.

5.3 Operational efficiency and frictionlessness

Operational efficiency of a market is related to the cost of transacting in that market\(^1\). The more it costs to make a transaction the less operationally efficient is that market. In capital markets, that cost typically consists of a commission paid to a broker and a tax\(^1\) on the value of the transaction. Taxes that, for example, discriminate against short holding periods make the transaction of recently bought stocks more expensive and thus also add to the other transaction costs.

Frictionlessness implies not only operational efficiency (no transaction costs) but also that no restriction exists on the price, volume or timing of any possible transaction\(^1\). In an ideal frictionless market any two parties would be able to transact at any price, for any number of securities, at any time they wished, at no transaction costs. This means that there would not exist regulations imposing the discretness of prices (for example, to within eights of one dollar in the New York Stock Exchange) or limiting the rate of change of prices (for example, the amount a stock price is allowed to vary during a certain time period, usually one day), or the volume of stocks transactioned (minimum or maximum number of shares allowed per transaction) or the time or timing of a transaction (rules mandating that shares of company B must be traded after those of company A, or that all trades must occur only once or twice a day, etc.).

The more frictionless capital markets are, the faster can prices adjust to their true value and the faster can subsequent necessary adjustments to consumption and production take place. Thus frictionlessness enhances allocational efficiency and welfare.

In the following chapters we will not take in consideration frictions like taxes or transaction costs.
5.4 Informational efficiency and perfect information

Informational efficiency has been defined in several different ways in section 4. However, a fairly simple and standard definition that we will use in the next chapters, requires that prices reflect completely and instantaneously all available relevant information.

Perfect information exists when information is complete (all relevant facts are made known), homogeneous (a piece of information means the same to everybody, that is to say, everybody evaluates information in the same way), free (no cost in producing, no price to pay) and received at the same time by all market participants.

Capital markets can possibly be informationally efficient without information being perfect. As long as prices fully reflect all information, information does not need to be perfect: Cornell and Roll (1981), for example, show that an informationally efficient market can exist with costly information: in equilibrium, those who pay for more information achieve higher gross rates of return but everyone earns the same net rate of return.

According to some interpretations, informational efficiency does not imply frictionlessness nor even operational efficiency. Fama (1970), for example, argued that as long as "transactors take account of all available information, even large transactions costs that inhibit the flow of transactions do not in themselves imply that when transactions do take place, prices will not fully reflect available information".

5.5 Efficient capital markets and perfect capital markets

Perfect capital markets were defined above as requiring perfect competition, frictionlessness, perfect information and rationality and it can be shown that they are instrumental in achieving allocative efficiency.

Efficient capital markets is an expression that can be used with two different meanings. One more specialized usage considers efficient capital markets as informationally efficient: the question is whether prices fully and accurately reflect all available relevant information or not (Fama (1970)). This is the usage found in most of the finance literature.
The other view of efficient capital markets is more comprehensive: the essential question is whether capital markets contribute positively to allocative efficiency or not, or to put it in another way, whether they have any role to play or any influence in the allocative process or not (Baumol (1965), Copeland and Weston (1988), Friedman and Laibson (1989)); perfect competition and operational and informational efficiency are just instrumental requirements for that end; profit and utility maximization and rationality are the other (behavioural) requirements.

The two meanings are not incompatible: actually one is used as a proxy for the other in the "measures" of how efficient real markets actually are. However, although the extensive use of "market efficiency" as informational efficiency is justified by the specialized and focussed character of most finance literature, it is poorer in economic meaning: if informational efficiency were not instrumental to the achievement of allocative efficiency it would not be of much interest to economic theory.

Even taking the broader meaning of efficient markets it is obvious that perfect and efficient capital markets are two different concepts. Let's make the distinction between the two clear.

First, imperfect competition in product markets does not impede the existence of efficient capital markets (Copeland and Weston (1988)): a firm can reap monopoly or oligopoly profits in the product market and still an efficient capital market determine the firm's stock price in a way that fully reflects the present value of the expected stream of monopoly profits. The allocative inefficiencies that arise in this situation are due to causes that have no direct relation with capital markets. As long as capital markets contribute positively to the general welfare by allowing the transfer of resources (not by making the transfer of resources actually happen) from low to higher productive uses they can be considered efficient. Thus, allocative inefficiencies in product markets are compatible with efficient capital markets.

Then, even if capital markets are not frictionless they can still be efficient (Fama (1970)): prices will reflect all available information and capital markets will perform their function of transferring funds even if there are costs associated with those transfers. However, limits to the quantity of funds (shares) traded or to their prices (or their rate of change) will limit both the ability of prices to reflect information and the ability of capital markets to transfer funds.

Finally, it is not necessary to have perfect information in efficient capital markets. For example, costly information is compatible with efficient capital markets (Cornell and Ross (1981)).

After seeing what differentiates perfect from efficient capital markets it is
convenient to see what brings the two concepts together. Perfect and efficient capital markets (efficiency can be taken in both the more specialized or the more comprehensive of the senses presented above) have in common the behavioural requirements they place on economic agents. These requirements are that agents should be profit and utility maximizers and rational in the pursuit of their objectives. If agents are not maximizers they are not economic agents, that is, they are not trying to get out the maximum they can from limited resources to try to fulfill their insatiable needs, and thus markets can not be either perfect nor efficient. The same can be said if agents do not act rationally in the pursuit of their objectives, even if they have them. Non rationality in the pursuit of an objective is equivalent to imperfection and inefficiency.

From now on when referring to efficiency we shall take the narrower meaning of informational efficiency.
CHAPTER 3

THEORY OF FUNDAMENTALS AND Bubbles

1. Introduction

As we have seen in Chapter 2 there are two main views of capital markets in general and of stock markets in particular.

One of these views holds that stock markets are necessary and useful institutions, instrumental in the transfer of funds between savers-lenders and investors-borrowers. This transfer allows the economy to be on its production-possibility frontier and not in a position inside it. The economy would be inside its production possibility frontier if investors were left in the situation of having investment possibilities with positive expected economic returns while savers were left with funds without any possibility of applying them in real or financial investments (Copeland and Weston (1988, p. 330), Brealy and Myers (1984, pp. 14-20)). This view that stock markets promote allocational efficiency depends on the efficiency of prices in conveying information between economic agents.

The other view is held not only by those who reject categorically markets as either indispensable or helpful institutions for any allocative process. It is also held by some who consider markets essential and of worth on the real side of the economy, but consider that capital markets, and specially stock markets, for some reason cannot perform well its functions (this seems to be the position of Shiller (1981 a),(1989)). The proponents of this view usually justify their position with the argument that stock prices actually do not transmit any useful information, and may actually convey misleading signals to the real part of the economy. It is argued that this can be inferred not only from several case studies of temporary past episodes of the ups and downs of stock prices without any apparent relation to what was happening in the real economy (De Bond and Thaler (1985), Brady (1988), Becketti and Sellon (1989), Mitchell and Netter (1989), Shiller (1987), (1989, Ch.2), Seyhun (1990), Garber (1990), White (1990), Levy and Yoder (1991), Bates (1991), Wei (1991)), but also from other empirical evidence that stock prices exhibit excess volatility, not just now and then, but usually.

The first view, that can be called the fundamentalist view, explains stock price volatility with recourse to the efficient markets and the rational expectations hypotheses. The efficient market hypothesis explains stock prices movements with the
arrival of new information about the conditions affecting a firm (or the market as a whole): any piece of information that is relevant should move stock prices up or down if the market is truly informationally efficient. The rational expectations hypothesis postulates that economic agents use all information available and a rational economic model when making expectations about future events: in this case price movements are a natural consequence of agents revising their expectations. However, unlike the efficient markets hypothesis, the rational expectations hypothesis is able to explain phenomena where prices reflect not only information about fundamentals but also information unrelated to those fundamentals.

In this chapter we will examine stock price formation from the rational expectations perspective (following Blanchard and Watson (1982) and Blanchard and Fisher (1989)). We will start by presenting, in the following two sections, a model of stock prices formation based on an arbitrage equation. This equation has two kinds of solutions: one is called fundamental, the other bubble, and they are presented respectively in sections 4 and 6. After presenting the bubble solution, we discuss briefly the processes of bubble formation, bubble bursting and the cases when bubbles can be ruled out also in section 6. In section 5 we will briefly examine what is the relation between the fundamental solution and the efficient markets hypothesis. We will defer until Chapter 5 in Part II the presentation of several tests to explore whether Japanese stock prices movements are better described by the fundamental or by the bubble solution to the arbitrage equation presented in this chapter.

2. Assumptions of a stock market model

Let us assume a stock market where shares can be traded at time t (t = 1, 2, ...). At the beginning of period or time t the share of company i is traded at price $p_{i,t}$ (for convenience, whenever i is a representative company, we drop the i subscript if no confusion results). The capital gain or loss for holding shares of company i for one period, between $t-1$ and $t$, is given by the value of $G_{i,t} = (p_{i,t} - p_{i,t-1})$. The non-negative dividend $d_{t}$ is also declared at the beginning of period $t$ but paid at the end of that period (or, what is equivalent, at the beginning of the next time period $t+1$) to individuals who held the share during time period $t$. We assume furthermore that the sequence of dividends $(d_1, d_2, ..., d_t, ...)$ follows an exogenously given stochastic process (that can, for example, be operationally dependent on the
company's output or on the demand for its output, etc; in other words, dividends are
determined by the real side of the company).

There is a finite set of individuals \( j = 1, 2, ..., J \), with rational expectations
and infinite lives, who can trade or transact the existing shares. Unless otherwise
stated, we assume that individual, investor, agent or trader \( j \) is risk neutral and that he
is also able to borrow and lend at the riskless interest rate \( r_t \). The individual \( j \) holding
of shares of company \( i \) at time \( t \) is \( j q_{i,t} \) (subscripts \( i \) and \( j \) are also dropped whenever
the meaning is clear from the context and no confusion results). The ex-post total
capital gain or loss, either potential or realized, of each individual \( j \) is given as the sum
of capital gains minus capital losses in the shares in his portfolio: \( j g_t = \sum_i (G_{i,t} \times
q_{i,t}) \). The utility of each individual is a concave function of \( j g_t \). Each individual
is also assumed to maximize his expected utility given the information he possesses.

Given the aggregate quantity of shares of company \( i, q_i \), we assume that
the condition \( \sum_j q_{i,t} = q_i \) holds always for all companies, that is,
no company issues new shares nor buys back in the secondary market shares it
issued.

Short sales are allowed so that \( j q_{i,t} \) can have both positive and negative
values. The market for a given share clears when at a certain price, after all trades
took place, no one wishes neither to buy nor to sell any more of those shares.

3. The arbitrage equation

We can use the following arbitrage equation between the return on a share
and the interest paid by a riskless asset as a behavioral specification for the stock
market model:

\[
\frac{\text{E}[p_{t+1} | t] - p_t}{p_t} + \frac{d_t}{p_t} = r_t
\]

where \( p_t \) is the price of a share or of an index or portfolio of shares at period \( t \), \( d_t \) is
the dividend of period \( t \) to be paid at the end of period \( t \) (because it is announced at
the beginning of \( t \) the dividend is known with certainty at that time), and \( r_t \) is the rate
of interest paid by the riskless asset on period \( t \), that for simplicity we assume to be
constant over time. The riskless interest rate \( r \) can also be viewed as the opportunity
cost of capital for the buyer of shares and the riskless asset can be thought of

consisting of government bonds. Finally, $E[p_{t+1} \mid t]$ denotes the expectation held at $t$ of the price of the share at $t+1$.

If risk neutral individuals can arbitrage between shares and the riskless asset, then the expected return on the share, which is equal to the expected rate of capital gain plus the dividend-price ratio on the left hand side of equation (3-1), must equal the riskless rate on the right hand side. By simple reorganization of terms of equation (3-1) we can get:

$$p_t = a E[p_{t+1} \mid t] + a d_t \tag{3-2}$$

where $a = (1 + r)^{-1} < 1$.

Equation (3-2) is an expectation difference equation where the current price depends on the current expectation of its value next period plus this period's dividend. Because the coefficient $a$ is equal to the one period discount factor and is less than one (as long as the interest rate is positive), this implies that that dependence is less than one for one. For reasons that will be apparent later, we assume that the dividends do not grow explosively over time, a reasonable assumption considering the life cycle every company seems subjected to. This is expressed by the following transversality condition:

$$\lim_{m \to \infty} a^m E[d_{t+m} \mid t] = 0 \tag{3-3}$$

To solve equation (3-2) for the behavior of $p_t$, we need to state explicitly how individuals form their expectations. Several theories on formation of expectations exist, but because of several advantages and due to the structure of our model we will assume the rational expectations hypothesis. Adaptive expectations for instance, although assume that individuals change or adapt their expectations to take account of past forecast errors, still accept that the future will be pretty much like the past. Rational expectations, in contrast, make no such assumption that the past will be projected into the future, but simply that individuals make use of all available information and in doing so they are on average correct (they make no systematic prediction errors). So, in this kind of phenomena where the present is more a function of the future than the future a function of the past, rational expectations seem to be the most appropriate model of expectations because of the exact mathematical relationship between actual and expected prices that it provides.
To make it operative we define rational expectations of $p_{t+m}$ as equal to the mathematical expectation of $p_{t+m}$ based on information available during period $t+n$ for $n < m$. We assume further that the individuals know the model (i.e., equation (3-2)) and its specification (i.e., parameter $a$) and also that all individuals have the same information at the same time. These are strong assumptions that, although not strictly necessary for the rationality of formation of expectations, are useful to keep the model simple and allow us to speak of a typical individual's expectation as the mathematical expectation based on the given information set. In practice, however, different people will be using different models and acquiring different information, and thus will be forming different expectations.

Let us define:

$$E[ p_{t+1} \mid t] = E[ p_{t+1} \mid \phi_t]$$ (3-4)

with $\phi_t = \{ p_{t-m}, d_{t-m}, x_{t-m}; m=0,\ldots,\infty \}$.

$E[ p_{t+1} \mid t]$ is equal to the mathematical expectation of $p_{t+1}$ based on the information set $\phi_t$. The information set contains current and lagged values of $p$, $d$, and $x$. $x$ is a variable summarizing other factors that though not present in equation (3-2) may help predict future values of $p$ and $d$. Since $\ldots \subset \phi_{t-1} \subset \phi_t \subset \phi_{t+1} \subset \ldots$, no memory is lost because what was known during period $t$ it is still known during period $t+1$.

### 4. Fundamentals

Linear rational expectations equations can be solved through the use of analytical methods (such as the undetermined coefficients and factorization methods). But the most convenient method for simpler cases and the one that we use here is repeated substitution.

We can write equation (3-2) at time $t+1$ and take expectations of both sides conditional on the information set at time $t$:

$$E[ p_{t+1} \mid \phi_t] = a E[ E[ p_{t+2} \mid \phi_{t+1}] \mid \phi_t] + a E[ d_{t+1} \mid \phi_t]$$

Using the law of iterated expectations $^2$ this equation becomes:
Replacing this result in equation (3-2) gives:

\[ p_t = a^2 E[ p_{t+2} \mid \phi_t ] + a^2 E[ d_{t+1} \mid \phi_t ] + a d_t \]

Repeating these steps once again for \( t+2 \), and then again up to time \( T \) we get:

\[ p_t = a^{T+1} E[ p_{t+T+1} \mid \phi_t ] + \sum_{m=0}^{T} a^{m+1} E[ d_{t+m} \mid \phi_t ] \quad (3-5) \]

For equation (3-5) to have a finite solution the second term on the right hand side needs to converge. The expected dividends average growth rate must be lower than the riskless interest rate, that is, the expected value of dividends should grow at a rate smaller than \( (1/a) - 1 \), ie., the interest rate. This is assured by the transversality condition in equation (3-3). Then, if:

\[ \lim_{T \to \infty} a^{T+1} E[ p_{t+T+1} \mid \phi_t ] = 0 \quad (3-6) \]

equation (3-5) reduces to:

\[ p_t = \sum_{m=0}^{\infty} a^{m+1} E[ d_{t+m} \mid \phi_t ] \quad (3-7) \]

Equation (3-7) is one of the solutions to equation (3-2). It gives \( p_t \), as the discounted sum of the future stream of \( d_{t+m} \), what means that the price of a share or portfolio of shares is equal to the present value of the expected perpetual stream of cash dividends of that share or portfolio of shares. This solution is usually called the fundamental solution because it reflects solely the basic or fundamental component of a share value: dividends.

Imposing the transversality condition (3-6) on equation (3-5) implies that (3-7) is a unique solution to equation (3-2). This is so as long as \( a < 1 \), that is, as long as the interest rate is positive. In equation (3-2) the effect of expected future prices on current prices has to be less than unity, what is a plausible restriction.
Another question is whether imposing condition (3-6) is justified or not. Since the implications of explosion of the endogenous variable $p$ are not inconsistent with the assumptions of our stock market behavioral model its imposition is arbitrary. Usually the justification for its imposition lies in whether the objective is to study phenomena other than bubbles. Actually, in most models of the stock market this kind of condition is only implicit.

So although this fundamental solution is the only one traditionally considered in financial economics, it is not the only solution to equation (3-2) as we will shortly see in section 6.

5. Fundamentals, efficient markets and stock prices' movements

From equation (3-7) it is apparent the way the fundamental model can explain the movements of stock prices: new information, as it is being incorporated in information set $\varphi$, causes expectations of future dividends to be revised, and in turn these revised expectations cause stock prices to move either up or down according to the nature of the revision. We examine next how new information may cause expectations to be revised.

We start by noting that what may prompt a change or revision of expectations according to the fundamental model is not the accumulated past information (which we assume is never forgotten) but the arrival of new information. This new information, needless to say, is not about future events themselves but about past or contemporaneous happenings that may affect future events (like future profits and dividends). The effect of past and contemporaneous happenings on future events could be considered to be either deterministic or probabilistic. Perfect knowledge about the actual state of the world would lead under the deterministic model to the perfect knowledge of the future states of the world. Imperfect knowledge under the deterministic model resembles a probabilistic model (and the distinction between the two, that is, which of them actually generates the observable data, is an interesting empirical problem; for literature on this topic see, for example Liu, Granger and Heller (1992) or Dechert and Gencay (1992)).

We will concentrate on probabilistic models of the world and begin by noting that the effect of the arrival of new information into the market is to change each agent's subjective probability distribution of future events or states of the world (Hirshleifer and Riley (1979)). This is what we mean when we refer to "revision of
expectations*. Assume there is a set of infinite possible states of the world \( p = (p^1, p^2, \ldots) \) for time period \( t+1 \), and that two typical individuals have the same initial beliefs at beginning time period \( t \) with prior probability distribution \( \pi_p \) concerning those possible states of the world. The investors can acquire information, receiving one of a set of infinite possible messages or news \( n = (n^1, n^2, \ldots) \) that will lead to a revision of their probability beliefs, and consequently, to a possible revised choice of action (i.e., portfolio revision). The information received is assumed to have different degrees of accuracy. To receive information with a greater degree of accuracy, investors have to pay an higher price. In Figure 1 we present an example of Bayesian recalculation of probabilities (i.e., revision of expectations) based on the accuracy of the message received. The possible states of the world (i.e., the possible stock prices) are represented as a continuum of values of \( p \) from zero to infinity. The prior probability distribution represented, \( \pi_p = \Pr[p] \), lies toward the right. This means that both individuals expect the stock price to be high next period. But as the depicted two conditional probability functions \( w^1_{n, p} = \Pr[n^1 \mid p] \) and \( w^2_{n, p} = \Pr[n^2 \mid p] \) (the likelihoods of receiving messages \( n^1 \) or \( n^2 \) for each state of \( p \)) show, the evidence received (news \( n^1 \) and \( n^2 \)) indicates that it is much more likely that the value of \( p \) at time period \( t+1 \) is a small value than a large value. Assuming that \( n^1 \) is more accurate than \( n^2 \), it is likely that the value of \( p \) will be much smaller than just somewhat smaller than the original probability. The posterior or revised probability distributions are averages of the prior probability distribution and the conditional probability distribution and are determined with the Bayes' theorem:

\[
\pi^1_{p, n} = \Pr[p^1 \mid n^1] = \frac{(\Pr[n^1 \mid p^1] \Pr[p])}{\Pr[n^1]} \quad (3-8a)
\]

\[
\pi^2_{p, n} = \Pr[p^2 \mid n^2] = \frac{(\Pr[n^2 \mid p^2] \Pr[p])}{\Pr[n^2]} \quad (3-8b)
\]

The posterior probability distributions are represented, in Figure 1, as \( \pi^1_{p, n} \) and \( \pi^2_{p, n} \) and both lie to the left of the original probability distribution \( \pi_p \) and to the right of their respective conditional probability distributions. The individuals' confidence in their initial expectations is indicated by the "spread" of their probability distribution: the more spreaded it is, the less confidence they place in it. The greater the confidence individuals have on their expectations or beliefs the lesser importance they will pay to new information. In an extreme case their prior expectation distribution would attribute probability of one to a certain state \( p^* \) and zero to all others. In this extreme case no
new information would be sought nor, if received, would it alter the prior beliefs. If the
prior expectation distribution is not as just described but individuals are somewhat
insure about the future state, we can think of another extreme case. This is when a
new piece of information $n^c$ being received which is of so high quality that the
occurrence of a particular state $p^*$ is taken as certain, in which case the revised
expectations distribution will assign probability of one to that state and zero to all other
states.

Although the efficient market hypothesis does not make any explicit
reference to the behaviour of individual investors, it also explains the movements that
stock prices exhibit with the arrival of new information to the market. This new
information, although usually non specified, is implicitly assumed by most authors to
be information that reflects the conditions that affect the value of the firms. That is,
information that reflects the changing conditions of the internal organization and
capabilities of the firms, as well as information concerning the state of the environment
that surrounds them. Therefore the use of the word "relevant" in the definition of
capital market efficiency by some authors. Thus Malkiel (1992):

*A capital market is said to be efficient if it fully and correctly reflects
all relevant information in determining stock prices.*

There are two main views as to how stock prices may reflect new
information. One is that they aggregate it, another that they average it. Information
aggregation seems to have been first suggested by Hayek (1945), and a mechanism
for aggregation has been suggested by Grossman (1976) and Grossman and Stiglitz
(1976). It proposes that stock prices reflect the full impact of the different pieces of
information held by different individuals. Fama's (1970) strong-form market efficiency
requires information aggregation. Information averaging proposes that stock prices
reflect only some average of the different pieces of information held by different
individuals. Little is known yet about how stock markets actually process incoming
information (Copeland and Weston (1988)): whether they aggregate it or average it.

The condition that requires stock prices to fully reflect available information
for capital markets to be efficient can be used to classify capital markets according to
the speed with which stock prices reflect the arrival of information, or to be more
precise, the promptness of stock prices in reflecting new information. In this way
capital markets can be either 1) efficient, if the effect of news on prices is
instantaneous and accurate, or 2) non-efficient, if the effect of the arrival of news on
prices is not immediate or not accurate. Non efficient markets can be divided further into a) **overreactive**, if stock prices react instantaneously to new information but over-react, b) **delayed**, if the reaction to news takes non negligible time to be reflected in stock prices, and c) **unresponsive**, if stock prices do not react to the arrival of relevant information. Delayed stock markets can in turn be classified as i) **strictly delayed**, if the effect of news takes non negligible time to be reflected appropriately in stock prices, or as ii) **delayed over-reactive**, if stock prices over-react, not immediately, but after a non negligible time span. Over-reactive markets and delayed over-reactive markets can still be classified further as **autocorrective**, if stock prices after over-reacting have the tendency of returning to their appropriate level, or non **autocorrective**, if stock prices in the absence of any other shocks (in the absence of any new information) stay at the level to which they over-reacted.

This classification of market informational efficiency can be presented in schematic terms in the following way:

1) Efficient markets
2) Non efficient or delayed markets
   a) Over-reactive (autocorrective and non autocorrective)
   b) Delayed
      i) Strictly delayed
      ii) Delayed over-reactive (autocorrective and non autocorrective)
   c) Unresponsive

The above classification can be made more clear with the help of Figures 2 to 6, that are adapted from Ross, Westerfield and Jaffe (1990). In the absence of any news from time period $t=0$ to time period $t=7$ the price of a certain share remains constant at 160. At time $t=8$ a piece of information that suggests that the appropriate price of the share should increase and be 180 for $t>8$ arrives to the market. It should also be noticed that the new information is information about an event that took place either before period $t=8$ or is happening during period $t=8$ and whose effects on fundamentals are expected to take place in the future ($t>8$). If the stock price reacts immediately to this news and moves instantaneously to 180 from 160 the market is efficient. In Figure 2 this is shown by the line that has a step at time $t=8$. If the stock price reacts at once to this news not to the appropriate level of 180 but to a higher level, say to 220, and then slowly autocorrects to 180 the market is over-reactive.
autocorrective. This is shown in Figure 3 by the line that goes up abruptly to 220 at
time t=8 and then reverts gradually to 180. If the stock price after reaching the
inordinately high level of 220 did not correct to 180 the market would be over-reactive
non corrective and this would be represented by a line with a step at time t=8, but a
step that would have been higher than in the efficient case. If the stock price starts to
increase gradually at time t=8 to reach the appropriate level several periods later the
market is strictly delayed. This is represented by the curve with an upward slope in
Figure 4. Finally, if the stock price starts to increase gradually, over-shots the
appropriate level, and reaches an inordinately high level the market is delayed
over-reactive. This is represented in Figure 5 by the line with the sharp upward edge
for the autocorrective case. Figure 6 depicts together the cases presented in Figures
2 through 5.

The above classification of market efficiency puts its emphasis on the
promptness of stock prices to adjust to the appropriate level suggested by new
relevant information. But by requiring that a market to be efficient the adjustment
should be immediate, it imposes a condition hard to be achieved in practice. There
is no reason to believe that the process that goes from the moment of public or private
perception of an event to the moment the appropriate changes in prices occur (what
we will call information processing) is or should be instantaneous: after the perception
of an event, investors have to draw what are the implications of that event for a certain
stock (recalculation of probabilities), contact their brokers to give their buy or sell
orders, etc, all of which are activities that take time. Also, the increase in welfare that
stock markets can achieve by providing both a mechanism of transfer of funds and a
mechanism for pricing capital does not require that the adjustment should be
immediate: investment decisions by firms and individuals also are not usually made in
a split of a second. Thus instead of the above classification, another can be made
where stock markets are said efficient or not efficient according to whether or not they
have a tendency to converge to the appropriate level suggested by the information sets
of investors. In schematic terms this classification is:

1) Efficient markets
   a) Short run efficient
   b) Long run efficient
2) Non efficient markets

Efficiency is here divided in short and long run according to whether stock
prices converge to the appropriate level immediately or not. Contrary to the previous classification, as long as stock prices converge to their appropriate level stock markets are considered efficient. Efficient markets in the short run correspond to efficient markets in the previous classification and are represented by Figure 2. Efficient markets in the long run are all those markets where the price has a tendency to converge to the appropriate level: it includes those markets previously classified as over-reactive autocorrective, strictly delayed and delayed over-reactive autocorrective, which are represented respectively in Figures 3, 4 and 5. Non efficient markets are all those where prices have not a tendency to converge to the appropriate level: unresponsive, over-reactive non autocorrective and delayed over-reactive non autocorrective markets.

It was noticed that the new information is information about an event that took place either before period t=8 or is happening during period t=8 and whose effects are expected to take place in the future (t>8). We can thus distinguish in the duration between the occurrence of an event and its impact in stock prices two time spans of different nature: 1) the time that takes an event to be perceived and 2) the time that takes from the perception of the event, that is, the arrival of the news to the investors, to the moment that stock prices begin to reflect that news. The first time span is the time that takes an event to become news and depends on a variety of factors: the degree of openess of public and private decisions, the efficiency of the media (that in turn depends on the state of the technology, working traditions and habits in the information industry, habits of the consumers of information), etc. The second time span is the time that takes stock prices to adjust to the news and depends on the market organization, on the ability of market's agents to process that information, etc. The second time span is the one to which the degree of informational efficiency of a capital market should be measured. However, the speed with which facts are made known is not of negligible interest in the study of capital markets efficiency. We can thus make another distinction in informational efficiency between 1) information channeling efficiency and 2) information reaction or information processing efficiency. This distinction applies for the three forms of efficiency defined by Fama (1970) and presented in the previous chapter: weak, semi-strong and strong forms.

Figure 7 presents schematically this distinction. Event is the happening that will have an effect on stock prices (because will affect expectations concerning future dividends, etc, in the way described in the beginning of this section). The perception of the event is the observation that the event happened and that it is relevant. Sometimes the perception of the event is contemporaneous with the event (usually
when the event is an human act or decision), other times it is posterior (when the
event is due to natural causes, like for example when lightning damages some
industrial instalations, or an act of man, that is, acts whose consequences were neither
fully averted nor desired, e.g. mistakes, inadvertences, etc). Then there is the public
announcement of the event that is logically posterior its perception but it can be
contemporaneous (if the president of a company dies or is killed during a press
conference the event, its perception and its "announcement" take place at the same
time). Then there is the public perception of the event which is the first time an
investor becomes aware of the fact (we assume that the people who work for the
media are not investors and would not phone to their brokers first and only then to
their redactors, what would be illegal; if they were also investors and engaged in this
kind of insider trading, the announcement of the event and its public perception would
be contemporaneous). The time span between the announcement and the public
perception of the event is the time it takes from the moment of the announcement to
the moment the first market participant hears from it. As already pointed it can be
contemporaneous with the announcement (as would be the case if an investor was
present at the press conference), or take minutes, hours, days or weeks depending of
the information technology available to bring the news to market participants (as would
be the case if the above mentioned press conference was taking place in some far
away island from where, due to government regulations, the only way of conveying the
news to the outside world was by surface mail). After the news has arrived to the
market (e.g., the moment an investor hears the news), market participants interact by
selling and buying and this induces stock prices to change (to "move"). Price changes,
although at the end of a long logical process, can be contemporaneous with logically
previous phases as would be the case if an investor were speaking in the phone with
its broker from the room the press conference was taking place when the president of
the company died and gave immediatly the order to sell (or buy, if the president had
already reached his incompetence level).

Information channeling efficiency and information processing (information
reaction) efficiency have different applications depending weak form, semi-strong form
or strong form efficiency is considered. Under strong form information efficiency,
channeling efficiency is related to the speed with which an event is perceived, no
matter by whom, and information processing efficiency is the speed with which stock
prices start to reflect that perception. Under semi-strong form information efficiency,
channeling efficiency depends on the speed with which an event is publicly perceived,
and information processing efficiency on the speed with which stock prices start to

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incorporate that perception. In this case, channeling efficiency includes the processes that take place since the event takes place until market participants become aware of it: the process from event occurrence to event perception, from perception to announcement, from announcement to public perception. Processing efficiency includes the process through which the information just perceived by the public of market participants is reflected in stock prices. Under weak form efficiency the information transmission process includes the process of transmission of information of events in the market itself (that is, stock prices changes, but not other kinds of information pertaining to the market like trading volumes, etc; we are following strictly Fama's (1970) definition of weak form efficiency "in which the information set is just historical prices") to the market participants (e.g., to public perception). Information processing efficiency depends on the process through which the information just perceived by actors that are the market participants is reflected in stock prices.

Some examples can make more clear the above distinction using the point of view of semi-strong form efficiency. Suppose that the Ministry of Finance begins to consider whether a tax increase or decrease might be appropriate for next year. After deciding that a change in a tax rate might be advisable it waits for the negotiations for next year's budget to start to announce its intentions. In this case, the event is the decision to alter the tax rate. The time span that goes between this event and the public perception of the MoF's intention we called information channeling efficiency. It includes the 1) time span between the decision and its perception by the insiders (the MoF's officials), that is zero in this case, the 2) time span between perception and the announcement during the budget negotiations, that can be from some days to some months, and the 3) time span between the announcement and the moment the best informed market participants hear from it, that with the actual state of the technology can take some minutes. The time that goes from this public perception by the best informed market participants to its reflection in stock prices we called information reaction (processing) efficiency.

Or suppose that a pharmaceutical company develops an anticancer medicine. The new medicine is tested and accepted by the competent certification government agency. During testing some patients die due to secondary effects of the medicine but those deads are attributed by all to third causes not related with the medicine. The medicine is a success, sells a lot, and the shares of the pharmaceutical company go up. Some time later the testing done to the new medicine is reappraised and the correct conclusion that the deads were due to the medicine is reached. When the company becomes aware of the fact that the medicine was the cause of the deads
it decides to announce the retirement of the medicine from circulation after consulting with the government agency (assume that none of the employees will sell their shares before the announcement is made). As soon as the announcement is made the share price drops. In this example, the fact that is going to make this pharmaceutical company to have to pay astronomical indemnifications to the families of the dead consumers is the wrong decision to introduce the medicine in the market. The time that goes from this event (the decision to introduce the product) to the awareness by the insiders, to the announcement and to the public perception that it was a wrong decision is the information channeling efficiency. It includes not only the time that took to perceive that the decision was wrong but also the time that goes from the apprehension of this fact to the public announcement. It also includes the time that took to the people of the media to transmit to their consumers (the participants in the stock market) what they heard at the press conference, and this depends on the technology utilized: if the press conference was in the morning, it will take five or six or more hours to the news appear in the evening edition of newspapers, two or three hours for it to appear in the next news bulletin in the radio or television, and from some minutes to about one hour for it to appear on the monitors of the subscribers to a news service. Finally the information reaction efficiency is related to the time that takes for stock prices to reflect this information from the moment it was transmitted by the fastest of all the media.

This example can also be presented from the strong form perspective: the information channeling efficiency depends on the time that goes from the introduction of the nocive product that creates a liability to the company to the moment when that liability is recognized by company insiders; the information processing efficiency depends on the time that goes from the time insiders became consious of the liability to the moment prices start to reflect their trading (because in this case there is no announcement, in Figure 7 an arrow goes directly from the perception of the event box the box labeled market). If the reaction of stock prices to insiders’ perception is so fast that insiders do not have time to trade profitably on that information, the stock market is strong form efficient. The more time insiders have to make their trades before stock prices adjust, the less strong form efficient is the markey. A weak form perspective is also possible: awareness that stock prices are falling comes from direct observation of changes in prices: this is information channeling and there are reasons to believe that it is a very fast process. Once market participants become aware that stock prices fell, their trading in the market will be at that price as long as no news reveal another price changes. This is information processing, a proceess that shold be quite efficient
because the information there is to process is minimal.

As should be apparent from the previous example, the division of informational efficiency between weak, semi-strong and strong forms of efficiency is nothing more than to move the market's information processing measure of efficiency upstream the information channelling system. When "available information" is that of the past sequence of prices the market is just processing information produced by itself (by the trading that takes place in it) and its information processing circuit is short and thus its efficiency should be high. When the "available information" is publicly available information (in the media, in financial publications and in the recommendations of brokerage houses) the information processing required of the the market to be considered efficient expands to include information outside the stock market itself and that was announced publicly to market participants though the media. When the "available information" is all information whether public or private, the information processing that leads prices to reflect new information is considered to start with the realization by someone that a relevant event to the evaluation of a share occured.

6. Bubbles

6.1 Definition

Because in equation (3-6) we imposed the condition that the expected value of the stock price in the far future does not explode, equation (3-7) was the only solution to the simple expectational difference equation (3-2). But once we lift this arbitrary condition, many other solutions to equation (3-2) are also possible. A possible solution is given by:

\[ p_t^f = p_t^{f^*} + b_t, \]  

where \( p_t^{f^*} \) will denote from now on the solution given by equation (3-7), that we call fundamentals or fundamental stock price, and provided that:

\[ \mathbb{E}[ b_{t+1} \mid \phi_t ] = a^{-1} b_t, \]  

Thus a possible solution to equation (3-2) may differ from the equilibrium
of equation (3-7) by a term \( b_t \) provided that \( b_t \) satisfies the condition of equation (3-10). If \( b_t \) is not identically zero, \( p_t \) may depend on extraneous variables, that is, on variables not affecting the fundamental value \( p_t^f \). A most important characteristic of \( b_t \) is that as \( a < 1 \), if \( b_t \) is not equal to zero the value of \( b_{t+m} \) increases to infinity as \( m \) increases; to \( +\infty \) if \( b_t > 0 \) or to \( -\infty \) if \( b_t < 0 \) as will be apparent from the next two examples. Then \( b_t \) can become an important component of \( p_t \) and because of these characteristics it is referred to as a bubble.

For better understanding of what a rational bubble is, the above equation (3-9) can be rewritten as:

\[
b_t = p_t - p_t^f
\]  

(3-11)

A bubble is thus simply the difference between the actual stock price and the fundamental one (defined in the above section as the present value of the expected future flow of dividends); furthermore, it should be increasing in accordance with the expression given in equation (3-10).

We can consider two families of bubbles: deterministic bubble and stochastic bubble. A deterministic bubble of the following type:

\[
b_t = b_0 a^{-t}
\]  

(3-12)

for arbitrary \( b_0 \) (\( b_0 \neq 0 \)), simply follows a time trend and is ever-expanding.

A stochastic bubble (West (1987), Diba (1990), Dwyer and Hafer (1990)) can have the following process:

\[
\begin{align*}
b_{t+1} &= (aq)^{-1} b_t + e_{t+1} \\
&\text{with probability } q \quad \text{with probability } (1-q)
\end{align*}
\]  

(3-13)

with \( E[e_{t+1} | \phi_t] = 0 \) and, as above, arbitrary \( b_0 \) (\( b_0 \neq 0 \)), and has each period the probability \( (1-q) \) of bursting (\( q \) is larger than zero but smaller than unity).

Let us assume for simplicity that dividends, and thus \( p_t^f \) are constant. If \( b_0 \) is positive, under the deterministic bubble the price of the share will increase exponentially to infinity although the dividends remain constant; under the stochastic bubble, as long as it does not burst, the price of the share will also increase exponentially to infinity but at a faster pace than the deterministic bubble to
compensate for the probability of a crash. The price in both cases will increase independently of any increase in dividends. Because individuals anticipate the possibility of a higher price for the share next period, they will buy it notwithstanding the price already being higher than the present value of the expected future dividends. This anticipation of ever-increasing prices satisfies the arbitrage condition and is self-fulfilling as long as the bubble does not burst. If $b_0$ were negative the price of the share would become negative in finite time. Although negative bubbles can exist in some models, we will not deal (for reasons given in sub-section 6.4 Ruling out bubbles) with them and will limit ourselves to the case of positive bubbles. If the stochastic bubble bursts, the disturbance $e_t$ allows for the formation of a new bubble. And although $e_t$ can have an independent distribution, it can also be correlated with any variable in vector $x_t$ (which is included in $\phi_t$) and still satisfy the condition that its conditional expectation be zero. So if individuals believe that sunspots affect the price, they will indeed affect it.

6.2 Bubble formation

For a probabilistic model of bubbles like equation (3-13), the disturbance $e_t$ allows for the stochastic formation of bubbles. But for a deterministic model like equation (3-12) it is difficult to specify a formation mechanism. Several reasons can be given to explain the emergence of bubbles in stock markets.

The first is simply a profit motive: anybody that has the chance of initiating a bubble (and then those who hold it immediately after him and before it bursts) can expect to receive large returns without engaging in any productive activity. This reason is also known as pyramiding or Ponzi game and to it was attributed the rise and fall of shares prices in Wall Street in 1928-29.²⁶

The second, usually known as the peso problem, is the response of market participants to inaccurate news. Even if their reaction to those news were rational the fact that the news were inaccurate (or false) leads them to follow a course of action that they would not follow had they had accurate information. For example, the announcement by an oil company of the discovery of a new large field can lead to an increase in the price of the shares of that company (a bubble), even if it turns out that the field was overestimated and the present discounted value of its production (the
fundamentals) was much lower. Because market manipulation through the spreading of inaccurate or false information can give rise to large and easy profits there is an incentive to engage in this activity. But because of the harmful consequences to other market participants, to the market, and to the economy as a whole (as stock prices no more can perform their role of providing a pricing mechanism for the capital represented by the stocks, nor of providing liquidity in an equitative way to the holders of stocks) stock market regulations that try to curb it usually exist.

A third reason for the formation of bubbles in stock prices are sunspots, that is, variables that affect prices only because individuals believe it does. If, for example, the market believes that companies that do more research should be valued more, the price may reflect the size of the research budget even if that research does not translate in practice in to any product improvement or whatever.

A fourth reason may be cross share-holdings. To see this assume there are two companies $i = 1, 2$. Consider also that, by definition, a share’s discounted expected future flow of dividends (it’s fundamental value), multiplied by the number of shares outstanding, must be equal to the net worth of that company:

$$ p_1 q_1 = W_1 \quad ; \quad p_2 q_2 = W_2 $$

(3-14)

where $p_i$, $q_i$, and $W_i$ stand, respectively, for the price of the share, the number of shares outstanding and the total net worth of a company. For simplicity we have assumed that until now no company has debt or financial assets so that total net worth is equal to total physical assets: $W_i$ finances only and completely the company’s productive activities.

We can now introduce financial assets in the form of a cross share-holding between the two companies. Equation (3-14) becomes then:

$$ p_1 q_1 = W^p_1 + W^c_1 \quad ; \quad p_2 q_2 = W^p_2 + W^f_2 $$

(3-15)

where $W^p$ and $W^f$ represent the part of net worth corresponding to productive (non-financial) and financial assets (or shares). If $v$ represents the proportion of the other company’s shares bought we have that:

$$ W^f_1 = v p_2 q_2 \quad ; \quad W^f_2 = v p_1 q_1 $$

(3-16)
Substituting $W_{1}^{p}$ and $W_{2}^{p}$ into equation (3-15) and rearranging the terms we get (for company 1):

$$p_{1} q_{1} = \left( W_{1}^{p} + v W_{2}^{p} \right) / (1 - v^2).$$

Differentiating we obtain:

$$\frac{d(p_{1} q_{1})}{dv} = \left\{ W_{2}^{p} (1 - v^2) - (-2v)(W_{1}^{p} + v W_{2}^{p}) \right\} / (1 - v^2)^2 > 0 \quad (3-17)$$

As can be seen from equation (3-17), an increase in the value of the cross share-holding implies an increase in the market value of both companies, even though the productive net worth of both companies remains unchanged. Cross share-holdings can then be one of the mechanisms of bubble formation.

A fifth reason that can lead to the formation of a bubble can be an institutional arrangement. This is the case of fiat money that has no intrinsic value. Nevertheless people accept it as having value because they have confidence that everybody else does the same due to some institutional arrangement (in most cases, a law that constrains the residents in a country to use the paper issued by some designated institution). Although fiat money is the best example of an institutional arrangement that leads to the formation and maintenance of a bubble, Malkiel (1985) describes the formation of the "South Sea Bubble" in the stock prices of The South Sea Company, 300 years ago in England, as due also to an institutional arrangement given force by the government.

### 6.3 Bubble bursting

Bubble bursting is probably the least understood phase of a rational bubble life (Evans (1989)). To begin with, the point in time when the bubble will burst and the bursting itself are uncertain. If investors knew with certainty that the bubble would burst at a certain point in time they would not want to hold it during the period before, neither during the period before that, nor during the previous one, ... all the way back to the present point in time. This type of reasoning is known as backward deductions.

One possible cause of the (possible) bursting of a rational bubble, is new information. Because the information set $\phi_{i}$ is a subset of the information set $\phi_{i+m}$.
(m > 0) we can have that:

\[ E[ p_{t+n} \mid \phi_{t+m} ] \neq E[ p_{t+n} \mid \phi_{t+m} ] \]

with \( n > m \), and where \( p \) denotes the stock price including a bubble as given by equation (3-9). That is, the expected value of a stock price at a certain future time period \( t+n \) may differ from period to period due to new information. This new information most certainly is not information concerning fundamentals, because rational bubbles are independent of fundamentals by definition. But it can perhaps be information related to the bubble itself or to a sunspot.

Another possible mechanism that can possibly trigger the bursting of a rational stochastic bubble (as it also can start the process of formation) is the the random term \( e_{t+1} \) of equation (3-13). If in a certain period the difference \( b_{t+1} - (aq)^{-1} b_t = e_t \) becomes a large negative value investors may conclude that the return they received did not match their required return (i.e., the riskless interest rate \( r_t \) of equation (3-1), section 3). If \( e_t \) is unacceptably large negative value this may start a process of revision of expectations, and lead to the conclusion that the expected return of a bubble will not match anymore the required return; and if investors start selling, this will depress further the price and the prospective returns and start a vicious cycle of selling and price decreases that becomes, like the bubble itself, a self-fulfilling prophecy.

A related cause that can lead to the bursting of a stochastic bubble is a decrease in the expected probability of continuation of the bubble, \( q \), that for some reason is not accompanied by the corresponding required increase in the rate of growth of the bubble. This would also start a vicious cycle of lower than expected returns, sales and lower prices.

### 6.4 Ruling out bubbles

One aspect that was not considered in the previous subsections should be dealt with now. It refers to the situations when bubbles can be ruled out from occurring in the stock market.

We can start, for example, by ruling out negative bubbles. Assets like shares of common stock were the liability is restricted to the value of the share itself
can not have a negative price\(^2\). Because a bubble must always be increasing in any dynamic model a negative bubble would imply a negative price in finite time, which is impossible (Diba (1990)). In a more general way, bubbles can be ruled out from occurring in assets that can be freely disposed when they become liabilities.

Another argument advanced to eliminate positive bubbles, in physical assets, is that, if a substitute exists in infinitely large supply, possibly at a very high price, then there cannot exist positive bubbles (Blanchard and Fisher (1989)). If a positive bubble would appear, then the expected price would go to infinity and consequently exceed the price at which the substitute was available, what is impossible in a world with rational agents.

A similar argument applied to the stock market would argue that as soon as a bubble would appear in a company's shares it would be in the interest of the initial shareholders to issue more shares and invest the proceeds as long as the bubble would exist. As more and more shares were issued at an increasing rate it can be argued that it is doubtful that the market would continue to absorb that ever increasing supply of shares unless their price decreased.

However, positive bubbles can not be ruled out when individuals have finite lives (exhibit myopic rational expectations), but they can be ruled out when individuals have fully dynamic rational expectations (what would happen with infinitely lived individuals) (Tirole (1982)).

We can also rule out bubbles in assets whose prices are subject to some terminal condition in the future, such as bonds. This is because their price at maturity is a fixed value (equal to their fundamental value) and no bubble can exist then. If no bubble exists at the terminal period it will also not exist the period before. Working backward in time we can see that a bubble can never exist in this kind of assets where there is a terminal condition on their price.

Concluding, bubbles may possibly exist (cannot be ruled out) when the two following conditions are present: (1) fundamentals are difficult to evaluate, and (2) individuals exhibit short run behavior (without infinite horizons rational expectations).
PART II
EVIDENCE
1. Introduction

In this first chapter of Part II we will examine a set of empirical studies that test whether stock prices behave in a way compatible with stock markets performing their function of transferring funds between economic agents who have them in excess to those who are short of them. Although the results of some of these tests are supportive of the weak form efficiency hypothesis, and thus of the view that stock markets are performing efficiently their role, the results of other tests are not so.

For stock markets to be able to perform their function, it can be shown that stock prices may move in certain ways but not in others. Economic justification can be given, for example, to stock prices that follow a martingale process, and also to stock prices that move in step with variables like dividends (or profits or earnings) that are the underlying basis for the value of share of stock.

But if stock prices' movements have no relation to the underlying economic variables then their economic raison d'être ceases to exist. Stock markets may then exist for other reasons but not as an instrument to an efficient allocation of resources. This would be the case if stock prices did not move enough (or did not move at all) or did move too much in relation to the movements of the fundamental variables.

In this chapter we will examine first, in the next three sections, the behaviour of stock prices per se. In section 2 we will give the basic definitions of random walks, martingales and fair games, and then, in the section that follows we will see the relation between martingales and the fundamental model introduced in Chapter 3. In section 4 we will present some statistical procedures whose objective is to test the weak form efficiency hypothesis.

Then in the last three sections we will present some evidence that seems to contradict the weak form efficiency hypothesis. In section 5 we will review weak form efficiency anomalies. In the following two sections we will critically make a brief examination of the excess volatility literature, variance bounds tests and orthogonality tests.
2. Definition of random walks, martingales and fair games

Random walks and martingales have long been used in financial economics to explain variables of interest such as stock prices (Working (1934), Kendall (1950), Fama ((1965) and (1970)), prices of bonds, etc. More recently their use has expanded to other areas of economics. For example, the use in macroeconomics of dynamic equilibrium approaches have imposed martingale restrictions on other time series. Two cases are Hall (1978), that shows cases where the marginal utility of consumption follows a martingale, and Barro (1981), that shows that tax rates also may follow this process.

A variable like stock price differences or rates of return follows a random walk process if its probability distribution given information set \( \phi_t \) is the same as its unconditional distribution. In formal terms, for stock price differences:

\[
f(\Delta p_{t+s}) = f(\Delta P_{t+s} | \phi_t)
\]  

(4-1)

for all \( s > 0 \), where \( f(\cdot) \) denotes a probability distribution function. This requires that each successive value of \( \Delta p_t \) (each successive return) is drawn independently from a probability distribution with zero mean and constant variance. Thus, \( p_t \) can be determined by the following process:

\[
p_t = p_{t-1} + \epsilon_t
\]  

(4-2)

with \( \mathbb{E}[\epsilon_t] = 0 \) and \( \mathbb{E}[\epsilon_t \epsilon_{t-s}] = 0 \) for all \( s \neq 0 \). Assuming that the information set \( \phi_t \) includes all past prices (and only that information), the conditional forecasts of \( p_{t+s} \) (\( s > 0 \)) (denoted by \( \hat{} \)) at time \( t \) are given by:

\[
\hat{p}_{t+s} = \mathbb{E}[p_{t+s} | p_t, p_{t-1}, ...] = p_t
\]

with variance of forecast error equal to:

\[
\sum_{k=1}^{s} \mathbb{E}[\epsilon^2_{t+k}] = s \sigma^2
\]
where \( \sigma^2 \) is the variance of \( \epsilon \). Thus, the standard error of forecast increases with the square root of \( s \). Because stock prices have a long run tendency to increase (have an upward trend) a drift \( d (d>0) \) can be added in equation (4-2):

\[
p_t = p_{t-1} + d + \epsilon_t
\]

Now the \( s \)-period ahead forecast is:

\[
\hat{p}_{t+s} = \mathbb{E}[ p_{t+s} | p_t, p_{t-1}, \ldots ] = p_t + s \cdot d
\]

with standard error of forecast remaining the same as before.

Stock prices \( p_t \) are defined to follow a martingale process with respect to information set \( \phi_t \) when:

\[
\mathbb{E}[ p_{t+1} | \phi_t ] = p_t \quad (4-4)
\]

A martingale process can also have a drift:

\[
\mathbb{E}[ p_{t+1} | \phi_t ] = p_t + d \quad (4-5)
\]

Equivalent to a martingale with drift is a submartingale process. Stock prices \( p_t \) follow a submartingale process when:

\[
\mathbb{E}[ p_{t+1} | \phi_t ] \geq p_t \quad (4-6)
\]

The price change \( \Delta p_{t+1} = p_{t+1} - p_t \) is a fair game if:

\[
\mathbb{E}[ \Delta p_{t+1} | \phi_t ] = 0 \quad (4-7)
\]

If the stochastic process \( \Delta p_{t+1} \) is a fair game then \( p_t \) is a martingale. Fair game and martingales make no restrictions on the variance of the distribution of stock prices and thus the nonstationarity of the variance of stock prices is not a problem. Random walks require the sequence of \( \Delta p_t \)'s to be independent events from the same distribution, what implies that serial covariances between price differences for
any lag must be zero; fair games and martingales do not make that restriction. Thus random walks make a stronger restriction on the time series behaviour than do martingales because higher-order dependencies among the residuals \( \epsilon_i \) may exist without violation of the martingale properties. Consequently a random walk is always a martingale but a martingale is not always a random walk.

Because they do not make any restrictions on the second and higher moments of the generating distribution, martingales correspond to a world of risk neutral agents. Risk neutral agents are indifferent to second and higher order dependencies among the residuals and thus do not bother to bid those dependencies away (LeRoy (1982)).

As a practical matter, however, it is not possible to differentiate between a random walk and a martingale because the regression methods that are usually used to test what process a time series follows assume serially independent errors. Thus the optimal properties of the statistical tests used to examine whether or not a series follows a martingale exist only under the stronger random walk restriction.

The above definitions of martingales (4-4), submartingales (4-6) and fair games (4-7) could have been made for other variables instead of stock prices.

### 3. The fundamental model and martingales

Expected rate of return is defined as being equal to the expected rate of capital gain plus the dividend yield:

\[
E[\rho_t | \phi_t] = \frac{E[p_{t+1} | \phi_t] - p_t}{p_t} + \frac{d_t}{p_t}
\]

If the stock market is weak form efficient and if risk neutral individuals can arbitrage between stocks and a riskless asset that pays a rate of return \( r_t \) (that sometimes is assumed to be constant over time: \( r_t = r \) for all \( t \)) then \( E[\rho_t | \phi_t] = r_t \) and the above equation becomes:

\[
\frac{E[p_{t+1} | \phi_t] - p_t}{p_t} + \frac{d_t}{p_t} = r_t
\]

Rearranging terms equation (4-9) becomes an expectational difference equation:
Equation (4-10) expresses that the stock price today equals the sum of the expected future price and dividends discounted back to the present at rate $r_t$. By writing equation (4-10) at time $t+1$, taking expectations of both sides conditional on the information available at time $t$, using the law of iterated expectations (which states that for any $p$: $E[ E[ p | \phi_{t+1} ] | \phi_t ] = E[ p | \phi_t ]$), substituting this result back in equation (4-10) and solving recursively up to infinity, results that stock prices equal the expected present value of future dividends:

$$p_t = \left( 1 + r_t \right)^{-1} \left( E[ p_{t+1} | \phi_t ] + d_t \right)$$

(4-10)

Equation (4-11) is also the result derived by Samuelson ((1965) and (1973)) that brings together the fundamental and martingale models, and holds when agents have common and constant time preference, have common probabilities and are risk neutral. It implies that if the sequence $\{d_t, d_{t+1}, \ldots\}$ has the martingale property of equation (4-4) (applied to dividends) then stock prices will also follow a martingale process.

If agents have common and constant time preference, have common probabilities and are risk neutral then they will prefer to hold the asset that has the highest expected return, ignoring differences in risk. Because in equilibrium all assets are held willingly any two assets must have the same expected rate of return. Thus, as implied in the above equation (4-9), holding stocks or holding a riskless asset is a fair game: no asset has higher expected return than the other. The difference between the rate of return of a stock $\rho_t$, and the rate of return of the riskless asset $r_t$ is then a fair game:

$$E[ \rho_{t+1} - r_{t+1} | \phi_t ] = 0$$

(4-12)
This is equivalent to say that prices plus reinvested dividends follow a martingale (LeRoy (1982),(1989)). To see this let $v_t$ be the discounted value at time zero of the value at time $t$ of an initial share plus its reinvested dividends, and $h_t$, the number of shares at time $t$ whose value is equal to the value of the initial share plus its reinvested dividends. Then, the value of $v_t$ is equal to:

$$v_t = p_t h_t / \prod_{i=0}^t (1 + r_i)$$  \hspace{1cm} (4-13)

and the value of $h_{t+1}$ satisfies:

$$p_{t+1} h_{t+1} = (p_{t+1} + d_t) h_t$$  \hspace{1cm} (4-14)

Let's consider now the expected value at time $t$ of $v_{t+1}$:

$$E[v_{t+1} | \phi_t] = E[p_{t+1} h_{t+1} / \prod_{i=0}^t (1 + r_i) | \phi_t]$$

$$= E[(p_{t+1} + d_t) h_t / \prod_{i=0}^t (1 + r_i) | \phi_t]$$

$$= p_t h_t / \prod_{i=0}^t (1 + r_i) = v_t$$  \hspace{1cm} (4-15)

In equation (4-15), after taking expectations to equation (4-13) at time $t+1$, equations (4-14) and (4-10) were used successively to show that a series of prices plus reinvested dividends follows a martingale.

As we have seen in the previous section, risk neutrality implies that if returns are a fair game (equation (4-12)) then prices plus reinvested dividends are a martingale, but it does not imply the more restrictive random walk model. If, as risk neutrality implies, individuals do not pay attention to the higher moments of their return distributions they will not bid away any serial dependence in the higher conditional moments of returns that might exist. Therefore risk neutrality is compatible with nonzero serial correlation in conditional variances because no one cares about these
variances.

4. Statistical tests of weak form efficiency

Many tests, that do not take risk in consideration, were utilized in testing the weak form efficiency hypothesis. Some are here reviewed.

(a) Linear relationships - The first tests examined whether price changes are linearly related over time. A possible specification of this test is:

$$p_t - p_{t-1} = \alpha + \beta (p_{t-T} - p_{t-1-T}) + \epsilon_t$$

where the term $\alpha$, called drift, measures the expected change in price unrelated to the the previous price change. As long as stocks have a positive expected capital gain $\alpha$ should be positive. Since usually real stock prices have a long run tendency to increase (Fama and French (1988), Francis (1991)) $\alpha$ should be included in linear relationship tests. The term $\beta$ measures the relationship between two price changes, between one occurring at time $t$ and another at time $t-T$. If $T=1$ then it measures the relationship between two consecutive price changes; if $T=2$ it measures the relationship between a price change and the one two periods before; and so on. If $\beta$ is not significantly different from zero then stock prices follow either a random walk or a martigale and this evidence can be interpreted in favor of market efficiency. Notice that if $\alpha$ was missing in the specification of equation (4-16) the long run stock price would be reflected instead in a positive $\beta$. Finally, $\epsilon_t$ is a random variable assumed to be normally independent and identically distributed with mean value of zero. It incorporates the variability of the current price change not related to the previous price change.

Other approaches to test the linear relationship between stock prices is to use either the rate of return (e.g., $(p_t - p_{t-1}) / p_{t-1}$) or the natural logarithm of price relatives instead of the first differences of real stock prices. The three approaches are equivalent (Granger (1975), Elton and Gruber (1987), Kariya, Tsukuda and Maru (1989)). The approach using the natural logarithm of relative prices is presented in next equation:

$$\ln \left( \frac{p_t}{p_{t-1}} \right) = \alpha + \beta \ln \left( \frac{p_{t-T}}{p_{t-1-T}} \right) + \epsilon_t$$

(4-17)
Using this later approach Fama (1965) found that there was “no evidence of substantial linear dependence between lagged price changes or returns” (Fama (1970,p.394) for several values of T for most of the 30 of the Dow Jones Industrials. This is interpreted as evidence in support of the efficient markets hypothesis. As Fama (1970) notes, this does not exclude the possibility of non linear relationships between lagged price changes or returns. In the 1960s and 1970s many other linear relationship studies about stock prices have been published. Many different securities, many different stock price indexes, many different laggs have been used, but they have failed to detect any significant patterns for daily, weekly or monthly data (Granger (1975)) what is interpreted as evidence in favour of the efficient market hypothesis.

However, using monthly data for the period 1926 to 1985 Fama and French (1988) found highly significant negative serial correlations in stock price returns that are lagged 3, 4, or 5 years. This is interpreted as being due to the relationship between stock markets and the business cycle (Francis (1991)).

(b) Runs - The previous test has an important drawback: estimated $\beta$ tends to be heavily influenced by extreme observations. An alternative test that eliminates the effect of very large observations consists of examining the sign of prices changes. Let a price increase is designated by +, a price decrease by - and a no change by 0. A sequence of the same sign is called a run. Then, if price changes are positively related, it is more probable that a + be followed by a + than by a - , and a - by a - than by a + . In fact, if there is a positive correlation between price changes then there should exist more long sequences of + and - than could be attributed to chance; thus, there should also be fewer runs than if price changes were generated by chance. If there is a negative correlation, then there should be too many short sequences and too many runs. Because runs tests depend only on the sign of the change, it is irrelevant whether price differences, or logarithms of price relatives or rates of return are used.

Fama (1965) found that for one-day intervals there were somewhat fewer runs than it would be expected if prices were random, and that for longer intervals the actual number of runs was in every case almost exactly equal to the theoretically expected number. This is interpreted as evidence of a small positive relationship between the log of successive price relatives. Fama (1970) reviews the runs tests performed by Niederhoffer and Osborne (1966).

(c) Spectral analysis - Using the Fourier transformation this method can be used to search for cycles in stock prices. Such cycles, if found, would constitute evidence against the efficient market hypothesis. Granger and Morgenstern (1963), who did most of the early work in this field, did find evidence of slight monthly and seasonal
cycles. Later works by Lo and MacKinlay (1988), Porteba and Summers (1988) and Durlauf (1991) have also used spectral analysis to test whether stock price returns follow a martingale. Their results, though negative to the efficient market hypothesis, do not seem to be strong.

(d) Filter rules - Filter rules, by assuming that there are systematic patterns in the movements of prices over time, specify buying and selling rules that are supposed to make profits to those who use them. A y% filter rule is: if the price of a stock rises y% buy and hold it until it falls y% and then sell and go short of it. Keep the short position until the price rises y% when the short position should be covered and a long position opened.

Fama (1970) reviews works by Alexander (1961) and Fama and Blume (1966). In general, these tests show that, even before subtracting transaction costs, filters greater than 1.5% do not do better than a simple buy-and-hold strategy; filters smaller than 1.5% make generally very small gross profits that are better than the market average because trading is very frequent; but net profits are null. Thus stock markets are not only informationally efficient but also operationally efficient. Also, because all the securities had average positive returns, another inference from the results of filter tests is that stock prices seem to follow a submartingale.

5. Weak form efficiency anomalies

Seasonal patterns that are inconsistent with the weak form of the efficient markets hypothesis have recently been found in stock prices. Following are some examples of calendar-based anomalies.

(a) The January effect - This anomaly is also known as "turn of the year effect". It is thus named because U.S. stock returns seem to be abnormally high during the first few days of January and especially so for small firms. This is a pattern that being nonstationary is inconsistent with both the martingale and the submartingale models and the efficient markets hypothesis (Rozeff and Kinney (1976), Reinganum (1981), (1982) and (1983), Roll (1983), Thaler (1987), Clark and Ziembra (1987)). This effect is also found in Japanese stock markets (Kato and Schallheim (1985), Hawawini (1991)).

(b) The weekend or day of the week effect - Average stock returns seem to be consistently negative from the close of trading on Friday to the close of trading on Monday (French (1980), Lakonishok and Levi (1982), (1985)), Rogalski (1984), Jaffe
and Westerfield ((1985a), (1985b)), Harris (1986), Smirlock and Starks (1986)), but the other days of the week seem to have positive returns of varying magnitude, Friday being the day with the higher average returns. Patterns are found on even a smaller time scale. Rogalski (1984), Smirlock and Starks (1986) and Harris (1986) established that the weekend and day of the week effects are concentrated in the first minutes of trading each day. Stock prices tend to drop during the first 45 minutes of trading on Monday morning, whereas on other weekday mornings they tend to rise. Also, stock prices usually increase during the last 15 minutes of trading on all days of the week. This behaviour of stock prices is inconsistent with the weak form of the market efficiency hypothesis. The day of the week effect seems also to be different between some markets: Pettway and Tapley (1984), Jaffe and Westerfield ((1985a) and (1985b)), Ikeda (1988), Kato (1991) and Kariya, Tsukuda and Maru (1989) report that in Japan Tuesday has the lowest average returns (that are negative) and Wednesday the highest.

Several explanations for this anomaly according to the efficient market hypothesis have been advanced. Gibbons and Hess (1981) advance the hypothesis that this anomalous pattern is due to settlement procedures and Lakonishok and Levi ((1982) and (1985)) add as a possible explanation the check clearing process. However, the results of the tests in neither paper support these hypotheses completely and Dyl and Martin (1985) provide evidence against it. Other possible explanations have been advanced: specialist related bias (Keim and Stambaugh (1984)) and measurement error (Gibbons and Hess (1981)) but both seem to have low explanatory power.

(c) The Wednesday effect - In order to facilitate the processing of orders at brokerage houses the New York Stock Exchange was closed on Wednesdays in 1968. It was found by French and Roll (1986) that the volatility of prices was lower from Tuesday to Thursday than over other two-day intervals. This suggests that stock prices fluctuate more when markets are open than when they are closed which is incompatible with the reasonable assumption that as much news about fundamentals is generated on Wednesdays as in any other weekday. Thus it seems that it is the trading process itself rather than the arrival of news about fundamentals that generates much of the prices changes.

(d) The turn of the month effect - It was found by Ariel (1987) that the returns are in average higher during the the last day and first days of each month.

Other anomalies are performance related:

(e) The P-E anomaly - This is considered to be one of the oldest anomalies.
It was discovered by Fritzemeier (1936) even before the efficient market hypothesis came into existence in the late 1960s. Besides Fritzemeier, Nichlson (1968), Basu (1977 and 1983), Reinganum (1981) and Dreman (1982) report that stocks with low price-earnings ratios appear to outperform systematically those with high price-earnings ratios.

(f) The losers anomaly - Stocks that suffer large drops in price appear to outperform systematically those who did not suffer drops in price (DeBondt and Thaler (1985) and (1987)).

(g) The price to book value anomaly - Stocks with low price to book value ratio seem to outperform those with higher ratios (Rosenberg, Reid and Lanstein (1985)).

(h) The small firm anomaly - Stocks of small firms appear to outperform systematically those of larger firms. Evidence was found that links this anomaly to the January anomaly (Keim (1983)).

These results may suffer from a selection bias problem (Merton (1987)): odd results are published because they are odd but confirmation of absence of an anomaly is not even submitted to publication because it does not constitute "new" or "original" research. Consider, for example, the following anomaly: stock prices depend on the weather. If no correlation is found between stock prices and the weather no results will be published; but if a correlation is found then it will (actually Sauders (1993) has found that the weather in Wall Street explains well stock prices movements in the New York Stock Exchange).

Other evidence that seems to contradict the weak form of the efficient markets hypothesis also exists. One of these is known as "winner's curse" (Thaler (1988), Miller (1977), Hendricks and Porter (1988)): in an auction market, those with inflated opinions of an asset's value will be the ones that will get it but they will also tend to overpay. This anomaly attacks the market efficiency hypothesis through the rationality side of the hypothesis. If market participants were rational, each bidder's strategy would make allowance for the possibility that his appraisal of the asset's value was biased.

Another fact also pointed as evidence against the efficient capital markets hypothesis (LeRoy (1989)) is the high volume of speculative trade in organized securities exchanges. In some stock markets trading volume is so high that that it exceeds the Gross National Product of the respective country. Thus, it is argued, only a small percentage of that trading can be attributed to portfolio adjustments (due to consumption or to correction of portfolios' risk exposure), the larger part of trading
being for speculative motives. But under certain assumptions, it can be shown (Tirole (1982)) that if rational agents with asymmetric information but common priors are risk averters then they will not trade securities based on a naive interpretation of their private information. This is because they are aware that someone else with different but perhaps equally accurate information is willing to take the other side of the trade. This kind of transactions is a zero-sum game for the typical individual and constitute pure risk uncompensated by positive expected gain.

6. Variance bounds tests

Besides the anomalies presented above other more serious evidence against the weak form of the market efficiency hypothesis exists: the results of several variance bounds tests (also called excess volatility or Shiller tests). These tests were developed independently by Shiller (1981a) and LeRoy and Porter (1981).

We can start by noticing that the expectational difference equation (3-7) (equation (4-11)) can be rewritten as (assuming that \( r_t = r \) for all \( t \)):

\[
p_t = (1 + r)^{-1} \left( p_{t+1} + d_t \right) - (1 + r)^{-1} \delta_{t+1} p_{t+1}
\]

where \( \delta_t \) is an innovation operator defined as:

\[
\delta_{t+1} z_{t+1} = E[ z_{t+1} | \phi_{t+1} ] - E[ z_{t+1} | \phi_t ]
\]

The second term of the right-hand side of equation (4-18) is the unexpected component of the one-period return on stock.

Then, replacing \( t \) by \( t + i \) in equation (4-18) and multiplying both sides by \( (1 + r)^{-i} \) we get:

\[
(1+r)^{-i} p_{t+i} = (1+r)^{-(i+1)} \left( p_{t+i+1} + d_{t+i} \right) - (1+r)^{-(i+1)} \delta_{t+i+1} p_{t+i+1} \tag{4-19}
\]

Summing equation (4-19) over \( i \) from zero to infinity and assuming convergence it results:

\[
p_t^* = p_t + e_t \tag{4-20}
\]
where

\[ p_t^* = \sum_{i=0}^{\infty} (1 + r)^{-i} d_{t+i} \quad \text{and} \quad e_t = \sum_{i=1}^{\infty} (1 + r)^{-i} \delta_{t+i} p_{t+i} \]

\( p_t^* \) is the perfect foresight (or, as Shiller calls it, the "ex-post rational") stock price, the price that would result if future dividends were perfectly forecastable. \( e_t \), the difference between the ex-post rational and the actual price, is equal to the discounted sum of the unexpected component of future returns. Equation (4-20) expresses \( p_t^* \) as the sum of a forecast, \( p_t \), and a forecast error, \( e_t \).

Taking conditional expectations, equation (4-20) yields:

\[ p_t = E[p_t^* | \phi_t] \quad (4-21) \]

what means that \( p_t \) is a forecast of \( p_t^* \) given the information the set \( \phi_t \) has.

Optimal forecasting implies that forecasts and forecast errors are uncorrelated, what implies that:

\[ \text{Var}(p_t^*) = \text{Var}(p_t) + \text{Var}(e_t) \quad (4-22) \]

Because variances are always nonnegative \( \text{Var}(p_t^*) \) constitutes an upper bound for \( \text{Var}(p_t) \):

\[ \text{Var}(p_t^*) \geq \text{Var}(p_t) \quad (4-23) \]

The above unconditional variance inequality (usually referred as the Shiller test) depends only on the dividends model and the discount factor, but not on the information sets \( \phi_t \) of the agents. Thus, it is argued that rejection of (4-23) unambiguously implies the rejection of the fundamental model irrespective of the specification of the information sets of the agents. Shiller (1981a) tested the above variance inequality for two annual series, i.e., the Standard and Poor Composite Stock Composite Index (1871-1979) and a modified Down Jones Industrial Average (1928-1979). He found that the inequality was reversed for the two data sets. In Figure 8 is reproduced his famous plot of prices \( p_t^* \) and \( p_t \) (for the Standard and Poor data) that shows that the later is more volatile than the former. LeRoy and Porter
(1981) using quarterly data (1955-1973) of the Standard & Poor's Composite Index and three large companies also found that (4-23) was violated when using earnings instead of dividends. This test is reproduced in Section 4 of Chapter 5 for the Japanese market using monthly data for the TOPIX from 1973 to 1992, with results similar to those of Shiller: the right hand side of (4-23) exceeds the left hand side by over 53 times. Ueda, Suzuki and Tamura (1986) using annual data (1949-1985) and Hoshi (1987) using monthly data (1952-1981) found that (4-23) was also violated for Japanese data (TOPIX).

However, the above variance bounds test suffers from serious econometric problems. Flavin (1983) demonstrated that small sample distributions led to downward bias of the estimated variances of both \( p_t \) and \( p_t \) in Shiller (1981), with the bias in the former exceeding that in the later. If the stock price time series is autorrelated the usual procedure of correcting the downward bias of the estimated variance due to the necessity of calculating the sample mean (reducing the number of degrees of freedom by one) is not enough. This problem leads to a high probability of occurrence of type I error: the Shiller test rejects the capital markets efficiency hypothesis with probability 0.999 even when the hypothesis is true.

Kleidon (1986a), focusing on the statistical consequences of violation of the stationarity assumption, showed that if dividends have unit roots problems similar to those pointed by Flavin (1983) could remain even in very large samples. The same point was made by Kunitomo (1990). Marsh and Merton (1986) also attack Shiller's tests by focussing the assumption of stationarity of the dividend process and construct an example of an efficient market that violates equation (4-23).

Shiller (1981b) developed also other two variance bounds tests whose results, reported in Shiller (1981a), were also negative to the capital markets efficiency hypothesis. They are (as for the derivation see Shiller (1981b)):

\[
\text{Var}(\triangle P) \leq \text{Var}(d) / 2r \quad (4-24)
\]

and

\[
\text{Var}(\triangle P) \leq \text{Var}(d) (1+2r) / 2r^3 \quad (4-25)
\]

where \( r_t = r \) for all \( r \).

In another influencial paper, Grossman and Shiller (1981) calculate the
perfect foresight stock prices $p_t^*$ with constant and variable discount rates. They assume a consumption utility function with constant relative risk-aversion:

$$U(c) = \left( \frac{1}{1 - A} \right) c^{1-A} \quad (4-26)$$

with $0 < A < \infty$, where $A$ is a coefficient of relative risk aversion, e.g., $A$ is a measure of the concavity of the utility function or the disutility of consumption fluctuations. Under the assumption that investors know the whole future path of consumption, Grossman and Shiller calculate the implied discount rates from equation (4-26) for different values of $A$ and attempt to infer which value makes the actual stock price series consistent with the market efficiency hypothesis. Their results are reproduced in Figure 9. The risk neutrality case ($A=0$), when investors have constant time preferences, gives a constant discount rate so that the plot of $p_t^*$ is much smoother than the plot of $p_t$. With risk aversion ($A=4$), discount rates are nonconstant, and the plot of $p_t^*$ becomes much more like that of $p_t$, at least up to about the mid-1950s. In this case "the rough correspondence between $p^*$ and $p$ (except for the recent data) shows that if we accept a coefficient of relative risk aversion of 4, we can to some extent reconcile the behavior of $p$ with economic theory even under the assumption that future prices are known with certainty" (Grossman and Shiller (1981)).

Another variance bounds test was developed by Mankiw, Romer and Shapiro (1985). This test, to avoid the small sample bias, tests the variance bounds inequality using variances around some other point than the sample means. Suppose that $p_t^0$ is some "naive" forecast of $p_t^*$:

$$p_t^0 = \sum_{i=0}^{\infty} \gamma^{i+1} F_t d_{t+i} \quad (4-27)$$

where $\gamma = 1/(1+r)$ and $F_t d_{t+i}$ denotes a naive forecast made at time $t$. This naive forecast need not be a rational one. By subtracting and adding $p_t$ to the difference between the perfect foresight price and its naive forecast we have the identity:

$$p_t^* - p_t^0 = (p_t^* - p_t) + (p_t - p_t^0) \quad (4-28)$$

Notice that $(p_t^* - p_t)$ equals $e_t$ (equation (4-20)) and thus is uncorrelated.
with information available at time t. Squaring both sides of equation (4-28) and taking expectations at time t (denoted by $E_t$) we get:

$$E_t (p_t^* - p_t)^2 = E_t (p_t^* - p_t)^2 + E_t (p_t - p_t^0)^2$$  \hfill (4-29)

Equation (4-29) in turn implies that:

$$E_t (p_t^* - p_t^0)^2 \geq E_t (p_t^* - p_t)^2$$  \hfill (4-30)

and

$$E_t (p_t^* - p_t^0)^2 \geq E_t (p_t - p_t^0)^2$$  \hfill (4-31)

Mankiw, Romer and Shapiro using the same data as Shiller (1981a) performed excess volatility tests using the above inequalities, and found that they were reversed. They concluded then that "the stock market simply does not accurately reflect underlying fundamentals". Hoshi (1987), using Japanese data, found that (4-30) was accepted and (4-31) rejected (although the degree of violation seemed to be small); he interpreted those results (together with those from several other tests) as not rejecting the efficient market hypothesis.

6. Orthogonality tests

Another test also used to test volatility is the orthogonality test. Assuming a constant discount factor ($r_t = r$, for all t) equation (4-11) can be rewritten as:

$$p_t = \sum_{i=1}^{\infty} (1 + r)^{-i} E[ d_{t+i} | \phi_i ]$$

Multiplying the above equation by any variable $z_t$ observed at time t and then taking expectations, it results:
\[ E[ z_t p_t ] = E[ z_t \sum_{i=1}^{\infty} (1+r)^{-i} d_{t+i} ] \] (4-32)

If \( z_t = p_t - E[p_t] \), equation (4-32) implies:

\[ \text{Var}(p_t) = \text{Cov} \left( p_t, \sum_{i=1}^{\infty} (1+r)^{-i} d_{t+i} \right) \] (4-33)

As the Shiller tests seen above, the tests using equation (4-33) resulted in rejections of the efficient markets hypothesis. However, they suffer from the same problems, notably, nonstationarity. This problem can be solved by dividing the constant discount factor version of equation (4-11) by dividends:

\[ \frac{p_t}{d_t} = E \left[ \sum_{i=1}^{\infty} \prod_{k=1}^{i} (1+r)^{-i} \eta_{t+k} \phi_i \right] \] (4-34)

with \( \eta_{t+k} = \frac{d_{t+k}}{d_{t+k-1}} \).

Test of equation (4-33) using the more stationary variables obtained with equation (4-34) still reject (Cochrane (1991)).

Another route was tried by allowing \( r_t \) to vary over time. A problem arises, however, because \( r_t \) is not directly observable and a proxy for it must be used. Campbell and Shiller (1988) added a constant risk premium to a real variable interest rate but still found a rejection (real interest rates do not vary much). Another approach was that of Grossman and Shiller (1981) seen already in the previous section.
CHAPTER 5
TESTING FOR FUNDAMENTALS AND BUBBLES IN JAPANESE STOCK PRICES

1. Introduction

In Chapter 3 we saw that the weak form efficiency and the rational expectations hypotheses can explain the formation of stock prices that have a meaningful economic function. These meaningful stock prices are determined according to a model, that we called fundamental, that rests on the values assumed by an underlying basic variable like dividends. We also saw that if we retain the rational expectations hypothesis but discard the weak form efficiency hypothesis, a solution can exist where stock prices do not reflect the value of their fundamentals but instead incorporate a bubble. If a bubble component is present in stock prices it is to be expected that as it increases it becomes the dominant part of the price. Thus stock prices with a bubble component lose their allocational function.

In the first case, when stock prices reflect their fundamentals, stock markets are an useful institution that promote the welfare of the economy. In the second case, when stock prices incorporate a bubble, stock markets lose their power for making an efficient allocation of resources, although the case can be made that bubbles in certain special cases can help bring the economy to equilibrium that otherwise would not be attained (Blanchard and Fisher (1989)).

In Chapter 4 we surveyed several empirical tests, some supportive of the first case, some that seem to indicate the possibility of the second. To try to explain the excess volatility found in stock prices outside the efficiency paradigm there are two main options. One is to try to keep the rationality paradigm, and thus a possible explanation to stock price volatility compatible with the rational expectations hypothesis has appeared: rational bubbles. A rational bubble is a self-confirming belief that stock prices depend on a variable, or a combination of variables, that is intrinsically irrelevant to the determination of the fundamental price. A bubble, or a succession of bubbles, plausibly can explain why stock prices seem to soar and then nose-dive. The second option is to discard also the rationality paradigm, either partially or completely. Rationality can be partially assumed by supposing that there are two types of agents in the market, some rational others irrational. In this case we have noise traders models (Black (1986), De Long, Bradford Shleifer, Summers and Waldmann (1990),
Rationality is rejected when all or almost all agents are assumed to behave non-rationally. In this case we can have either fashions or fads, or any non-rational behavior explanation like non-rational bubbles, etc.

The main object of this chapter is to examine the admissibility of rational bubbles explaining the volatility of stock prices in Japan better than fundamentals alone. After Japanese stock prices began to fall in 1990, it has become habitual in Japan both in the press and in academia, to call the period of stock price increase that ended in 1989 the "bubble period". The starting point of this possible bubble is not clearly defined but 1986 (after the "high-yen recession period") and 1982 are two popular possibilities. Although during the 1980s Japan exhibited meager consumer price inflation, it suffered accentuated asset price inflation with sharp rises not only in stock prices, but also in bonds, land and art prices.

Whether there is a bubble in stock prices or not can be tested through an examination of the time series of stock prices. In principle, if a rational bubble is present, stock prices should not follow a random walk process insofar as a rational bubble should be increasing at a constant exponential rate. Inversely, if stock prices follow a random walk process we should be able to conclude that no rational bubble is present.

Depending on the different specifications of the models for fundamentals and bubbles the corresponding appropriate statistical tests should be used. As we will see later, slightly different hypothesis can call for different testing procedures.

Evidence in support of the presence of a bubble in a certain data set is, in a way, evidence against the fundamental hypothesis, and vice-versa. But for sake of clarity we will classify some as tests for fundamentals and others as tests for bubbles.

We describe, in section 2, the various sets of data used to perform the following tests. Then we test the weak form efficiency hypothesis, in section 3, using the empirical tests already in use for a long time (Fama (1970)). These tests in a way can be considered as tests to see if the fundamental model holds, that is, if the fundamental model adequately describes the behavior of a given time series of stock prices. In section 4 we replicate the variance bounds test of Shiller (1981). To test the fundamental model once again we proceed to use unit root tests (Dickey and Fuller (1979)), in section 5. Then we test if stock prices are cointegrated with profits (that we use as a proxy for dividends), in section 6. Failure to reject the fundamental model by these tests is an indication that probably a rational bubble is not present in the data tested.

As we will demonstrate later, if a rational bubble is present the rate of
increase in stock prices should be increasing through time. This characteristic that bubbles imprint in the stock prices time series should provide us with a rationale for testing their possible existence in section 7. We will do this with two tests: time trend regression tests and autoregression tests. Should these tests indicate the presence of a bubble and then the fundamental model is being rejected at the same time. Conversely, if these tests do not indicate the presence of a bubble then our confidence in the fundamental hypothesis becomes reinforced.

2. The data

Four data sets are used in this chapter, in our query to test whether stock prices behaved efficiently or bubbles existed in Japanese stock prices in the recent past. We will present them briefly here, but more detailed information relating to the first three sets can be found in my Master Thesis (Dos Santos (1992)).

The first data set is composed of the daily series of the Nikkei 225 index from September 29, 1987, to January 22, 1990, but the tests performed are for the period between November 12, 1987 and December 29, 1989, respectively the minimum after the October 1987 crash and the maximum before the 1990 crash. The Nikkei 225 is used as a measure of the level of Japanese stock prices and is chosen because of its popularity and wide usage in the financial press and academic circles, and because of its easy availability. The reported values of the index are their values at the end of a trading day and all trading days are included (including those Saturdays when trading also took place).

The second data set is composed of the daily Nikkei 225 index (for Japanese stock prices), the Dow 30 (for US stock prices) and the yen-dollar exchange rate time series for a period ranging from September 29, 1987, to February 5, 1990. The reported values of the Nikkei 225 and the yen-dollar exchange rate are their values at the end of the trading day in Japan and the reported values of the Dow 30 is its values at the end of the trading day in New York. All days in which trading occurred in all three markets (Tokyo stock and foreign exchanges and New York stock exchange) are included. But when no trading occurred in one market no trading is reported for all markets. This was done to keep a correspondence between the values of the three series.

The third data set consists of the quarterly averaged TOPIX, call rate, GNP deflator and business profits (seasonally adjusted) series, from the first quarter of 1968
to the last quarter of 1989. As it will be explained later, the business profits series is intended to be used as substitute for dividends, an use justified by the institutional characteristics of Japanese economic life that do not make dividends a good measure of the performance and future prospects of Japanese companies. Because business profits includes the profits of all companies, the more broad measure of Japanese stock prices, TOPIX, is used instead of the Nikkei 225.

The fourth data set, used in section 4, is consists of four monthly average series for the period from December 1973 to December 1992. They are: TOPIX, Total Dividends Paid in the Tokyo Stock Exchange, Ratio of TOPIX to Market Capitalization and Consumer Price Index. The values of the first three data sets were collected from the the Geppou published by the Tokyo Stock Exchange and the Consumer Price Index from the NEEDS electronic data base.

3. Weak form efficiency tests

We have already seen in Chapter 3 that a bubble, either of the deterministic type of equation (3-12) or of the stochastic type of equation (3-13), should increase at an exponential rate. Consequently, from either of these equations and from equation (3-9) we should expect that if a bubble exists in stock prices then those stock prices should increase exponentially. Moreover, if stock prices are increasing exponentially, present price increases should be explained, at least in part, by past price increases. Or putting it in another way, past price increases should be able to predict future price increases, with certainty in the case of a deterministic bubble, and with a certain probability in the case of a stochastic bubble.

But the ability to predict future price increases based on the examination of the sequence of past prices contradicts the efficient markets theory in its weak form. Then, one of the consequences of the existence of a rational bubble is that the weak form of the efficient markets theory should not hold. This has strong implications for all modern financial theory and it is one of the reasons that attracted so much attention to rational bubbles during the past ten years.

The hypothesis that past returns cannot be used to predict future returns cannot be proven since there are an infinite number of ways that the sequence of past prices can be used to forecast future prices. All that can be done is to test particular ways of combining past price data to predict future returns. However, a very large number of tests of alternative price patterns have already been made using data for
different countries and different periods of time (for examples see Fama (1970), Granger (1975)). And the conclusion of these studies is that information in the past price series is already incorporated in the present stock price.

One of the most used process to test the weak form efficient markets hypothesis is to see if stock prices follow a random walk (actually a martingale, but for the reason presented in the previous chapter we are reduced to test random walks). If in fact they follow a random walk then the weak form of efficient markets hypothesis can not be rejected, and it follows that a rational bubble should not be present. As we already saw in Section 3 of Chapter 4, this random walk test can be done, among other ways, by examining the correlation between past price changes and future price changes as in equation (4-16) reproduced here again:

\[
p_t - p_{t-1} = \alpha + \beta (p_{t-1-T} - p_{t-2-T}) + \epsilon_t \tag{5-1}
\]

As we already saw, the drift term \( \alpha \) measures the expected change in price unrelated to the previous price change. As long as stocks have a positive capital gain return, the drift term should be positive. Also, the term \( \beta \) measures the relationship between two prices changes, between the one occurring at \( t \) and the one occurring at \( t-1-T \). If \( T=0 \) then it measures the relationship between two consecutive price changes; if \( T=1 \) it measures the relationship between a price change and the price change that occured two periods previously; and so on. \( \epsilon_t \) is a random variable assumed to be normally independent and identically distributed with mean value of zero. It incorporates the variability of the current price change not related to the previous price change.

Denoting a price change during period \( t \) by \( \Delta p_t \), the above equation can be written as:

\[
\Delta p_t = \alpha + \beta \Delta p_{t-1-T} + \epsilon_t \tag{5-2}
\]

Estimation of equation (5-2) for \( T=0 \) using the Nikkei data for the period from November 12, 1987 to December 29, 1989, gives:

\[
\begin{align*}
\Delta p_t &= 29.609 + 0.042993 \Delta p_{t-1} \\
&= \frac{29.609}{3.4571} + \frac{0.042993}{1.0278} \\
\text{Adjusted } R^2 &= 0.0019, \ D-W &= 2.0239
\end{align*}
\]

where the values in parenthesis are the \( t \)-values of the estimated parameters. Both
in the case when \( T=0 \) as well in the cases when \( T = 1, 2, 3, 4 \) (reported in Dos Santos (1992), Table 1), \( \beta \) is not significantly different from zero. From this test the random walk hypothesis cannot be rejected.

Not all random walk tests have been done by utilizing equation (5-2). Instead of the first differences of stock prices, stock returns can be used. Also, another popular approach has been to test the logarithm of price relatives as in the following equation:

\[
\ln\left( \frac{p_t}{p_{t-1}} \right) = \alpha + \beta \ln\left( \frac{p_{t-1-T}}{p_{t-2-T}} \right) + \epsilon_t, \tag{5-3}
\]

The natural logarithm of the ratio of two consecutive stock prices is approximately equal the rate of return of holding a share. These two approaches are equivalent.

4. Volatility in stock prices

4.1 A measure of volatility in Japanese stock prices

By the end of the 1970s the efficient market hypothesis was a well established theory considered to be useful and well supported by evidence. The two twin ideas behind it were very simple: prices reflect all available and relevant information and are optimal forecasts of the future flow of real dividends. Not only this, but the efficient markets hypothesis was fundamental to most of financial theory.

But in 1981 two controversial papers by LeRoy and Porter (1981) and Shiller (1981a) argued that stock prices were actually too volatile to be in fact determined by the present value of the optimal forecast of the future flow of real dividends. As seen already above in Chapter 4, their basic argument is as follows.

Let \( p_t \) denote the actual price of an asset at time \( t \), and \( p_t^* \) denote the discounted value of the future flow of dividends at time \( t \). This \( p_t^* \) is what Shiller calls "ex-post rational price". If, as the fundamental solution of equation (3-7) implies, price \( p_t \) is the rational expectation of \( p_t^* \) (\( p_t = E[p_t^* | \phi_t] \), the expected present value of future dividends), then stock prices and discounted dividends must satisfy the variance inequality (4-23): \( \text{var}(p_t^*) \geq \text{var}(p_t) \). This is because since \( p_t \) is known at time \( t \) we have that \( p_t^* = p_t + e_t \), where \( e_t \) is a forecast error. And because a forecast error must be uncorrelated with the corresponding forecast, so \( e_t \) must be uncorrelated with
Therefore it follows that \( \text{var}(p_t, *) = \text{var}(p_t) + \text{var}(e_t) \). Since variances are nonnegative the above variance inequality (4-23) results. Since Shiller (1981) and LeRoy and Porter (1981) found that the above inequality was violated for the U.S. data, the excess volatility debate has continued. One of the explanations initially offered for the excess volatility found by LeRoy and Porter and Shiller was the possible existence of a rational bubble.

Before doing a replication for the Japanese stock market of the variance bounds test performed by Shiller, we performed a preliminary test that consists in comparing the following coefficient of variation \( v = \frac{\sum (X_i - \bar{X})^2}{\sum X_i^2} \) for the series of the actual price (for what we used the Topix index series) and for the ratio earnings/nominal interest rate. This ratio earnings/nominal interest rate is used as a proxy of present value of the expected future flow of dividends (for a more detailed rationale for this procedure see Funaoka (1990)). We used for earnings a seasonally adjusted series of business profits, and for the nominal interest rate a series of average call rates. This comparison was performed for the period from 1968 to 1989 using quarterly data and the results were that the coefficient of variation for stock prices was 0.4778 and that the coefficient of variation for the earnings/interest rate ratio was 0.3835. These results seem to show that in fact stock prices in Japan are more volatile than a measure of their fundamental value, which calls for an explanation. One hypothesis advanced is the existence of a rational bubble, a possibility we test in Section 7. Other explanations for crashes and excess volatility that do not assume rationality also exist (for example, see Shiller (1988)). But a very simple example can show that even minimal changes in expectations about interest rates can have a massive effect on prices. Let us assume that investors expect a dividend of a certain stock to be 5 yen at the end of this year and that it will increase thereafter at the rate of 3% per year. If they expect the interest rate to remain constant at 5%, then the value of this stock will be 250 yen. However, if the expectations about the interest rate change, let’s say to 4%, while the expectations about the company future prospects remain unchanged, then the value of the stock will become 500 yen, a 100% change! Because small changes in interest rates can have large repercussions in stock prices it can be argued that it is surprising that stock prices are not even more volatile.
4.2 A variance bounds test

Using monthly data for the Topix for the period from December 1973 to December 1992 inequality (4-23) was found to be violated. First, the "Total Dividends Paid in the Tokyo Stock Exchange" time series was deseasonalized using a Fourier transformation (employing "RATS Ver 4.1"). The deseasonalized Total Dividends series was then multiplied by the mensal series consisting of the ratio of Topix to Market Capitalization to get a dividend index comparable to Topix. Both the dividend index and Topix were then deflated using the Consumer Price Index to obtain real dividends and real stock prices. The long-run exponential growth path of real stock prices was found by regressing their natural logarithm on a constant and time and then it was used to detrend both real stock prices and dividends. An average discount rate was estimated by dividing the average real dividend by the average real price \( r = 0.9\% \) in annual terms. The terminal value of the ex-post rational stock price \( p_t^* \) (December 1992) was assumed to equal the terminal detrended real stock price \( p_t \). \( p_t^* \) was then calculated backwards from the terminal date using the relation 
\[
p_t^* = (1+r)^{-1}(p_{t+1}^* + d_t).
\]

The standard deviation was then computed to the \( p_t \) and \( p_t^* \) series yielding respectively 6.777 and 0.1259, resulting in the violation of inequality (4-23). The plot of \( p_t \) and \( p_t^* \) (presented in Figure 10) shows clearly that \( p_t \) is more volatile than \( p_t^* \).

5. Tests for fundamentals

5.1 Preliminaries

Let us now remember the stock price determination model presented in Chapter 3:

\[
p_t = a E [ p_{t+1} | \phi_t ] + a d_t \tag{5-4}
\]

where \( a = (1 + r)^{-1} < 1 \), with the further assumption that the riskless interest rate remains constant \( (r_t = r, \text{ for all } t) \). As assumed before, investors are risk neutral and have rational expectations. If the transversality condition in equation (3-6) holds then a solution to equation (5-4) is given by:
This $p_t^f$ we called the fundamentals or fundamental stock price. Equation (5-5) gives the fundamental stock price as the present value of the expectation of the future stream of dividends discounted at the constant riskless interest rate.

To see if the fundamental solution is a reasonable representation of the actual process of price formation we can test it using unit root tests. But before we can do this we need to specify a model for dividend behavior. Different models for dividends have different implications and require the use of different testing procedures. Once having specified a model for dividends we can test jointly that dividends' model and the above fundamental model by using the appropriate unit root tests.

5.2 Deterministic model for dividends

We can have two classes of dividends' models: deterministic and stochastic. In the deterministic category we can have a relation of the following type:

$$d_t = (1 + g^d)^t d_0$$

(5-6)

for an arbitrary $d_0$, and where $g^d$ is a constant growth rate. Substitution of equation (5-6) into equation (5-5) gives:

$$p_t^f = \sum_{m=0}^{\infty} a^{m+1} d_{t+m}$$

and calculation of the proportional change in the fundamental price brings:

$$\Delta p_{t+1}^f / p_t^f = g^d$$

(5-7)

Equation (5-7) implies that if prices are determined according to the fundamental solution and dividends are determined as in equation (5-6) then the rate
of change in stock prices should equal the rate of growth of dividends. To test if this hypothesis holds, two tests (in every aspect equivalent to each other) can be performed. Noticing that $\Delta p_{t+1}^f / p_t^f$ is approximately equal to $\ln(p_{t+1}^f / p_t^f)$ we can transform equation (5-7) into:

$$\ln \left( p_{t+1}^f / p_t^f \right) = g^d$$

(5-8)

We can test equation (5-8) using the following regression:

$$\ln p_{t+1} = g^d + \beta \ln p_t + e_t$$

(5-9)

where $e_t$ is assumed to be white noise ($e_t \sim iid N(0,1)$).

Estimation of equation (5-9) using the Nikkei 225 daily series between November 12, 1987 and December 29, 1989 gives:

$$\ln p_{t+1} = 0.022764 + 0.99789 \ln p_t$$

(1.0430) (470.70)

Adjusted $R^2 = 0.9975$, D-W = 1.8944

The regression corresponding to equation (5-9) has an estimated $\beta$ coefficient that is approximately one and significant. This means that the natural logarithms of stock prices seem to follow a random walk. This being the case it seems that no bubble is present in stock prices, and thus this result allows us not to reject (accept) the hypothesis tested. Namely, that stock prices are determined according to the fundamental model of equation (5-5) with dividends following a deterministic relation of the type of equation (5-6) as summarized by equation (5-8), and under the assumption that the approximation used is reasonable.

5.3 Stochastic model for dividends

Another class of dividends' models are stochastic models. One specification for this kind of models can be:

$$d_{t+1} = (1 + g^d) d_t + \eta_{t+1}$$

(5-10)
where $\eta_t$ is the unexpected part of dividends growth in period $t$ and it is assumed to be distributed with mean of zero. The difference between the deterministic model of equation (5-6) and this model resides in that this one dividends follow a stochastic process due to the inclusion of the random term $\eta_t$.

In a simple stochastic dividend model as the one of equation (5-10) we can add further assumptions about the term $\eta_t$. As we said above $\eta_t$ is distributed with mean zero. But besides this basic assumption we can assume that $\eta_t$ is either (i) normal independent identically distributed with mean zero and variance one, so that we have: $\eta_t \sim iid N(0,1)$, or that (ii) it follows a stochastic process of the type: $\eta_{t+1} = \eta_t + \epsilon_t$, with $\epsilon_t$ normal independent identically distributed with mean zero and variance one ($\epsilon_t \sim iid N(0,1)$). However, choice of either (i) or (ii) will be irrelevant in what follows.

Substitution of equation (5-10) into equation (5-5) and then calculation of the proportional change in the fundamental price brings:

$$
\Delta p_{t+1}^f / p_t^f = g^d + (\eta_{t+1} / p_t^f) \sum_{m=0}^{\infty} a^{m+1} (1 + g^d)^m
$$

(5-11)

Denoting the second term of the right hand side of equation (5-11) by:

$$
\eta_{t+1}^* = (\eta_{t+1} / p_t^f) \sum_{m=0}^{\infty} a^{m+1} (1 + g^d)^m
$$

we can write equation (5-11) as:

$$
\Delta p_{t+1}^f / p_t^f = g^d + \eta_{t+1}^*
$$

(5-12)

Independently of the assumptions relative to $\eta_{t+1}$ made above, the term $\eta_{t+1}^*$ will always be serially correlated.

Using the approximation $\Delta p_{t+1} / p_t = \ln(p_{t+1} / p_t)$, equation (5-12) can be transformed to:

$$
\ln p_{t+1} = g^d + \rho \ln p_t + \eta_{t+1}^*
$$

(5-13)
Because of the serial correlation of \( \eta'_{t+1} \), we cannot regress equation (5-13) directly to test the possible existence of an unit root \( \rho = 1 \). But we can make some simplifying assumptions about the behaviour of \( \eta'_{t+1} \). One possible assumption is that it follows a stochastic process of the type:

\[
\eta'_{t+1} = \delta \eta'_t + \epsilon_t \tag{5-14}
\]

with \( 0 < \delta < 1 \) (this ensures that the process is a stable, non-explosive process) and \( \epsilon_t \) normal independent identically distributed with mean zero and variance one (\( \epsilon_t \sim \text{iid } \text{N}(0,1) \)).

Then if we subtract from equation (5-13) written at \( t+1 \) the same equation written at \( t \) multiplied by the factor \( \delta \) of equation (5-14) we can get:

\[
\Delta \ln p_{t+1} = (1 - \delta) g^d - (1 - \delta)(1 - \rho) \ln p_t - \delta \rho \Delta \ln p_t + \epsilon_t
\]

where \( \epsilon_t \sim \text{iid } \text{N}(0,1) \) is not serially correlated. Making \( \alpha_0 = (1 - \delta) g^d \), \( \alpha_1 = -(1 - \delta)(1 - \rho) \), and \( \alpha_2 = -\delta \rho \) we can write the above equation as:

\[
\Delta \ln p_{t+1} = \alpha_0 + \alpha_1 \ln p_t + \alpha_2 \Delta \ln p_t + \epsilon_t \tag{5-15}
\]

To see whether the model behind equation (5-12) (that is, the fundamental price determination model of equation (5-5) and the dividend formation process of equations (5-10) and (5-14)) holds or not, we want to test whether the value of \( \rho \) in equation (3-13) is one or not. Because we cannot test this directly in equation (5-13) we need to test whether \( \alpha_1 \) in equation (5-15) is zero or not. As we assume in equation (5-14) that \( 0 < \delta < 1 \) we can accept without any further ado that if \( \alpha_1 = 0 \) then \( \rho = 1 \). If we do not impose on equation (5-14) the stability condition that \( \delta < 1 \) then even if \( \alpha_1 = 0 \) we cannot be sure that \( \rho = 1 \): we can have in this case that either \( \delta = 1 \) and \( \rho \neq 1 \), or \( \delta \neq 1 \) and \( \rho = 1 \), or \( \delta = 1 \) and \( \rho = 1 \). One possible solution to distinguish between these three cases would involve first to test whether the constant term is \( \alpha_0 = 0 \) or not. According to the result of this test we would have that either a) \( \delta \neq 1 \) or b) \( \delta = 1 \). If a) were the case then we could proceed to test whether \( \alpha_1 = 0 \), and if this hypothesis could not be rejected then we would be able to conclude that \( \rho = 1 \). But if b) were the case, then no test whether \( \alpha_1 = 0 \) would allow us to infer that \( \rho = 1 \).
Estimation of equation (5-15) using the Nikkei 225 daily series between November 12, 1987 and December 29, 1989 gives:

\[
\Delta \ln p_t = 0.023629 - 0.0022005 \ln p_{t-1} + 0.070468 \Delta \ln p_{t-1}
\]

\(\text{Adjusted } R^2 = 0.0034, DW = 2.0318\)

In the above regression the test statistic is the reported t-ratio. If the estimated t-ratio is greater than the critical value, we cannot reject the hypothesis that the series contains a unit root. If the t-ratio is smaller than the critical value, then we can reject that hypothesis (Dwyer and Hafer (1990)). When the sample has 500 observations the critical value is -2.87 at the 5% level of significance (Fuller (1976)). Thus we cannot reject the null hypothesis \(H_0: \alpha_1 = 0\). Assuming that \(\delta < 1\) we cannot reject the hypothesis that \(\rho = 1\) and thus should be able to infer that Japanese stock prices between November 12, 1987 and December 29, 1987 followed the fundamental price determination model and that no bubble was present.

The results of this test, that is, the values of the t-ratios obtained for the estimated values of \(\alpha_1\), as well as those of the tests performed by Dwyer and Hafer (1990) for levels and changes for other two periods also with daily data are summarized in Table 1 in the Appendix.

The test statistics reported above for the levels indicate that we cannot reject the hypothesis \(H_0: \alpha_1 = 0\) or \(\rho = 1\) of an unit root in the in the series for the three sample periods. The test statistics for the changes, however, rejects the the hypothesis of a second unit root.

6. More tests for fundamentals: stock prices and profits

We will now analyse the joint behavior of stock prices and profits under the assumptions of (1) constant and (2) variable real interest rates.

6.1 Constant expected real interest rate

Let us continue to assume that stock prices are determined according to the fundamental solution given by equation (5-5), with a constant real interest rate \(r\). Let
us also assume that dividends follow a stochastic process such as represented by the general equation (5-16):

\[ d_{t+1} = \mu + (1 + g^d) d_t + \eta_{t+1} \]  

(5-16)

with \( \mu = 0 \) and \( \eta_{t+1} \) itself following a stochastic process of the type:

\[ \eta_{t+1} = \delta \eta_t + \epsilon_{t+1} \]  

(5-17)

with \( 0 \leq \delta \leq 1 \) and \( \epsilon_{t+1} \sim \text{iid } N(0, \sigma^2) \). Notice that when \( \beta = 1 + g^d = 1 \) and \( \delta = 0 \) the process has a unit root \( \beta = 1 \) and dividends follow a random walk. The restriction imposed on equation (5-16) that there is no drift \( (\mu=0) \) is not essential and can be lifted if it is convenient.

Let us now introduce the concept of cointegration. If both of two variables each has a unit root, then we know that they are non stationary. And if the residuals of the simple linear equation characterizing the relationship of the two variables is stationary, then the two variables are said to be cointegrated. For example, if both stock prices and dividends have an unit root, and if they are regressed one in another and the resulting residuals do not have an unit root (are stationary) then stock prices and dividends are said to be cointegrated. The meaning of cointegration is that even if two variables follow a non stationary process there is still a significant relationship between them. In the previous example of stock prices and dividends, even if both stock prices and dividends follow a random walk, if they are cointegrated they move together.

If stock prices and dividends each follows a random walk process and each has a unit root, then, according to the fundamental stock price determination model, they should be cointegrated. That is, a regression of stock prices on dividends of the form of:

\[ p_t = \alpha_0 + \alpha_1 d_t + \epsilon_t \]  

(5-18)

should have a residual that is stationary (does not have a unit root). This can be demonstrated as follows. Equation (5-5) for the fundamental price can be rewritten as:
\[ p_t \ = \ d_t \sum_{m=0}^{\infty} a^{m+1} + \sum_{m=0}^{\infty} a^{m+1} \left( E[d_{t+m} | \phi_t] - d_t \right) \]  

(5-19)

The expression inside parenthesis is equal to:

\[ E[d_{t+m} | \phi_t] - d_t = \sum_{k=1}^{m} \left( E[d_{t+k} | \phi_t] - E[d_{t+k-1} | \phi_t] \right) \]

Substitution of this equation into equation (5-19) gives:

\[ p_t \ = \ r^{-1} E[d_{t+1} | \phi_t] + r^{-1} \sum_{m=1}^{\infty} a^{m} \left( E[d_{t+m+1} | \phi_t] - E[d_{t+m} | \phi_t] \right) \]  

(5-20)

If dividends follow a random walk process and have a unit root as we are assuming, the difference \( E[d_{t+m+1} | \phi_t] - E[d_{t+m} | \phi_t] \) does not have one unit root because its expected value \( E[\epsilon_{t+m+1}] = 0 \) for all \( m \). Because, in addition, the discount factors \( a^{m+1} \) form a set of geometrically declining coefficients on these already stationary values this implies that the second term of the right hand part of equation (5-20) does not have an unit root. And because this term of equation (5-20) corresponds to the residuals of equation (5-18), these residuals should not have an unit root if the expected real interest rate is constant and fundamentals determine stock prices

The above conclusions do not apply for the case when a bubble is present on stock prices. This is because when a bubble is on, the bubble part of the stock prices increases over time, and since the bubble component is independent of the dividends by definition, it should appear in the residuals \( \epsilon \), of equation (5-18). And if it appears in those residuals their estimated value should increase with time and have a root greater than one \( (1 + r / q) \). Thus, if a rational bubble exists and is an important component of the observed stock prices then the stock prices will not be cointegrated with dividends.

To perform the cointegration test between stock prices and dividends we will have to make three unit root tests: one for stock prices, another for dividends and a last one for the residuals of the regression between stock prices and dividends. In these unit root tests we will always be testing the null hypothesis \( H_0: \beta = 1 \) against
H₁: β ≠ 1. We can perform the cointegration test having two different basic assumptions about the process followed by dividends. These are that:

(A) dividends do indeed follow a simple random walk process (δ = 0 in the above equation (5-14));

(B) dividends follow the stochastic process described by the above equation (5-14) with 0 < δ < 1.

Let Yₜ denote the variable being tested for an unit root (stock prices, dividends or residuals), φ(L) denote a lag polynomial with lag operator L, and μ denote a constant term (the rate of growth of dividends). β, δ, εₜ, and ηₜ are as defined before.

Let us then consider (A). In this case we have the general expression:

\[ φ(L) [(1 - βL) Yₜ - μ] = εₜ \tag{5-21} \]

Let us take the simple case where the lag polynomial φ(L) = 1 - φ₁L. Making the appropriate transformations we get:

\[ ΔY = (1 - φ₁) μ - (1 - φ₁)(1 - β) Yₜ₋₁ - φ₁ β ΔYₜ₋₁ + εₜ \]

or, by making α₀ = (1 - φ₁), α₁ = -(1 - φ₁)(1 - β), and α₂ = -φ₁β this becomes:

\[ ΔY = α₀ μ + α₁ Yₜ₋₁ + α₂ ΔYₜ₋₁ + εₜ \tag{5-22} \]

We can now consider two cases:

(A1) φ₁ = 0 , and

(A2) 0 < φ₁ < 1.

In case (A1) with φ₁ = 0 we have that α₀ = 1, α₁ = β - 1, and α₂ = 0, and equation (5-22) becomes:

\[ ΔYₜ = μ + (β - 1) Yₜ₋₁ + εₜ \tag{5-23} \]

Equation (5-23) corresponds to equation (5-9) we saw above in Section 5.2. We test, in this equation, the existence of an unit root using the null hypothesis
\( H_0: \alpha_1 = \beta - 1 = 0 \). The t-ratios for \( \alpha_1 \) are presented in Table 2 in the Appendix for stock prices, dividends and residuals of the regression between the two (for the later we made \( \alpha_0 = 0 \), since there is no reason to include an intercept term in the case of residuals).

Then, in case (A2), with \( 0 < \phi_1 < 1 \), we test with equation (5-22) the existence of a unit root with the null hypothesis \( H_0: \alpha_1 = - (1 - \phi_1)(1 - \beta) = 0 \). Because \( \phi_1 < 1 \) this is equivalent to the hypothesis that \( \beta - 1 = 0 \). Because we are making the same basic assumptions in cases (A1) and (A2), the two are equivalent and the results we will get from the two should be approximately similar (provided that \( \phi_1 \) is in fact less than one, a question we already discussed at some length in Section 5.3. The t-ratios for \( \alpha_1 \) are also given in Table 2 in the Appendix for stock prices, dividends and the residuals of the regression between the two.

Let us now consider (B). In this case we have the general expression:

\[
\phi(L) \left[ (1 - \beta L) Y_t - \mu - \eta_t \right] = 0
\]  
(5-24)

Equation (5-24) differs from equation (5-21) in that the residuals \( \eta_t \) are considered to be autocorrelated in opposition to the normal independent identical distribution assumed for \( \varepsilon_t \). Taking the case where the lag polynomial is \( \phi(L) = 1 - \phi_1 L \) and making \( \phi_1 = \delta \) and after doing the appropriate manipulations we get:

\[
\Delta Y = (1 - \phi_1) \mu - (1 - \phi_1)(1 - \beta) Y_{t-1} - \phi_1 \beta \Delta Y_{t-1} + \varepsilon_t
\]

and from this equation the above equation (5-22) follows.

In performing the cointegration test for Japanese stock prices and dividends, we used quarterly data between the first quarter of 1968 and the last quarter of 1989 for the Topix, a stock price index including all companies listed in the first section of the Tokyo stock exchange based in January 4, 1968, and a seasonally adjusted series for the business profits of all industries. The use of business profits as a proxy for dividends calls for a few words of explanation.

The model behind equation (5-5) is based on the hypothesis that individuals are concerned with dividends plus capital gains. Also, it assumes that capital gains are just a reflection of information about future dividends. Profits or earnings, on the other hand, are statistics that can provide an indicator of how well a company is doing but, theoretically, there is no reason why price per share should be the discounted
value of expected future profits per share if some profits are retained to finance the future expansion of the company. Such a present value formula would entail a sort of double counting. It is incorrect to include in the present value formula both profits at time $t$ and the later profits that accrue when time $t$ earnings are reinvested. However when firms pay no dividends (or when these dividends are such a small amount of profits that they can be considered negligible) then the price of a share of stock cannot be determined by the present value of dividends given by equation (5-5). And, as it has been already shown by Campbell and Shiller (1988), profits, when averaged over the years, are a good substitute for dividends. In conclusion, our use of business profits instead of dividends is justified because dividends in Japan are, as a rule, of very small value and are not a good indicator of profitability - dividends in Japan remain unchanged for long periods of time irrespective of profitability, that is, they do not reflect fundamental changes in the prospects of companies when those changes happen.

Another problem with the series of business profits is that it includes the profits of all companies in Japan (not only the companies listed in the first section of Tokyo's stock exchange whose price Topix measures) and is not in per share values. This is a problem that could not be avoided due to the difficulty of obtaining data to construct a series only for those companies listed in the first section of Tokyo's stock exchange. This series should then be taken only as an approximation of the real profits per share series. Also, this series is composed only by the operating profits not including profits arising from speculative investments in the stock market and land.

Performing the unit root test for the logarithms of stock prices, the logarithms of business profits and the residuals of the regression between the two for the period between the first quarter of 1968 and the fourth quarter of 1989 yields the results presented in Table 2 in the Appendix.

The results for cases (A1) and (A2) and (B) are similar as we expected. They show that we cannot reject the null hypothesis that stock prices, dividends (profits) and the residuals of the regression between the two had a unit root. The conclusion is then that with constant interest rates, stock prices and dividends (profits) are not cointegrated variables. This is evidence against the fundamental model and in favor of the possible existence of a rational bubble.
6.2 Variable expected interest rate

Let us remember that the arbitrage between the return on a share of stock and the interest paid by a riskless asset can be expressed by the above equation (5-4). Assuming that the expected interest rate is variable, the mentioned equation (5-4) can be written thus:

\[ p_t = a_t \ E[p_{t+1} \mid \phi] + a_t \ d_t \]  

(5-25)

where \( a_t = (1 + r_t)^{-1} < 1 \).

The interest rate as well as the discount factor have now a time subscript to indicate their nonconstancy. Because we will have to deal with rather complicated equations and to avoid cumbersomeness we will from now on use a simpler notation for expectations: \( E_t y_{t+k} \) will from now on represent the rational expectations at time \( t \) of variable \( y \) at time \( t + k \). The above equation (5-25) will accordingly be written as:

\[ p_t = a_t \ E_t p_{t+1} + a_t \ d_t \]

The fundamental solution when the interest rate is variable is found, as before, by writing equation (5-25) at time \( t + 1 \) and taking expectations of both sides conditional on the information set at time \( t \):

\[ E_t p_{t+1} = E_t (a_{t+1} E_{t+1} p_{t+2} + a_{t+1} d_{t+1}) \]

Replacing this result in equation (5-25) gives:

\[ p_t = a_t E_t (a_{t+1} E_{t+1} p_{t+2} + a_{t+1} d_{t+1}) + a_t d_t \]

Repeating these steps again for \( t + 2 \), and so forth, we get:

\[ p_t = a_t d_t + a_t E_t (a_{t+1} d_{t+1}) + a_t E_t (a_{t+1} E_{t+1} (a_{t+2} d_{t+2})) + ... \]

Assuming that the discount factors and dividends have independent distributions, and that the discount factors are not serially correlated, if we use the law of iterated expectations the resulting expression can be written as:
\[ p_t = \sum_{m=0}^{\infty} \gamma_{t+m} \Delta t + \mu \]  

(5-26)

with \( \gamma_{t+m} = \frac{1}{h} \sum_{h=0}^{\infty} E_t a_{t+h} \), where \( E_t a_t = a_t \).

Equation (5-26) can be rewritten as:

\[ p_t = d_t \sum_{m=0}^{\infty} \gamma_{t+m} + \sum_{m=0}^{\infty} \gamma_{t+m} (\Delta t + \mu) \]  

(5-27)

Assuming that the expected real interest rate is constant over time (that is, assuming that the term structure is flat), then:

\[ \gamma_{t+m} = a_t \]

This simplifies equation (5-27) to:

\[ p_t = \left( \frac{d_t}{r_t} \right) + \sum_{m=0}^{\infty} \left( E_t d_{t+m} - E_t d_t \right) \]  

(5-28)

Substitution into equation (5-28) of:

\[ E_t d_{t+k} - E_t d_t = \sum_{k=1}^{m} \left( E_t d_{t+k} - E_t d_{t+k-1} \right) \]

gives, after simplification:

\[ p_t = \left( \frac{d_t}{r_t} \right) + \left( \frac{1}{r_t} \right) \sum_{m=1}^{\infty} a_t^k \left( E_t d_{t+m+1} - E_t d_{t+m} \right) \]

Assuming that the expected one-period change in dividends is a constant, \( c \), we have:
Equation (5-29) is the basis for the cointegration tests between stock prices and the ratios of dividends to the variable interest rate and a constant to the square of the variable interest rate. The t-ratios of the unit root tests for the residuals of equation (5-29) when the interest rate used is both in real and in nominal terms are presented in Table 3 in the Appendix.

The critical value for the size of the sample used and for a 10% significance level is -2.57 according the table given by Fuller (1976) and reproduced by Yamamoto (1989, p.330). This means that, when real interest rates are allowed to fluctuate, the hypothesis that stock prices and profits are cointegrated cannot be rejected for that significance level for the period considered. However, the same does not happen when the interest rate used is in nominal terms: in this case the hypothesis that stock prices and profits were cointegrated can be rejected. The evidence seems thus somewhat inconclusive to whether stock prices followed fundamentals or included a bubble component.

7. Tests for bubbles

7.1 Preliminaries

To test for the existence of rational bubbles, besides assuming the previous models for fundamentals and dividends as given by equations (5-5) and (5-10), let us remember the definition of rational bubbles given first in Chapter 3 and reproduced here as equation (5-30):

$$p_t^b = p_t^f + b_t$$  \hspace{1cm} (5-30)

with:

$$E[b_{t+1} | \phi_t] = a^{-1} b_t$$  \hspace{1cm} (5-31)

Also, the process for a stochastic bubble is given by:
\[ b_{t+1} = (aq)^{-1} b_t + e_{t+1} \text{ with probability } q \]
\[ b_{t+1} = 0 \text{ with probability } q - 1 \quad (5-32) \]

As long as the bubble is on, the bubble part of the price grows at a rate \( g^b \):

\[ g^b = (aq)^{-1} - 1 = ((1 + r) / q) - 1 = r + (1 + r) (1 - q) / q > r \quad (5-33) \]

Because \( q \) is larger than zero and smaller than one the bubble part of the price grows at a rate larger than the riskless interest rate \( r \). When the bubble bursts its value becomes zero. A simplifying assumption used here is that when the bursting occurs the bubble value immediately becomes zero implying at that point of time a negative infinite growth rate. This obviously is probably not true of a real bubble whose value perhaps becomes zero after a certain number of trading sessions take place. When the bubble component of the stock price becomes zero the stock price is solely determined by the fundamentals and its growth rate by their growth rate because the expected growth rate of the bubble component becomes zero.

But as long as there is a bubble in stock prices the proportional changes in stock prices should be predictable for any given finite period. If the expected dividend grows at a constant rate as either of the above equations (5-6) and (5-10), then the proportional change in the stock price that, as in equation (3-8), includes both a fundamental and a bubble component should be:

\[ \left( \frac{\Delta p_{t+1}^b}{p_t^b} \right) = g_t^f \left( \frac{p_t^f}{p_t^b} \right) + g^b \left( \frac{b_t}{p_t^b} \right) \quad (5-34) \]

Rearranging this equation, we can get equation (5-35):

\[ \left( \frac{\Delta p_{t+1}^f}{p_t^b} \right) = g_t^f \left( \frac{p_t^f - b_t}{p_t^b} \right) + g^f \left( \frac{b_t}{p_t^b} \right) \]

\[ = g_t^f + (g^b - g_t^f) \left/ \left( 1 + \frac{p_t^f}{b_t} \right) \right. \quad (5-35) \]

According to equation (5-35) the rate of change of the stock price when it includes a bubble can be divided into two different parts. The first part is the rate of growth of fundamentals \( g_t^f \) (that should be equal to the rate of growth of dividends as we saw above). The second part is due to the bubble and itself has two parts: the first is the difference of the growth rates of the bubble component and fundamental
component of price, \( g^b - g^f_t \), and should have a positive expected value since \( g^b > r \) (from equation (5-33)) and the expected value of \( g^f_t \) is \( g^d < r \) (due to the imposition of the transversality condition). The second part of the term due to the bubble is an increasing function of time because the ratio \( p^f_t / b_t \) is a decreasing function of time. The ratio of the fundamental price to the bubble part of the price is given by the following equation (5-36):

\[
( p_t^f / b_t ) = \left( (1 + g^f_t)^t / (1 + g^b_t)^t \right) ( p_0^f / b_0 ) \tag{5-36}
\]

with period zero being the first period when the bubble is on and \( p_0^f / b_0 \) being constant for all \( t > 0 \). As time goes to infinity, the ratio in equation (5-36) goes to zero because \( g^d < g^b \) and the bubble part of the price eventually dominates the fundamental component in the stock price.

Equation (5-36) implies that, for any finite period when the bubble is on, the proportional change in stock prices is an increasing function of time. Consequently, the proportional change in observed stock prices should be predictable from its own past values. This is not the case for the Nikkei 225 daily series between November 12, 1987, and December 29, 1989 as we saw in the weak form efficiency tests in Section 3. But two more tests can be done to try to detect the existence of a possible bubble. They are time trend regressions and autoregressions.

7.2 Time trend regressions

From equation (5-35) we can see that if a bubble is on, the proportional increase in stock prices is an increasing function of time. The reason for this is that the growth rate of the bubble component of price is larger than the growth rate of the fundamental component of price, and the bubble component increasingly dominates the stock price. Whether a bubble is present or not in a series of stock prices can then be tested by regressing the proportional changes in stock prices on time to see if there is, say, a linear or quadratic relationship between those proportional changes and time. The existence of such a relationship would provide evidence in support of the existence of a rational bubble.

The Table 4 in the Appendix provides the results of regressions of proportional changes in stock prices on time for three sample periods, the first two
performed by Dwyer and Hafer (1990) and the third by us. For each period two regression results are reported: one in which time enters only as a linear term and another that adds a quadratic term.

In all the three cases the regressions with time entered only as a linear function do not indicate the proportional change in stock prices to have any significant trend (for a 95% confidence interval the appropriate t-statistic is approximately ±1.96). And adding a quadratic term does not alter this finding. In conclusion, for these three time periods, according to these time regressions tests, the hypothesis that a bubble was present can not be accepted.

7.3 Autoregressions

The presence of bubbles can also be tested by examining the results of autoregressions of the proportional change in stock prices. If bubbles are present then observed changes in stock prices should exhibit significant serial correlation. To test this Dwyer and Hafer (1990) regressed, for two sample periods, the proportional change in stock prices on a constant and twenty five lagged values of that change. In addition we regressed for a third sample period the proportional change in stock prices on a constant and seven lagged values of that same change. If the lag coefficients are not positive and significant we can reject the hypothesis that a bubble is present in the data tested.

The test statistics are reported in Table 5 in the Appendix. To test the hypothesis that autoregressive parameters are significant as a group $\chi^2$-statistics were calculated for the first two sample periods. Significance is denoted by the asterisk *. The autoregressive parameters are found to be significant as a group for the second sample period but the sum of coefficients is negative, contrary to the hypothesis being tested. For the other two samples, besides the sum of coefficients being negative they are not significant either individually and/or as a group. For the third group the chi-square statistic was not calculated but the F-statistic ($F(7,550) = 1.76$) shows that we can not reject the null hypothesis that the sum of the estimated coefficients is not significantly different from zero. For more detailed results concerning the first two samples see Dwyer and Hafer (1990), pp. 42 - 46, and for the third one see my master thesis (Dos Santos (1990)).

In conclusion, the autoregressive tests realized are largely inconsistent with the hypothesis of presence of a rational bubble in the sample data.
CHAPTER 6

BOUNDED TIME HORIZONS AND EFFICIENT MARKETS

1. Introduction

Shiller's (1981) empirical tests of the efficiency of the U.S. stock market, using data spanning from 1871 to 1979, are well known: their results, as we saw in Chapter 4 and also in Chapter 5 reproduced for Japanese stock prices, were negative to the hypothesis of efficiency of stock markets in the U.S. and Japan. These tests, also referred as variance bounds tests, are joint tests of the following four hypothesis:

1. efficiency of the stock market;
2. linear trend;
3. constant discount rate;
4. rational expectations.

In other words, variance bounds tests besides testing hypothesis (1) assume that (2), (3) and (4) hold.

The results of these variance bounds tests as well the statistical procedures required to perform them and involving assumptions (2) and (3) have since attracted the interest of a large number of economists (Marsh and Merton (1983), (1986), Kleidon (1986), etc).

In this chapter we will focus our attention on the detrending process used by Shiller (1981) and on the usual assumption that the market can somehow have expectations for future dividends up to an infinite time horizon (assumptions (2) and (4) respectively).

Variance bounds tests usually assume a simple trend model for stock prices and dividends. However, the assumption that the variables in question are generated by a stationary process around a linear trend is a very strong assumption when using data covering long spans (more than 100 years in Shiller's case) to perform these tests. If a stable economic structure is an essential premise to the use of a linear trend, then stock prices and dividends' series lasting for over a century, when several major changes affecting that structure took place, cannot be appropriately represented by a linear trend. Also, as Nelson and Plosser (1982) pointed out, the use of the assumption of a linear trend means that the concerned variable suffers only stationary
and temporary shocks, and is never affected by permanent shocks. Thus, concerning the dividends’ trend that determines the fundamental stock prices, we adopt in this chapter Harvey and Todd's (1983) and Clark's (1987) basic structural model (stochastic trend model). By adopting this basic structural model we can use smoothing estimation procedures of a state space model to estimate the variable (stochastic) trend of dividends. By the use of this more general basic structural model we can then capture both temporary as well as permanent shocks.

However, by the use of the basic structural model we create a new problem. This new problem is that when dividends are assumed to follow the basic structural model, the degree of integration of their trend must be I(2). This makes the conditional variance of future dividends increase with time, that is, the more distant from the present is a certain point in time the larger the conditional variance of expected dividends at that time. This makes fundamental stock prices extremely volatile.

Thus in this chapter we assume that the stock market does not try to evaluate the flow of future dividends up to an infinite horizon but only up to a finite horizon. We attempt to estimate empirically that time horizon and the corresponding fundamental stock prices. To evaluate the goodness of this model we investigate how much these time horizon bounded fundamental stock prices explain of the behaviour of the actual stock prices. To do this we test if the two are cointegrated or not. We also verify that when the actual stock prices deviate from the fundamental stock prices with bounded time horizons they have the tendency to revert to these fundamental stock prices, what is to say that the error correction mechanism is at work.

We also test whether the time horizon of the stock market is variable. For example, when the economic situation is good and the economy is growing in a stable and sustainable way stock prices have the tendency to rise. If this rising tendency is also stable and market agents have no reason for believing it will be reversed, then probably the time horizon of the market will increase. However, when the economic situation deteriorates and there are no clear prospects for future recovery, then probably the time horizon of the stock market gets shorter. In this chapter we also investigate how much of the actual stock prices' behaviour is explained by this variability of time horizons.

It should be noted that in this chapter we inquire whether stock markets are, in the terminology we used in Chapter 3 (p. 39), long run efficient markets. In other words, we investigate whether or not actual stock prices have the property of a stable long run relationship with their fundamentals (whether or not the two are cointegrated); and also whether or not the error correction mechanism works if a shock occurs that
breaks in the short run that relationship between actual stock prices and their fundamentals.

We start by arguing, in the next section, that changes in the economic structure are not captured by a linear trend. Then we will develop our model in Sections 3 and 4. In Section 3 we explain the basic structural model adapted from Harvey and Todd (1983), and in Section 4 we present the bounded time horizon valuation model of expected future dividends. In Section 5, based in the analysis of Sections 3 and 4, we test empirically the ability of bounded horizons fundamental stock prices to explain the movements of actual stock prices.

2. Changes in the economic structure

In order to test "the simple efficient markets model", Shiller ((1981), (1989)) hypothesizes that the logarithms of dividends have a linear trend. We can borrow the argument of Nelson and Plosser (1982) concerning the business cycle, and point that this assumption is equivalent to the view that the movements of dividends and stock prices are temporary disturbances (like, for example, monetary disturbances) around a linear trend. However, when testing the efficient market hypothesis Shiller uses time series spanning for over one hundred years. The assumption of a linear trend for such long series is similar to the assumption that the structure of the U.S. economy did not change. Such an assumption is not easy to justify.

It can be argued both from observation of the dividends' and stock prices' time series and from economic history that the structure of the U.S. economy did in fact change.

Figures 11 and 12 present the real stock prices and real dividends time series. From these figures it is possible to distinguish several different periods corresponding to different economic situations. There is first the period before World War I, then the period of World War I and the ensuing depression, followed by the period of the roaring twenties. An uncharacteristic period of ups and downs lasting for about two decades encompasses the thirties and forties of bad memories. It is followed by a prosperous period of about twenty years during which dividends and stock prices increase to new levels never before reached. This prosperous period comes to an end with the slump in the early 1970s. Stock prices and dividends keep falling during the 1970s. Finally, the last years shown, already in the 1980s, present a steady increase in both prices and dividends.
Figures 13 and 14 present possible trends for real stock prices and dividends based on the above described historical periods. Table 6 presents the results of simple regressions of respectively real stock prices and real dividends on a constant and on time and their coefficient of determination. It can be seen that the coefficients for time vary widely, taking positive and negative values, and in general are statistically significant. Moreover, except for the period of the thirties and forties when stock prices and dividends moved violently down and up, and also for the last period of fifteen or so years when large falls were followed by large increases, the coefficient of determination is reasonably high.

The same picture is presented by Figures 15 and 16. Here are depicted the recursive residuals of real stock prices and real dividends from the CUSUM test. For real stock prices, for example, the recursive residuals are about zero from 1882 to about 1912, assume increasingly negative values from 1913 to the mid-twenties, increase for about seven years after 1925, show a slightly decreasing but undulating behaviour during the thirties and forties, start in the early fifties to increase, become positive in the early sixties and reach a maximum in the early seventies. During the seventies and early eighties the recursive residuals steadily decrease to resume increasing in the last few years of the sample. This is a pattern that closely matches the patterns described in the previous two paragraphs for stock prices and dividends. The recursive residuals from real dividends show the same behaviour, perhaps even more pronouncedly. It should be noted, however, that recursive residuals of real stock prices and real dividends never break their upper or lower limits, what would indicate more clearly the variability of the parameter.

Figures 17 and 18 present the squared recursive residuals from the CUSUMSQ test applied to real stock prices and real dividends. In the movements of the squared recursive residuals it is somewhat more difficult to distinguish the above mentioned six historical periods than in the movements of recursive residuals. However, the squared recursive residuals for stock prices cross their lower limit twice thus indicating that the parameter for the variable time is not constant.

It is arguable that these probable changes in the time parameter reflect changes in the economic structure. The division of the over one hundred years into about six different periods made above can be justified through economic history arguments that show that the economic environment has changed. Although this is not the place for long digression in this field, it should suffice to point that the first period, before World War I, when there was steady growth in both real stock prices and dividends is a period of strong capital accumulation and population growth in the
U.S., and also of free trade in the world and of the gold standard. Then there is the period of the first World War and the ensuing recession that corresponds to the suspension of the gold standard, the disruption of world trade, and steep decline in the rate of growth of the U.S. population because movements into the U.S. come to a near halt. The roaring twenties saw the freeing of the economy from the restrictions imposed during the war, a partial return to the gold standard and a short period of free trade policy by the U.S., the combination of all of which resulted in high economic growth. The period of the great depression and World War II is more difficult to characterize but it corresponds to financial anarchy, an increase of the government intervention in the economy that culminated during World War II, suspension of the gold standard, commercial and other wars, weather calamities like droughts, etc. The fifties and sixties saw among other important changes in the economic structure a new international monetary system, increased trade in the world and a new way of thinking and conducting economic policy. Finally the seventies brought the oil crisis, the substitution of the international monetary system and yet other new ways of thinking and conducting economic policy.

3. The basic structural model for dividends

From the above arguments, the use of more realistic models able to capture both permanent shocks and changes in the economic structure (for example, ARIMA models) in the analysis of movements of dividends and stock prices seems preferable. Thus concerning the dividends' trend that determines the fundamental stock prices we adopt Harvey and Todd's (1983) and Clark's (1987) basic structural model (stochastic trend model). That is, the dividends' generation process is as follows:

\[
\begin{align*}
d_t &= T_t + \epsilon_t, \\
T_t &= \delta_t + \eta_t, \\
\delta_t &= \delta_{t-1} + \xi_t.
\end{align*}
\]

Here \(d_t\) is the observed dividend at time period \(t\), and \(T_t\) and \(\epsilon_t\) are respectively a trend component and a stationary residual also at time period \(t\). The trend \(T_t\) follows a random walk with drift \(\delta_{t-1}\), which itself also follows a random walk. This means that, not only the levels but also the slope of the forecast function
of the dividends' trend changes with time. Thus the level of integration of dividends is I(2). \( \mathbf{\epsilon}, \mathbf{\eta}, \) and \( \mathbf{\xi} \) are independent white noise processes with distributions of NID(0, \( \sigma^2 \)) and NID(0, \( \sigma^2 \)), respectively. The independence between \( \mathbf{\epsilon}, \mathbf{\eta}, \) and \( \mathbf{\xi} \) is a necessary condition to the identification of this model. Model (6-1) captures both temporary shocks (\(\mathbf{\epsilon}\)) and permanent shocks (\(\mathbf{\eta}, \mathbf{\xi}\)).

Technically, the use of this model allows the easy estimation of the variable (stochastic) trend through the use of smoothing estimation of the state space form of this model as we will see in Section 4.

4. Fundamental stock prices and the boundedness of the span of time horizons

4.1 Fundamental stock prices

As we saw in Chapter 3, according to "the simple efficient markets model", the price of a share tends to equal the fundamental value of that share. In turn, the fundamental value of a share of at the beginning of period \( t \) is usually equated to the discounted value of the expected future dividends. Thus the fundamental stock price \( p_t^f \) is defined as follows:

\[
p_t^f = \sum_{k=0}^{\infty} \gamma^{k+1} \mathbb{E}(d_{t+k} | \Omega_t), \quad 0 < \gamma < 1.
\]  

(6-2)

Here \( p_t^f \) is the fundamental stock price at the beginning of time period \( t \), \( \gamma \) is a constant discount factor with \( \gamma = 1/(1+r) \), with \( r \) a real interest rate where an appropriate risk premium is added, and \( \mathbb{E}(\cdot | \Omega_t) \) denotes the conditional expectation at the beginning of time period \( t \) given the information set \( \Omega_t \). We assume also that dividends are paid at the end of the corresponding period.

If we assume that the disturbance components \( \mathbf{\epsilon}, \mathbf{\eta}, \) and \( \mathbf{\xi} \) take place at the end of period \( t \), then the information set at the beginning of that period will include the past values of \( \mathbf{T} \) and \( \mathbf{\delta} \) so that:

\[
\Omega_t = (T_{t-1}, \delta_{t-1}, T_{t-2}, \delta_{t-2}, ...).
\]  

(6-3)

Under this information set \( \Omega_t \) and the dividends formation process of equation (6-1)
it can be shown that the fundamental stock prices will be determined according to the following equation:

\[ p_t^f = \{\gamma/(1-\gamma)\}T_{t-1} + \{\gamma/(1-\gamma)^2\}\delta_{t-1} \]  \hspace{1cm} (6-4)

The fundamental stock price \( p_t^f \) depends not only on the level of the dividends' trend, represented by the first right hand side term of equation (6-4), but also on the change of that trend, represented by the second term. When dividends simply follow a random walk process this second term is not captured.

Shiller (1989, page 27) emphasizes that the correlation between \( (p_t/d_{t-1}) \) and \( p_t \) and \( d_{t-1} \) is high so that the behaviour of stock prices can be best described as an overreaction to dividends. This can be interpreted, according to Shiller, as evidence against the efficiency of stock markets. However, under the dividend formation model of equation (6-1), the overreaction of stock prices to dividends is not at odds with the efficiency of stock markets.

We can explain the correlation between \( (p_t/d_{t-1}) \) and \( p_t \) at the light of the efficient markets model and the dividend formation model of Section 2. Under the efficient markets hypothesis the actual stock price \( p_t \) equals the fundamental value or price \( p_t^f \). Due to the realization of \( \xi_{t-1} \) and through the consequent change in the drift of the trend \( \delta_{t-1} \) (equation (6-1)) the stock price \( p_t \) changes (equation (6-4)) but the dividend \( d_{t-1} \) remains unchanged. In other words, from equations (6-1) and (6-4) we have that:

\[ \frac{d(p_t/d_{t-1})}{d\xi_{t-1}} = \frac{d(p_t/d_{t-1})}{d\delta_{t-1}} > 0. \]  \hspace{1cm} (6-5)

Under model (6-1), one unit increase in \( \xi_{t-1} \) is transmitted through one unit increase in \( \delta_{t-1} \) to the expectation that future dividends will increase one unit each future period. As a consequence of this, one unit increase in \( \xi_{t-1} \) implies an \( \gamma/(1-\gamma)^2 \) increase in stock prices \( p_t \). At the same time one unit increase in \( \xi_{t-1} \) has no effect whatsoever in \( d_{t-1} \).

This overreaction of stock prices to dividends is due to fundamental stock prices being very sensitive to changes in the drift \( \delta_{t-1} \) of the trend of dividends. Changes in stock price fundamentals as defined in equation (6-4) reflect changes in the drift \( \delta_{t-1} \) in an amplified manner.
The above argument with the appropriate changes can be applied to explain the correlation between \( \frac{p_t}{d_{t-1}} \) and \( d_{t-1} \) with recourse to changes in \( \eta_{t-1} \).

4.2 The boundness of the span of time horizons in the stock market

The dividends’ formation model of equation (6-1) implies that the dividends’ time series should be a non-stationary integrated process of order two. Thus, the conditional expectations \( E(d_{t+j} | \Omega_t) \) and conditional variance \( \text{var}(d_{t+j} | \Omega_t) \) of future time period \( t+j \) (\( j > 0 \)) are:

\[
E(d_{t+j} | \Omega_t) = T_{t-1} + (1+j) \delta_{t-1} \tag{6-6a}
\]
\[
\text{var}(d_{t+j} | \Omega_t) = (1+j) \sigma \eta^2 + \left( \sum k^2 \right) \sigma \varepsilon^2 \tag{6-6b}
\]

As can be seen from the above equation, the further away into the future is some time period the larger is the conditional variance of expected dividends: the conditional variance of dividends of periods infinitely far away is infinite. We can then say that:

1. the degree of confidence of the forecast of future dividends decreases exponentially with the distance of the future period;
2. if \( \delta_{t-1} < 0 \) there is a certain period \( t+j \) after which the conditional expectation of dividends becomes negative.

Thus, from now on, we will consider a more general model for the evaluation of future dividends that allows for the time horizon of the stock market to be finite. Based on this model we redefine fundamental stock price \( p^f_t(k) \) as follows:

\[
p^f_t(k) = \sum_{j=0}^{k} \gamma^{j+1} E(d_{t+j} | \Omega_t) + \gamma^{k+1} p^f_{t+k+1}^o. \tag{6-7}
\]

Here, \( p^f_t(k) \) is the fundamental stock price when the stock market time horizon is \( k \) periods long, and \( p^f_{t+k+1}^o \) is the forecast (estimate or anticipation) at period \( t \) of the valuation price of the stock at period \( t+k+1 \). We can think this later value to be, at the level of each individual investor, an estimate of the selling price of the stock during period \( t+k+1 \).
We assume this valuation price to be equal to the present fundamental stock price:

\[ tP_{t+1}^f = p_t^f(k). \]  

(6-8)

This assumption is an ad hoc use of static expectations made mainly to simplify the algebra of the analysis of what follows. As our purpose is to investigate the consequences of restricting the time horizon of expectations formation, this assumption has the additional advantage of not depending on expectations of what happens after \( t + k + 1 \), in other words, of allowing all expectations to be contained in the limited time horizon of the market under consideration.

According to equation (6-7), the formation of expectations is qualitatively different depending whether expectations concern periods up to \( t + k \) or \( t + k + 1 \). Up to \( t + k \), forecasts of dividends are made according to rational expectations. On the other hand, the valuation of the stock price at \( t + k + 1 \) is made according to static or adaptive expectations. Thus the valuation model originating from equations (6-7) and (6-8) can be called a partial rational expectations or a bounded rationality of expectations formation model.

Substitution of equation (6-8) into equation (6-7) yields:

\[ p_t^f(k) = (1 - \gamma^{k+1})^{-1} \sum_{j=0}^{k} \gamma^{j+1} E(d_{t+j} | \Omega_t) \]  

(6-9)

According to the above equation (6-9), the fundamental stock price is determined by the expectations concerning the value of dividends of only \( k \) periods ahead. This means that the stock market in valuating a stock only considers the forecasts for the next \( k \) periods. When \( k=0 \) we have that:

\[ p_t^f(0) = \gamma E(d_t | \Omega_t) / (1 - \gamma) = E(d_t | \Omega_t) / r \]  

(6-10)

This result is similar to the obtained when dividends are assumed to stay at the same level for ever after the current period and the interest rate is assumed to be constant. However, here we interpret this result as meaning that the stock market makes the valuation of a stock based only on the forecast of the dividend to be received at the end of this period \( t \).
Thus through the use in equation (6-8) of static expectations we are able to capture the boundness that might exist in the stock market's time horizon.

If we substitute the model for dividends of equation (6-1) into equation (6-9) and rearrange we can get the following equation:

\[ p_t^f(k) = \left(\frac{\gamma}{1 - \gamma^k}\right) \cdot T_{t-1} + \omega(k) \cdot \delta_{t-1}, \]  
\[ \omega(k) = \left(\frac{\gamma + 2 \gamma^2 + (1+k) \gamma^{k+1}}{1 - \gamma^k}\right). \]

Here \( \omega(k) \) is an increasing function of \( k \) (\( \lim_{k \to \infty} \omega(k) = \gamma / (1 - \gamma^k) \)).

4.3 The variability of the span of time horizons of the stock market

From the analysis above the main cause of movements in fundamental stock prices can be attributed to changes in the drift \( \delta_{t-1} \) of the trend of dividends.
However, another cause of those movements can be associated with the variability of the span of time horizons in stock markets. In other words, the time horizon $k$ may not be constant but change with time ($k_t$ instead of $k$), with the consequent increased volatility of fundamental stock prices. For example, during the so called bubble period in Japan that lasted from 1987 to 1989 we can admit as reasonable the hypothesis that the span of the time horizon of the stock market was longer than average and increasing (because of the feeling among investors, or in other words, because of the forecast by investors that the stock prices would be able to sustain past growth). In a similar way when the bubble "burst" in 1990 it is admissible that the span of the time horizon of the stock market was reduced (because the crash was expected to be temporary).

Here not only we do not consider the determination of the span of the time horizon of the stock market as a maximization problem to be solved by the investors (we do not offer a micro analysis of this question) but also we will not present a rational explanation for the variability of the time horizon. Thus there are various interpretations of stock price volatility that are compatible with the variability of the span of time horizons, one of those being fashions, fads and other psychological factors suggested by Shiller (1989).

It should be noted however that the variability of the span of the time horizon of the stock market should not be thought of as depending of the risk associated with of the future flow of dividends. This is because, as it is apparent from equation (6-6b), the risk associated with dividends and expressed as the conditional variance of future dividends does not depend on time but only on the distance between two points in time. The contribution to the risk of a certain stock is greater the farthest away from the present moment is a certain point of time. But a certain stock, according to the above model, faces always the same level of risk independently of the current time period. Thus, from the point of view of the risk associated with the flow of future dividends, once a certain time horizon is determined there is no reason for the span of that time horizon to change.

We will also investigate the possible variability of the span of the time horizon of the stock market in the empirical analysis in the next section.
5. Empirical results

5.1 State Space Model

In this Section we investigate the validity of the theory developed in the above Sections 3 and 4. First we estimate the basic structural model for dividends: using Shiller's (1989) data (pp.439-446, Data Series, Tables 26.1-26.2) we estimate for the model represented by equation (6-1), the trend component $T_t$ and its drift $\delta_{t-1}$. Following Harvey and Todd (1983) and Clark (1987) we rewrite the dividend generation process as a state space model:

\[ d_t = A \theta_t + \epsilon_t \]  
\[ \theta_t = B \theta_{t-1} + \nu_t \]

Equation (6-13a) is usually referred as the measurement equation and equation (6-13b) as the state equation. Here $A = (1, 0)$, $B$ is a two by two matrix composed of two line vectors $(1, 1)$ and $(0, 1)$. Also, $\theta = (T_t, \delta_t)'$, and $\nu_t = (\eta_t, \xi_t)'$.

In the estimation of the values of the state variable $\theta_t$, we used smoothing estimation that makes use of all available information in the sample. To use smoothing estimation it is necessary to have estimates of the variances of the unknown parameters $\epsilon_t$, $\eta_t$, and $\xi_t$: $\sigma^2$, $\sigma^2$, and $\sigma^2$. To this purpose we used the maximum-likelihood method.

Assuming the normality of the distributions of $\epsilon_t$, $\eta_t$, and $\xi_t$, the log likelihood for the sample $d_t$ ($t = 1, 2, \ldots, T$) is proportional to:

\[ L = -\sum_{t=1}^{T} \ln (s_t) - \sum_{t=1}^{T} (e_t)^2 / s_t \]

In the above equation, $L$ can be appraised for any values of the variances of $\epsilon_t$, $\eta_t$, and $\xi_t$, and maximized by a nonlinear search of the parameter space. Here $e_t$ is the forecast error (the difference between the values of the smoothing forecast for $d_t$ and the actual $d_t$) and $s_t$ is its variance. In order to maximize $L$ with respect to the unknown parameters a Nelder-Mead simplex routine is usually used, but in this case
a simple grid search was used instead.

The estimated values of $T_t$ and $S_t$ estimated according to the procedures described above are presented in Figures 19 and 20 respectively.

5.2 Fundamental stock prices

5.2.1 Estimation of the discount factor

Making use of the estimated values of $T_t$ and $S_t$ in equation (6-4) we estimated the fundamental stock prices when the span of the time horizon is infinite. In order to make this estimate, we needed to have an estimate of the constant discount factor $\gamma$. To get an estimate of $\gamma$ we used three different methods.

The first method was to assume that (1) the span of the time horizon of the stock market is constant, and (2) the drift $\delta_{t-1}$ of the trend of dividends in the sample period is zero. Under the hypothesis that when these assumptions hold the actual stock prices equal fundamental stock prices we have that the following equation holds:

$$\bar{P} = \left\{ \frac{\gamma}{1 - \gamma} \right\} \bar{T}, \quad \gamma = \frac{T}{(\bar{P} + \bar{T})}. \quad (6-15)$$

Here, $\bar{P}$ and $\bar{T}$ are respectively the entire sample period (from 1872 to 1987) average stock price and average value of the dividends' trend. Computation of this value yields: $\gamma = 0.958$. This method is basically similar to the one used by Shiller (1981) in his computation of the discount factor.

Another method is to use the fact that $\gamma = 1 / (1 + r)$, where $r$ is the real interest rate plus an appropriate risk premium, to calculate the discount factor from the real interest rate. Since in the Data Series in the Chapter 26 of Shiller (1989) a nominal interest rate and a price index series are given we could calculate the average real interest rate for the sample period (from 1872 to 1987). Since the real interest rate is 2.72%, if the risk premium is assumed to be zero the discount factor is 0.974. Taking in consideration the possibility of the risk premium being positive, and assuming that the average value of the sum of the real interest rate and the risk premium is at maximum 5% (thus the average risk premium being at most 2.28%), the corresponding discount factor is 0.952. Thus, according to this method the discount factor is in the
interval between 0.952 and 0.974.

Still another method is to estimate $\gamma$ directly from the sample data under the assumption that the actual stock prices equal fundamental stock prices. When the span of the time horizon of the stock market is infinite as defined in equation (6-4), and if the relation $p_t = p_t^f$ holds, then we have the following equation:

$$p_t = \frac{\gamma}{1-\gamma} T_{t-1} + \frac{\gamma}{(1-\gamma)^2} \delta_{t-1}. \quad (6-16)$$

Because equation (6-16) is non-linear in the parameters, we used least squares non-linear estimation to calculate $\gamma$. The estimated value was $\gamma = 0.934$. In equation (6-16) it is assumed that the span of the time horizon of the stock market is infinite, that is $k = \infty$. But when that span is assumed to be one period, from the beginning to the end of the present period, that is $k = 0$, then the following equation holds:

$$p_t = \frac{\gamma}{1-\gamma} T_{t-1} + \frac{\gamma}{(1-\gamma)} \delta_{t-1}. \quad (6-17)$$

As above, using non-linear least squares estimation we estimated $\gamma$ to be $\gamma = 0.934$. With the use of this method the estimated value of $\gamma$ is then in the range between 0.934 and 0.961.

From the use of the three methods above we are lead to think that a reasonable value for the discount factor is between a lower floor of $\gamma = 0.934$ and an higher limit of $\gamma = 0.974$. From now on we will assume that the discount factor is the middle value of these two extreme values or $\gamma = 0.954$.

### 5.2.2 The fundamental stock prices

Using as discount factor the value of $\gamma = 0.954$, we computed the fundamental stock prices both when $k = \infty$ (when the span of the time horizon of the market is infinite) and when $k = 0$ (when the span of the time horizon is just up to the end of the current period) for the already estimated values of the trend component of dividends and its variable drift. These computed fundamental stock prices are presented in Figure 21 for $k = \infty$ and in Figure 22 for $k = 0$. From Figure 21 it is evident that when the market has an horizon with infinite time span the fundamental
stock price is very volatile. From Figure 22 it can be seen that when the stock market
as an horizon with a very short span the fundamental stock price seems to be less
volatile than the actual stock price presented in Figure 11. Clearly then, the volatility
of actual stock prices is between these two extreme cases.

Logically the next step is to find from among all possible stock market time
horizons the one that best explains the movements of actual stock prices. To find this
span we used three different methods.

**Simple Regression**

We regressed first the actual stock price with the variable trend component
of dividends $T_{t-1}$ and its variable drift $\delta_{t-1}$. The result of this estimation for the
period from 1872 to 1987 was:

$$p_t = 23.6 \ T_{t-1} + 46.0 \ \delta_{t-1}, \quad (6-18)$$

$$\text{(40.2)} \quad \text{(3.71)}$$

Adjusted $R^2 = 0.854$, D-W = 0.355, S.E. = 22.6.

The figures inside brackets below each estimated parameter are the
 corresponding t-values, Adjusted $R^2$ is the multiple coefficient of determination
corrected for the degree of freedom, D-W is the Durbin-Watson statistic and S.E. is the
standard error.

Since the parameter of $T_{t-1}$ is $\gamma/(1-\gamma)$ we have that $\gamma/(1-\gamma) = 23.6$ and
thus the discount factor is $\gamma = 0.959$. In this case, from the definition of $\omega(k)$ in
equation (6-11b) and from the estimated parameter of $\delta_{t-1} = 46.0$ we can compute
the span of the stock market time horizon to be about $k=2$, that is to span for two
years after the end of the current period. Thus, using simple regression we find that
the fundamental stock price explains the actual stock prices the best when the span
of the stock market is two years. To test the stationarity of the residuals of equation
(6-18) we used the Dickey-Fuller (1981) test and confirmed that indeed they were
stationary. Thus $p_t$ and the fundamental stock price defined as $p_{f,t} = 23.6T_{t-1}
+ 46.0 \delta_{t-1}$ are cointegrated.

Figure 23 presents a comparison of the fundamental stock prices, when
$k = 2$, and the actual stock prices.
\[ \nabla^2 p_t = 25.8 \nabla^2 T_{t-1} + 117.2 \nabla^2 \delta_{t-1} \]  
\( (2.39) \quad (4.98) \)

Adjusted \( R^2 = 0.179 \), S.E. = 18.6.

As for the two previous cases, from the coefficients of equation (6-21) we can estimate the span of the horizon of the stock market to be around 8 years.

From the analysis above, the fundamental stock price defined under the hypothesis that the span of the time horizon of the stock market is limited, i.e., that it can be somewhere between 2 and 8 years long, has a long run stable relationship with the actual stock prices. It should be especially noted the high explanatory power of the Error Correction Model. Even if the actual stock prices temporarily diverge from fundamental stock prices, in the long run their movements show a stable relationship with the fundamental stock prices defined for constant limited time horizons.

5.3 Variability in the span of time horizons

The span of the time horizon \( k \) of the stock market is not necessarily constant. For example, in times of continuous prosperity and rising stock prices, the span of the time horizon of the stock market should become longer. But if a change in economic conditions occurs and the prospect of the future state of the economy becomes unclear then the span should become smaller.

If the span \( k \) of the time horizon of the stock market is not constant, then also the parameter \( \omega(k) \) of the variable \( \delta_{t-1} \) in equation (6-11b) is not constant. Another way of saying this is to write \( \omega(k) = \omega \), instead of \( \omega(k) \). In what follows we estimate the fundamental stock price when the span of the time horizon changes with time. To estimate \( \omega \), we specify its stochastic process as:

\[ \omega_t = \omega_{t-1} + \nu_t \]  
\( (6-22) \)

where \( \nu_t \) is white noise with distribution NID(0, \( \sigma^2 \nu \)). We are thus assuming that \( \nu_t \) follows a random walk process. Although the value of \( \omega \), assumes discrete values function of the value of \( k \), we approximate it to a continuous function. We make the following assumptions concerning the determination of \( k \) and \( \omega \): the value of
**Error Correction Model**

Based on the results obtained above for the simple regression tests and on the hypothesis that the actual stock price $P_t$ has two components (i.e., fundamental stock price $p_{t}^f +$ stationary residual $u_t$), we estimated the following AR(1) model on the residual ($u_t = \rho u_{t-1} + \xi_t$; $|\rho| < 1$, $\xi_t$ is white noise):

\[
\begin{align*}
  P_t &= 24.2 \, T_{t-1} + 72.8 \, \delta_{t-1}, \quad \rho=0.837, \\
    & \quad (13.8) \quad (5.75) \quad (16.7) \\
  \text{Adjusted } R^2 &= 0.938, \quad \text{S.E.}=12.7.
\end{align*}
\]

Performing the Dickey and Fuller (1979) unit root test we confirmed that the residuals in equation (6-19) are stationary. Because $P_t$ and $p_{t}^f$ under the assumptions of our model are non-stationary processes of order 1(2) and because $u_t$ is a stationary time series, rearranging terms on equation (6-19) yields the following Error Correction Model:

\[
\begin{align*}
  \Delta p_t &= -(1-\rho) (p_t - p_{t-1}^f) + \Delta p_{t-1}^f \\
  &= -0.163 (p_{t-1} - p_{t-1}^f) + \Delta p_{t-1}^f
\end{align*}
\]

(6-20)

The high explanatory power of equation (6-19) means that this error correction model also has high explanatory power.

As above for the simple regression, because $\gamma/(1-\gamma) = 24.2$ and $\omega(k) = 72.8$ we get that $k = 4$. Figure 24 presents a comparison between the stock price as defined by the above error correction model and the actual stock price.

**Second Differences**

It should be noted that neither $T_{t-1}$ nor $\delta_{t-1}$ are stationary processes. From the dividend model of equation (6-1) we can see that if we take differences twice to $T_{t-1}$ and $\delta_{t-1}$ both processes become stationary: $\nabla^2 T_{t-1} = \xi_{t-2} + \eta_{t-1} - \eta_{t-2}$, and $\nabla^2 \delta_{t-1} = \xi_{t-1} - \xi_{t-2}$. Regressing the second differences of $p_t$ on the second differences of $T_{t-1}$ and $\delta_{t-1}$ we get:
k during period t, k<sub>t</sub> is determined based in the value of k during period t-1, k<sub>t-1</sub>, plus an effect upwards or downwards due to the white noise component υ<sub>t</sub>.

Based on equation (6-22) we estimated the following model using smoothing estimation of a Kalman filter:

\[
\begin{align*}
    p_t &= \alpha_t \cdot T_{t-1} + \omega_t \cdot \delta_{t-1} + \upsilon_t \\
    \alpha_t &= \alpha \\
    \omega_t &= \omega_{t-1} + \upsilon_t
\end{align*}
\]  

(6-23a)  
(6-23b)  
(6-23c)

where \( \alpha = \{ \gamma / (1 - \gamma) \} \), and \( \upsilon_t \) is white noise with distribution NID(0, \sigma^2). The estimated \( \omega_t \) is presented in Figure 25. In this estimation the values of the unknown parameters \( \sigma^2 \) and \( \sigma^2 \) were chosen so that the estimated values of \( \omega_t \) would fall within reasonable limits.

As is apparent from Figure 25, the estimated \( \omega_t \) shows a increasing value after World War II, showing that the span of the horizon of the stock market was increasing. According to this model, the increase of the span of the time horizon of the stock market means that the fundamental stock price is receiving the effects of \( \delta_t \) more strongly.

It should be noted however that the consideration of the possibility of the time horizons changing does not increase much the explanatory power by the fundamental stock prices of the behaviour of the actual stock prices. In fact, when \( \omega_t \) is allowed to vary, the power to explain the behaviour of actual stock prices by fundamental stock prices is according to the simple regression method Adj. \( R^2 = 0.86 \): this is about the same as when we did not allow \( \omega_t \) to vary. However, if we lift the restriction on the values that \( \omega_t \) may take the explanatory power rises to Adj. \( R^2 = 0.93 \). But in this case \( \omega_t \) takes extremely volatile values, sometimes even negative values (see Figure 26).

The result we got was that although the span of the time horizon of the stock market is indeed variable, its consideration does not increase considerably the power to explain real stock prices.
<table>
<thead>
<tr>
<th>Author</th>
<th>Sample</th>
<th>Source</th>
<th>Change</th>
<th>Participactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscar A. Harris</td>
<td>Jan 4, 1980</td>
<td>0.365</td>
<td>1.655</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>Jan 11, 1980</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trevor &amp; Miller</td>
<td>Jan 1, 1980</td>
<td>0.21</td>
<td>2.42</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mar 11, 1980</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Scientist</td>
<td>Nov 23, 1981</td>
<td>-1.395</td>
<td>-17.60</td>
<td>2.02</td>
</tr>
<tr>
<td></td>
<td>Nov 20, 1981</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Table 1: Unit Root Tests

<table>
<thead>
<tr>
<th>Author</th>
<th>Sample</th>
<th>t-value</th>
<th>Durbin-Watson</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Levels</td>
<td>Changes</td>
</tr>
<tr>
<td>Dwyer &amp; Hafer</td>
<td>Aug 4 1986</td>
<td>0.208</td>
<td>-7.952</td>
</tr>
<tr>
<td></td>
<td>Jun 11 1987</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dwyer &amp; Hafer</td>
<td>Jan 2 1987</td>
<td>-0.637</td>
<td>-6.258</td>
</tr>
<tr>
<td></td>
<td>Jun 11 1987</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dos Santos</td>
<td>Nov 12 1987</td>
<td>-1.039</td>
<td>-17.306</td>
</tr>
<tr>
<td></td>
<td>Dec 29 1989</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 2  COINTEGRATION BETWEEN STOCK PRICES AND PROFITS

<table>
<thead>
<tr>
<th>Variable</th>
<th>t-value on lag variable</th>
<th>Durbin-Watson</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Levels</td>
<td>Changes</td>
</tr>
<tr>
<td>(A1) Stock Prices</td>
<td>0.6073</td>
<td></td>
</tr>
<tr>
<td>Profits</td>
<td>-1.2213</td>
<td>1.9361</td>
</tr>
<tr>
<td>Residuals</td>
<td>-1.5170</td>
<td>1.9190</td>
</tr>
<tr>
<td>(A2) and (B) Stock Prices</td>
<td>0.3513</td>
<td>-6.5311</td>
</tr>
<tr>
<td>Profits</td>
<td>-1.2200</td>
<td>-4.3718</td>
</tr>
<tr>
<td>Residuals</td>
<td>-1.5667</td>
<td>-5.3444</td>
</tr>
</tbody>
</table>
## TABLE 3  COINTEGRATION TESTS BETWEEN STOCK PRICES AND PROFITS WHEN INTEREST RATE IS VARIABLE

<table>
<thead>
<tr>
<th>Variable</th>
<th>t-value on lag variable</th>
<th>Durbin-Watson</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Levels</td>
<td>Changes</td>
</tr>
<tr>
<td>Real interest rates</td>
<td>-2.7249</td>
<td>-9.8011</td>
</tr>
<tr>
<td>Nominal interest rates</td>
<td>-2.1679</td>
<td>-4.6803</td>
</tr>
<tr>
<td>Author</td>
<td>Sample*</td>
<td>Const</td>
</tr>
<tr>
<td>------------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>Dwyer &amp; Jul 30 1986</td>
<td>0.001</td>
<td>1.176x10^{-5}</td>
</tr>
<tr>
<td></td>
<td>Hafer Jun 6 1987</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dwyer &amp; Jan 2 1986</td>
<td>0.004</td>
<td>-7.80x10^{-6}</td>
</tr>
<tr>
<td></td>
<td>Hafer Jun 6 1987</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dos Nov 12 1987</td>
<td>0.001</td>
<td>-6.67x10^{-7}</td>
</tr>
<tr>
<td>Santos Dec 29 1989</td>
<td>1.86</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(\*) Sample endpoints represent peaks in stock price indexes
<table>
<thead>
<tr>
<th>Author</th>
<th>Sample</th>
<th>Sum of Coefficients</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dwyer &amp; Hafer</td>
<td>Sep 3 1986</td>
<td>$25 \sum \beta_i = -1.40 \times 10^{-1}$</td>
<td>24.001</td>
</tr>
<tr>
<td></td>
<td>Jun 11 1987</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dwyer &amp; Hafer</td>
<td>Jan 2 1987</td>
<td>$25 \sum \beta_i = -1.108$</td>
<td>41.819*</td>
</tr>
<tr>
<td></td>
<td>Jun 11 1987</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dos Santos</td>
<td>Nov 12 1987</td>
<td>$7 \sum \beta_i = -1.07 \times 10^{-3}$</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Dec 29 1989</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 6 POSSIBLE TRENDS OF REAL STOCK PRICES

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Real Stock Prices</th>
<th>Real Dividends</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time***</td>
<td>R²</td>
</tr>
<tr>
<td>1871-1912</td>
<td>1.191</td>
<td>0.869</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td></td>
</tr>
<tr>
<td>1912-1921*</td>
<td>-0.965</td>
<td>0.895</td>
</tr>
<tr>
<td></td>
<td>(0.602)</td>
<td></td>
</tr>
<tr>
<td>1921-1929**</td>
<td>8.803</td>
<td>0.869</td>
</tr>
<tr>
<td></td>
<td>(1.206)</td>
<td></td>
</tr>
<tr>
<td>1930-1951</td>
<td>-1.534</td>
<td>0.354</td>
</tr>
<tr>
<td></td>
<td>(0.464)</td>
<td></td>
</tr>
<tr>
<td>1951-1971</td>
<td>8.306</td>
<td>0.907</td>
</tr>
<tr>
<td></td>
<td>(0.611)</td>
<td></td>
</tr>
<tr>
<td>1971-1987</td>
<td>-2.438</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>(1.972)</td>
<td></td>
</tr>
<tr>
<td>1871-1987</td>
<td>1.100</td>
<td>0.550</td>
</tr>
<tr>
<td></td>
<td>(0.093)</td>
<td></td>
</tr>
</tbody>
</table>

(*) For dividends 1912-1920
(**) For dividends 1920-1930
(*** Numbers in parenthesis are standard errors
FIGURES
Figure 1 - Recalculation of probabilities
Types of possible reactions to the arrival of new information at $t=8$

Figure 2 - Efficient stock price reaction to the arrival of news
Figure 3 - Over-reactive reaction of stock prices to the arrival of news
Types of possible reactions to the arrival of new information at t=8

Figure 4 - Delayed reaction of stock prices to the arrival of news
Types of possible reactions to the arrival of new information at $t=8$

**Figure 5** - Delayed over-reactive reaction of stock prices to the arrival of news.
Types of possible reactions to the arrival of new information at $t=8$

Figure 6 - Comparison of some possible reactions of stock prices to the arrival of news
EVENT

PERCEPTION OF THE EVENT

ANNOUNCEMENT OF THE EVENT

PUBLIC PERCEPTION OF THE EVENT

EXPECTATIONS FORMATION: RECALCULATION OF PROBABILITIES

PRICE CHANGES

Information channeling

Information processing

Information channeling

Information processing

Semi-strong form

Weak form

Figure 7 - Information channeling and information reaction

125
Figure 8 - Actual and ex-post rational prices

Real Standard and Poor's Composite Stock Price Index (solid line \( p \)) and ex-post rational price (dotted line \( p^* \)), 1871-1979, both detrended by dividing by a long-run exponential growth factor. (From Shiller (1981)).
Figure 9 - Actual and adjusted ex-post rational prices

Real Standard and Poor's Composite Stock Price Index (solid line $P$) and ex-post rational price (dotted line $P^*$), where $A$ is a coefficient of risk aversion. With $A=0$ (dotted line) the discount rates are constant, with $A=4$ (dashed line) they vary with consumption. 1889-1979.

(From Grossman and Shiller (1981).)
Figure 10 - Actual and ex-post rational prices in Japan

Real TOPIX (solid line $p$) and ex-post rational price (dotted line $p^*$), December 1973 - December 1992, both detrended by dividing by a long-run exponential growth factor.
Figure 11 - Real Stock Prices Time Series
Figure 12 - Real Dividends Time Series
Figure 13 - Real Stock Prices with different possible trends reflecting different possible economic structures
Figure 14 - Real Dividends with several possible different trends reflecting possible different economic structures
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Figure 19 - Trend T1
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Figure 26 - Unrestricted $\omega(t)$
NOTES

We can distinguish two basic meanings for the word "market". One is that of exchange or exchange mechanism or process, the sense we are dealing with, and the sense we use when we say "the capital market", "the foreign-exchange market", or "the gold (sugar, loan, etc.) market". This sense has evolved from the one cited in note (1), that of "public space where buying of goods could take place. Another meaning is that of potential demand or opportunity to supply a good (in this sense Adam Smith in his An Inquiry into the Nature and Causes of the Wealth of Nations (1776) writes: the division of labour depends on the extent of the market), or of a geographical area or group of people in which there is demand for a certain good (in this sense Karl Marx in Capital (1867) asserts: "at that moment (...) the Indian and Chinese market is again extended by the discoveries of the British cotton manufacturers"). Connecting these two meanings is a third one that of the meeting of the demand and supply so that prices and quantities exchanged are determined and tend to equilibrium.

Usually we refer together three of the above three meanings at mind when using the word. "Market" does not seem to be referring to the kind of the above meanings but only to the second and third. Marshall's distinction affects the first as well, that is consistent with what he wrote open.
NOTES

1 The English economist William Stanley Jevons in his *Theory of Political Economy*, Ch. IV, (1871) alludes to this definition in the following terms:

*Originally a market was a public place in a town where provisions and other objects were exposed for sale.*

2 Alfred Marshall, 1890, *Principles of Economics*, Book V, Ch. 1. This quotation is from Augustin Cournot, *Recherches sur les principes mathematiques de la theorie des richesses*, Ch. IV.

3 We can distinguish two basic meanings for the word "markets". One is that of exchange or exchange mechanism or process, the sense we are dealing with, and the sense we use when we say "the capital markets", "the foreign exchange market", or "the gold (sugar, loan, etc) market". This sense has evolved from the one cited in note (1), that of a public space where trading of goods could take place. Another meaning is that of potential demand or opportunity to supply a good (in this sense Adam Smith in his *An Inquiry Into the Nature and Causes of the Wealth of Nations* (1776) writes "the division of labour depends on the extent of the market"), or of a geographical area or group of people in which there is demand for a certain good (in this sense Karl Marx in *Capital* (1867) scrawls "at this moment (...) the Indian and Chinese market is again overstocked by the consignments of the British cotton manufacturers"). Connecting these two meanings is a third one, that of the meeting of the demand and supply so that prices and quantities exchanged are determined and tend to equilibrium.

Usually authors have more than one of the above three meanings in mind when using the word. Cournot does not seem to be referring to the first of the above meanings but only to the second and third. Marshall’s translation affords the first as well, that is consistent with what he writes next.
Great, complex and wealthy civilizations have existed in which markets did not exist and where commerce was not practiced. For example, the old Persians had no market places and their economic life was subordinated to the indications of the political authorities, who decided what to produce, collected it and then distributed what they had collected. The whole network of supply and distribution was operated through the administrative system.

Herodotus notes in *The History* (c. 445 B.C.) that when the king of the Persians Cyrus received a Greek embassy warning him against invading some Greek cities in Asia Minor he told the Greek herald (embassador):

"I have never yet been afraid of any men who have a set place in the middle of their city, where they come together to cheat each other and forswear themselves. If I live, the Spartans shall have troubles enough of their own to talk of, without concerning themselves about the Ionians." Cyrus intended these words as a reproach against all the Greeks, because of their having market-places where they buy and sell, which is a custom unknown to the Persians, who never make purchases in open marts, and indeed have not in their whole country a single market-place.

The implication in Cyrus words is that market places involve cheating and cheating makes a group (nation) weak (less perfect, inefficient), a view still held by some in the 20th century. Thus the confrontation between market economies and command economies started at least 2500 years ago.

This statement of the aim (or function, purpose, role, etc) of financial (or capital, securities, etc) markets can be found with some slight differences in wording in many finance and money and banking text books. Two examples are Copeland and Weston (1988), and Mishkin (1986).

In Japan depository institutions are classified into private depository institutions (that include city (commercial) banks, regional banks, and specialized foreign exchange banks), foreign banks, long-term credit banks, trust banks, credit associations, rural financial associations, etc.
7 In Japan contractual savings institutions are divided into life insurance companies, non-life insurance companies, securities investment trust companies, securities finance companies, etc.

8 In Japan in this category is composed by securities houses, short-term finance companies, public finance organizations and others.

9 Definitions of the spans of short, medium and long-term can vary, but usually up to one year is short-term, from one to three years is medium-term and more than three years is long term.

10 Already at the end of the 19th century Marshall considered a large portion of the world to constitute one integrated large stock market. In his Principles of Economics, Book V, Ch. 1, he writes:

    The whole Western World may, in a sense, be regarded as one market for many kinds of stock exchange securities, for the more valuable metals, and to a less extent for wool and cotton and even wheat.

11 For a treatment of this topic see, for example, Copland and Weston (1988) or Hirshleifer (1970).

12 The almost impossibility of having one without the other makes difficult the development of stock markets in developing countries and also in countries trying to change their economic systems from centrally planned economies to market economies. Which should be encouraged first and more actively, the primary or the secondary market?
Like that of gambling.

The famous passage is:

Every individual endeavors to employ his capital so that its produce may be of the greatest value. He generally neither intends to promote the public interest, nor knows how much he is promoting it. He intends only his own security, only his own gain. And he is in this led by an invisible hand to promote an end which was no part of his intention. By pursuing his own interest he frequently promotes that of society more effectually than when he really intends to promote it.


Miller and Modigliani (1961) demonstrate that in evaluating the value of a firm the dividend approach is equivalent to the present value of the net cash flows into the firm (the so called earnings-investment approach).

The continuation of Marshall's passage quoted in note 10 reads as follows:

Proper allowance being made for expenses of transport, in which may be included taxes levied by any customs houses through which the goods have to pass. For in all these cases the expenses of transport, including customs duties, are not sufficient to prevent buyers from all parts of the Western World from competing with one another for the same supplies.

For an analysis of how the friction caused by the presence of taxes affects the valuation of assets see Litzenberger and Sossin (1977), Prisman (1986) or Ross (1987).
Speaking of markets in general, Cournot, 1838, *Mathematical Principles of the Theory of Wealth*, Ch. 1, refers to "circulation" as the "possibility to find means to exchange [goods and services] for other objects of equal value" and depending of "the state of commercial relations and civil institutions".

He goes on writing that actual markets enjoy different degrees of friction:

> Of all the things on which we set a price, or to which we attribute a value in exchange, there are none always exchangeable at will for any other commodity of equal price or value. In the act of exchange, as in the transmission of power by machinery, there is friction to be overcome, losses which must be borne, and limits which cannot be exceeded.

He gives some examples:

> The proprietor of a great forest is only rich on condition of managing his lumbering with prudence, and of not glutting the market with his lumber; the owner of a valuable picture gallery may spend his life in the vain attempt to find purchasers; while, on the other hand, in the neighbourhood of a city the conversion of a sack of grain will only require the time necessary to carry it to the grain market; and at great commercial centres a stock of coffee can always be sold on the exchange.

and then explains that although always present in the real world (as opposed to the theoretical world of abstract models), friction diminishes with the development of markets that thus tend to the ideal world of abstract conditions:

> The extension of commerce and the development of commercial facilities tend to bring the actual conditions of affairs nearer and nearer to this order of abstract conceptions, on which alone theoretical calculations can be based, in the same way as the skilful engineer approaches nearer to theoretical conditions by diminishing friction through polished bearings and accurate gearing. In this way nations are said to make progress in the commercial or mercantile system.
It is also evident that Cournot was aware of two antagonistic views of market because he adds:

> These two expressions [commercial and mercantile] are etymologically equivalent, but one is now taken in a good and the other in a bad sense, as is generally the case, according to Bentham, with the names of things that involve advantages and evils of a moral order.

Marshall already considered (in *Principles of Economics*, Book V, Ch. 1,) that at the end of the 19th century the information technology then existent, the telegraph and the printing press, allowed for almost instantaneous dissemination of information around the world so that the prices of the same security would be the same everywhere:

> But the strongest case of all is that of securities which are called "international," because they are in request in every part of the globe. They are the bonds of the chief governments, and of very large companies such as those of the Suez Canal and the New York Central Railway. For bonds of this class the telegraph keeps prices at almost exactly the same level in all the stock exchanges of the world. If the price of one of them rises in New York or Paris, in London or in Berlin, the mere news of the rise tends to cause a rise in the other markets; and if for any reason the rise is delayed, that particular class of bonds is likely soon to be offered for sale in the high priced market under telegraphic orders from the other markets, while the dealers in the first market will be making telegraphic purchases in other markets. These sales on the one hand, and purchases on the other, strengthen the tendency which the price has to seek the same level everywhere; and unless some of the markets are in abnormal condition, the tendency soon becomes irresistible.

Karl Marx did not believe that any individual could have complete information about what he is transacting, and criticised those economists who assumed otherwise. For example, in his 1867 tome he scribbles:
In bourgeois societies the economical fictio juris [legal fiction] prevails: that every one, as a buyer, possesses an encyclopaedic knowledge of commodities.

22 Financial return, or just return, is the profit provided by an asset net of all expenses necessary to generate that profit. Economic return is return (financial return) after deduction of opportunity costs.

23 If individuals were not risk neutral as we assumed in section 2. Assumptions of a Stock Market Model, the interest rate of the riskless asset would have to be adjusted with a risk premium. For a presentation along this line see Keizai Hakusho (1991).

24 If \( \Omega \) is an information set and \( \omega \) a subset of this information set, then for any variable \( y \):

\[
E\left[ E[y \mid \Omega] \mid \omega \right] = E[y \mid \omega]
\]

25 There is a kind of opportunity cost in not receiving new information as fast as the investor that receives that receives it fastest.

26 For example, by John Kenneth Galbraith (in "The 1929 Parallel", "The Atlantic Monthly", January 1987). He writes:

In the months and years to the 1929 crash there was a wondrous proliferation of holding companies and investment trusts. The common feature of both the holding companies and the trusts was that they conducted no practical operations; they existed to hold stock in other companies, and these companies frequently existed to hold stock in yet other companies. Pyramiding, it was called. The investment trust and the utility pyramid were
the greatly admired marvels of the time. ( ...).

[This leveraged process] meant that any increase in the earnings of the ultimate companies would flow back with geometric force to the originating company. ( ...)

It was a grave problem, however, that in the event of failing earnings and values, leverage would work fully as powerfully in reverse.

27 Jevons seems to be the first economist to express the idea that sunspots can influence agricultural output and thus also other economic activities. However, the meaning of sunspots in economics nowadays is that of a variable that affects equilibrium only because individuals believe it does.

28 Common stock holders have residual claims on the assets of a company after all creditors (employees, suppliers, banks bond holders, etc) have been paid. These residual claims are expressed by the concept of net worth and their fundamental value is equal to the present value of expected future dividends.

29 This does not happen with proprietorships and partnerships that have unlimited liability. "The owners' of proprietorships and partnerships typically have unlimited liability, which means that business creditors can look for repayment beyond the business entity's assets to the owners' personal assets." (from Horngren and Sanders (1987), "Introduction to Financial Accounting")

30 We had better say these hypotheses since, as said previously, we are making two hypothesis: 1) that stock prices are determined according to the fundamental solution (equation (5-5)), and 2) that dividends follow the deterministic model of equation (5-6).

31 This can be seen by substitution of the fundamental price $p_t^*$ by its definition as given by equation (5-5) and the dividend's process specification of equation (5-10):
The test for unit roots was performed for levels and changes: for the former, as explained, the first differences in the logarithms of stock prices (changes) are regressed on a constant, the lagged value of the logarithms of stock prices (levels) and the lagged value of the first difference in logarithms of stock prices; for the later, the second differences in logarithms of stock prices are regressed on a constant, the lagged first difference of the logarithms of stock prices and the lagged value of the second difference in the logarithms of stock prices, or:

\[ \Delta^2 \ln p_{t+1} = \alpha_0 + \alpha_1 \Delta \ln p_t + \alpha_2 \Delta^2 \ln p_t. \]

The test statistic is in the first case the reported t-ratio for the lagged level, and in the second case the reported t-ratio for the lagged first difference. For reference, if in the test for changes we omit the constant term \( \alpha_0 \), the t-ratio would be -16.851 instead of -17.306.

Two or more, but for simplicity and because the case at hand involves only two variables, we will refer only to the two variables case.

Equation (5-18) is the correct specification when \( \mu \neq 0 \), in equation (5-16). When there is no drift and \( \mu = 0 \) then \( \alpha_0 = 0 \) and the correct specification of equation (5-18) becomes:

\[ p_t = \alpha_1 d_t + \epsilon_t. \]
If $\beta = 1$ and $\delta = 0$ and dividends have a unit root, then equation (5-16) is:

$$d_{t+1} = \mu + d_t + \epsilon_{t+1}$$

If we take expectations at time $t$ on the above equation we have that

$$E[d_{t+1} | \phi_t] = \mu + d_t$$

Also if we take expectations on:

$$d_{t+m+1} = \mu + d_{t+m} + \epsilon_{t+m+1}$$

we can have that

$$E[d_{t+m+1} | \phi_t] - E[d_{t+m} | \phi_t] = \mu$$

Substitution of these two results into equation (5-20) gives that:

$$p_t = r^{-1} \mu + r^{-1} d_t + r^{-1} \mu \sum_{m=0}^{\infty} a^m$$

or

$$p_t = r^{-1} \mu + r^{-1} d + r^{-2} \mu$$

Making $\alpha_0 = r^{-1} \mu + r^{-2} \mu$ and $\alpha_1 = r^{-1}$ we arrive at:

$$p_t = \alpha_0 + \alpha_1 d_t + \epsilon_t$$

If there no drift in equation (5-16), that is $\mu = 0$, then we have that:

$$E[d_{t+m+1} | \phi_t] - E[d_{t+m} | \phi_t] = 0$$

and
\[ p_t = \alpha + d_t + \epsilon_t \]
as already said in note 34.

Assuming that \( \gamma = 0.954 \) we calculated \( \omega(k) \) for different values of \( k \). The value \( \omega(k) \) which is closest to the estimated parameter of \( \delta_{t-1} \) was chosen as giving the span of the stock market horizon.
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