Number and Calculations, Number and Algebraic Expressions

KOYAMA Masataka, Hiroshima University

Abstract: The purpose of this paper is to summarize the changes of teaching and learning contents in the areas of "Number and Calculations" and "Number and Algebraic Expressions" in the Course of Studies for elementary and secondary school mathematics after 1945 in Japan. As a result, the followings are found out as three main characteristics. (a) There is a remarkable difference between the period of "Life-Unite Learning" (1945 – 1957) and the period of "Systematic Learning" (1958 – 1967) regarding to the teaching and learning contents and their sequences in these areas shown in the Course of Studies. (b) The contents and sequences shown in the Course of Study (1958) has been the prototype of those shown in the following Course of Studies (1968, 1977, and 1989). In the new one (1998), however, we notice that calculations are rather lightened; decimal numbers and fractions are introduced one year later than the before (1989); the using of algebraic expressions with alphabetic letters is shifted from elementary to lower secondary school mathematics. (c) The dealing with equations and inequalities in secondary schools is gradually lightened in recent years.

1. Introduction

The purpose of this paper is to summarize the changes of teaching and learning contents in the areas of "Number and Calculations" and "Number and Algebraic Expressions" in elementary and secondary school mathematics after 1945 in Japan. In doing so, we refer to the Course of Studies that have been revised every about ten years by the Ministry of Education.

2. Changes in "Number and Calculations"

The tables 1, 2, 3, and 4 show the summary of changes of the teaching and learning contents regarding to the number and calculations shown in the Course of Studies (1947, 1951, 1958, 1968, 1977, 1989, and 1998).

In the tentative Course of Study (1947), the four operations with fractions were dealt with in the sixth grade. In the tentative one (1951), the timing of dealing with teaching and learning contents in this area was generally one- or two-year later than the before. For example, multiplication facts was in third grade; division by two-digit numbers was in fifth grade; multiplication and division with fractions were in the first grade at lower secondary level; and four operations with positive and negative numbers were in the second at lower secondary level (Table 1).

In the period of "Systematic Learning" (1958 – 1967), the idea and structure of number sets was introduced as new teaching and learning content in the second grade (Table 2). The contents and sequences shown in the Course of Study (1958) has been the prototype of those shown in the following Course of Studies (1968, 1977, and 1989).

In the period of "Modernization" (1968 – 1976), at elementary school level no remarkable changes were made except completing the four operations with fractions in elementary school. On the other hand, at lower secondary level, the idea and structure of number sets was introduced as new teaching and learning content in the second grade (Table 2).

In the period of "Back to Basics" (1977 – 1988), however, the idea and structure of number sets was eliminated (Table 3). In this period, the need for careful selection of the teaching and learning contents in mathematics was emphasized. But we cannot find out the remarkable changes in this area of number and calculations. It means that the area seems to be basic and important in school mathematics.

In the Course of Study (1989), there were no changes in this area at elementary level. At secondary level, some
Table 1. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 1)

<table>
<thead>
<tr>
<th>Grade</th>
<th>Tentative Course of Study (1947)</th>
<th>Tentative Course of Study (1951)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Counting, reading, and writing numbers up to 100</td>
<td>Numbers up to 50</td>
</tr>
<tr>
<td>2nd</td>
<td>Numbers up to 1000</td>
<td>Principle of place unit and half of unit</td>
</tr>
<tr>
<td>3rd</td>
<td>Unit fractions</td>
<td>Multiplication facts</td>
</tr>
<tr>
<td>4th</td>
<td>Numbers less than one trillion</td>
<td>Understanding of meanings of equal division and inclusive division</td>
</tr>
<tr>
<td>5th</td>
<td>Multiple relations of numbers and fractions</td>
<td>Numbers up to 5000</td>
</tr>
<tr>
<td>6th</td>
<td>Fractions</td>
<td>Mixed calculations</td>
</tr>
<tr>
<td>1st</td>
<td>Positive and negative numbers</td>
<td>Numbers up to 1000</td>
</tr>
<tr>
<td>2nd</td>
<td>Square numbers and square root numbers</td>
<td>Numbers up to 5000</td>
</tr>
<tr>
<td>3rd</td>
<td>History of numbers and computations, and its relation to our daily life</td>
<td>Using the table of square root numbers</td>
</tr>
<tr>
<td>UPPER S.</td>
<td>Complex numbers</td>
<td>Computations with square numbers &amp; square root numbers</td>
</tr>
</tbody>
</table>

Table 2. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 2)

<table>
<thead>
<tr>
<th>Course of Study (1923)</th>
<th>Course of Study (1958)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>Addition &amp; Subtraction</td>
</tr>
<tr>
<td>1st Grade</td>
<td>Counting, reading, and writing numbers up to 100</td>
</tr>
<tr>
<td>2nd Grade</td>
<td>Reading and writing of numbers up to 1000</td>
</tr>
<tr>
<td>3rd Grade</td>
<td>Decimal positional notation</td>
</tr>
<tr>
<td>4th Grade</td>
<td>Units of million and billion</td>
</tr>
<tr>
<td>5th Grade</td>
<td>Properties of natural numbers (odd and even numbers)</td>
</tr>
<tr>
<td>6th Grade</td>
<td>Approximate numbers</td>
</tr>
<tr>
<td>1st Grade</td>
<td>Positive and negative numbers &amp; negative numbers</td>
</tr>
<tr>
<td>2nd Grade</td>
<td>Fractions</td>
</tr>
<tr>
<td>3rd Grade</td>
<td>Approximate numbers</td>
</tr>
<tr>
<td>UPPER S.</td>
<td>Approximate numbers</td>
</tr>
</tbody>
</table>
Table 3. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 3)

<table>
<thead>
<tr>
<th>Course of Study (1977</th>
<th>Course of Study (1989)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number</strong></td>
<td><strong>Addition &amp; Subtraction</strong></td>
</tr>
<tr>
<td>1st Grade</td>
<td>Representing the number and the order of digits with numbers up to 14 digits (whole numbers)</td>
</tr>
<tr>
<td>2nd Grade</td>
<td>Thousand positions, integer, order of numbers up to 4-digit numbers</td>
</tr>
<tr>
<td>3rd Grade</td>
<td>Ten Thousand positions, integer, order of numbers and fractions, with simple 2-digit numbers</td>
</tr>
<tr>
<td>4th Grade</td>
<td>Integers and rational numbers</td>
</tr>
<tr>
<td>5th Grade</td>
<td>Situations of Addition &amp; Subtraction with simple 2-digit numbers</td>
</tr>
<tr>
<td>6th Grade</td>
<td>Situations of Addition &amp; Subtraction with simple 2-digit numbers</td>
</tr>
</tbody>
</table>

Table 4. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 4)

<table>
<thead>
<tr>
<th>Course of Study (1989)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number</strong></td>
</tr>
<tr>
<td>1st Grade</td>
</tr>
<tr>
<td>2nd Grade</td>
</tr>
<tr>
<td>3rd Grade</td>
</tr>
<tr>
<td>4th Grade</td>
</tr>
<tr>
<td>5th Grade</td>
</tr>
<tr>
<td>6th Grade</td>
</tr>
</tbody>
</table>

Table 3. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 3)

<table>
<thead>
<tr>
<th>Course of Study (1977)</th>
<th>Course of Study (1989)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number</strong></td>
<td><strong>Addition &amp; Subtraction</strong></td>
</tr>
<tr>
<td>1st Grade</td>
<td>Representing the number and the order of digits with numbers up to 14 digits (whole numbers)</td>
</tr>
<tr>
<td>2nd Grade</td>
<td>Thousand positions, integer, order of numbers up to 4-digit numbers</td>
</tr>
<tr>
<td>3rd Grade</td>
<td>Ten Thousand positions, integer, order of numbers and fractions, with simple 2-digit numbers</td>
</tr>
<tr>
<td>4th Grade</td>
<td>Integers and rational numbers</td>
</tr>
<tr>
<td>5th Grade</td>
<td>Situations of Addition &amp; Subtraction with simple 2-digit numbers</td>
</tr>
<tr>
<td>6th Grade</td>
<td>Situations of Addition &amp; Subtraction with simple 2-digit numbers</td>
</tr>
</tbody>
</table>

Table 4. Changes of the Teaching and Learning Contents in "Number and Calculations" (Part 4)
changes were made. For example, the teaching and learning of real numbers and complex numbers was set in an optional mathematics subject at upper secondary school level.

Then in the new Course of Study (1998), the total number of school hours for mathematics is inevitably decreased by the implementation of the five-day schooling system. As a result, calculations are rather lightened; decimal numbers and fractions are introduced one year later than the before (1989); the using of algebraic expressions with alphabetic letters is shifted from elementary to lower secondary school mathematics. In contrast, the meanings of number and four operations, the making use of properties of four operations to construct and learn algorithms of complicate calculations (Table 4).

3. Changes in “Algebraic Expression and Operations”

The table 5 shows the summary of changes of the teaching and learning contents regarding to the algebraic expression and operations. In this table, not only the content of algebraic expressions but also the content of symbolic expressions at elementary level and the content of equation and inequalities at secondary levels are included. In addition, there were no explicit mentions about algebraic expressions in the tentative Course of Studies (1947 and 1948). So the contents in this area in the Course of Studies (1951, 1958, 1968, 1977, 1989, and 1998) are summarized in the table 5.

As you can see in the table, regarding to the algebraic expression and operations, the contents and sequences shown in the Course of Study (1958) has been the prototype of those shown in the following Course of Studies (1968, 1977, and 1989). According to the emphasis of mathematical thinking, even at elementary school level, the following contents were gradually introduced; presenting the quantitative relations in symbolic expressions and interpreting these expressions in some real contexts; presenting and using mathematical formulas; using alphabetic letters and symbols such as $x$ and $\square$. At the lower secondary level, basing on the experiences in elementary school, the rule of writing algebraic expressions, the multiplication and division with algebraic expressions, and the linear equations and quadratic equations were set as the teaching and learning contents. In the Course of Study (1960) for upper secondary mathematics, the fractional and irrational expressions, the higher-order equations, the quadratic inequalities were set as the teaching and learning contents.

In the Course of Study (1968), at elementary school, using symbols such as $\square$ and $\triangle$ to represent numbers and quantities was set in the third grade; and the using alphabetic letters such as $a$ and $x$ instead of words and symbols of $\square$ and $\triangle$ was set in the fifth grade. At the lower secondary level, the addition and subtraction with linear expressions and the properties of equality and linear equations were set in the first grade; the properties of inequality and linear inequalities were set in the second grade; the multiplication and division with linear expressions and the quadratic equations were set in the third grade as main contents. The one prominent characteristic in this period is emphasizing the integrated view on equations and inequalities.

In the Course of Study (1977), the more careful attention was paid to introducing of symbols and alphabetic letters gradually in order to make a connection with algebraic expressions in elementary school as follows; using the symbol of $\square$ in the third grade; using the symbols of $\square$ and $\triangle$ was in the fourth grade; and using the alphabetic letters such as $a$ and $x$ in the fifth grade. On the other hand, at lower secondary level, linear inequalities with two unknowns was eliminated.

In the Course of Study (1989), at elementary level, the activity of representing and interpreting symbolic expressions of four operations with whole numbers was put more emphasized. At lower secondary school, the simultaneous system of linear equation and inequality was eliminated. Moreover, at upper secondary level, the dealing with equations and inequalities was lightened.

Then in the new Course of Study (1998), as the total number of school hours for mathematics is decreased, some remarkable changes are made. At elementary school, using the sign of inequality is eliminated; using the symbols of $\square$ and $\triangle$ is introduced one year later than the before (1989). On the other hand, the activity of representing and interpreting symbolic expressions is put more emphasized. The using of algebraic expressions with alphabetic letters is shifted from elementary to lower secondary school mathematics. Moreover, at lower secondary level, the properties of inequality and linear inequalities are eliminated; the quadratic equations are limited to the simple and the solution formula of quadratic equations is not dealt in lower secondary school. These contents eliminated in the
Table 5. Changes of the Teaching and Learning Contents in "Algebraic Expression and Operations"

<table>
<thead>
<tr>
<th>Primary School</th>
<th>Lower Secondary School</th>
<th>Upper School</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Grade</td>
<td>2nd Grade</td>
<td>3rd Grade</td>
</tr>
<tr>
<td>Writing and reading symbolic expressions by using signs of +, -</td>
<td>Representing and interpreting situations of addition and subtraction with symbolic expressions</td>
<td>Representing and interpreting situations of division with symbolic expressions</td>
</tr>
<tr>
<td>Representing and interpreting quantitative relations (&gt;, &lt;, =)</td>
<td>Representing and interpreting the things and relations (&gt;, &lt;, =)</td>
<td>Representing and interpreting situations of multiplication</td>
</tr>
<tr>
<td>Representing and interpreting situations of addition and subtraction with symbolic expressions</td>
<td>Representing and interpreting situations of division with symbolic expressions</td>
<td>Representing and interpreting situations of division</td>
</tr>
<tr>
<td>Using symbolic expressions</td>
<td>Putting the quantitative relations into formulas and using them</td>
<td>Putting the quantitative relations into formulas and using them</td>
</tr>
<tr>
<td>Representing and interpreting quantitative relations (&gt;, &lt;, =)</td>
<td>Putting the quantitative relations into formulas and using them</td>
<td>Putting the quantitative relations into formulas and using them</td>
</tr>
<tr>
<td>Representing and interpreting situations of division with symbolic expressions</td>
<td>Representing and interpreting situations of division</td>
<td>Representing and interpreting situations of division</td>
</tr>
<tr>
<td>Representing and interpreting situations of division</td>
<td>Representing and interpreting situations of division with symbolic expressions</td>
<td>Representing and interpreting situations of division</td>
</tr>
<tr>
<td>Representing and interpreting situations of addition and subtraction with symbolic expressions</td>
<td>Representing and interpreting situations of division</td>
<td>Representing and interpreting situations of division</td>
</tr>
<tr>
<td>Representing and interpreting situations of division with symbolic expressions</td>
<td>Representing and interpreting situations of division</td>
<td>Representing and interpreting situations of division</td>
</tr>
<tr>
<td>Representing and interpreting situations of division with symbolic expressions</td>
<td>Representing and interpreting situations of division</td>
<td>Representing and interpreting situations of division</td>
</tr>
</tbody>
</table>

4th Grade
- Order of Calculation
- Making symbolic expressions with ( )
- Properties of signs

5th Grade
- Using expressions to solve problems
- Making division by using symbols and numbers

6th Grade
- Applying formulas in case of functions and interpreting the relations expressed more generally
- Using formulas in case of functions and interpreting the relations expressed more generally

1st Grade
- Representing quantities and the relations expressed by using algebraic letters
- Rules for writing algebraic expressions and finding out the value of unknowns

2nd Grade
- Representing quantities and the relations expressed by using algebraic letters
- Rules for writing algebraic expressions and finding out the value of unknowns

3rd Grade
- Linear equations
- Simultaneous system of linear equations
- Multiplication with linear equations

4th Grade
- Dividing and evaluating algebraic expressions

5th Grade
- Using algebraic expressions with ( )

6th Grade
- Using algebraic expressions with ( )

[Analysis 1] Four operations with integral expressions and fractions
- Four operations with integral expressions and fractions
- Inversion expressions
- Interal expressions
- Quadratic equations (discriminant)
- Fractional equations & rational equations
- Higher-order equations
- Simultaneous system of quadratic equations with two unknowns
- Linear inequalities & quadratic inequalities

[Mathematics 1] (1960) Four operations with integral expressions and fractions
- Interal expressions
- Quadratic equations
- Equations & Factor theorem
- Simple absolute inequalities
- Quadratic inequalities

[Mathematics 1] (1970) Four operations with integral expressions and fractions
- Interal expressions
- Quadratic equations
- Simple higher-order equations
- Simultaneous system of equations
- Simple absolute inequalities
- Quadratic inequalities

[Mathematics 2] (1975) Quadratic equations & inequalities
- Simple higher-order equations
- Simultaneous system of equations
- Simple absolute inequalities
- Quadratic inequalities

[Mathematics 3] (1980) Quadratic inequalities
- Simple absolute inequalities
- Quadratic inequalities

- Simple absolute inequalities
- Quadratic inequalities

- Simple absolute inequalities
- Quadratic inequalities

- Simple absolute inequalities
- Quadratic inequalities
lower secondary are shifted to and set in the mathematic subject at upper secondary school level.

4. Conclusion

The purpose of this paper was to summarize the changes of teaching and learning contents in the areas of “Number and Calculations” and “Number and Algebraic Expressions” in the Course of Studies for elementary and secondary school mathematics after 1945 in Japan. To do so, the tables of 1 to 5 were shown.

As a result of overview and some considerations, the followings are found as three main characteristics. (a) There is a remarkable difference between the period of “Life-Unite Learning” (1945 – 1957) and the period of “Systematic Learning” (1958 – 1967) regarding to the teaching and learning contents and their sequences in these areas shown in the Course of Studies. (b) The contents and sequences shown in the Course of Study (1958) has been the prototype of those shown in the following Course of Studies (1968, 1977, and 1989). In the new one (1998), however, we notice that calculations are rather lightened, decimal numbers and fractions are introduced one year later than the before (1989); the using of algebraic expressions with alphabetic letters is shifted from elementary to lower secondary school mathematics. (c) The dealing with equations and inequalities in secondary schools is gradually lightened in recent years.

Generally speaking, education is the dynamic system with purposes and designs to meet and supply the needs of society and students through long generations, not excepting mathematics education. In this sense, it is important for us as mathematics researchers/educators to rethink the teaching and learning contents and their sequences in the areas of “Number and Calculations” and “Number and Algebraic Expressions” in one whole vision including their aims, teaching methods, and the ways of evaluation in light of historical lessons.

References

(Books and articles marked by * are written in Japanese.)


Ministry of Education: (*)

The Tentative Course of Study (1947).
The Tentative Course of Study (1948).
The Tentative Course of Study for Elementary (1951).
The Tentative Course of Study for Secondary (1951).
The Course of Study for Elementary (1958).
The Course of Study for Elementary (1968).


---

*References (Books and articles marked by * are written in Japanese.)*


Ministry of Education: (*)

The Tentative Course of Study (1947).
The Tentative Course of Study (1948).
The Tentative Course of Study for Elementary (1951).
The Tentative Course of Study for Secondary (1951).
The Course of Study for Elementary (1958).
The Course of Study for Elementary (1968).


---