## 1DOF Examples of Pose Representation

### Approx. by Eigenimages

\[ x = \sum c_i e_i \]

### Approx. by training images

\[ x = \sum b_i x_i \]

<table>
<thead>
<tr>
<th>Representation</th>
<th>Training</th>
</tr>
</thead>
<tbody>
<tr>
<td>angles</td>
<td>10 = ( F x_{10} ) ...</td>
</tr>
<tr>
<td>sin,cos</td>
<td>( \sin(10^\circ) \cos(10^\circ) = F x_{10} ) ...</td>
</tr>
</tbody>
</table>

### 3DOF Pose Representations: Comparison

<table>
<thead>
<tr>
<th>Fixed angles (roll, pitch, yaw)</th>
<th>Euler angles</th>
<th>Angle-axis</th>
<th>Unit quaternions</th>
<th>Rotation Matrix</th>
<th>Spherical representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuity (one-to-one)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Bijection (one-to-one)</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Higher Freq.</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Proposed method

\[ x = \sum c_{l1l2l3} Y_{l1l2l3}(\theta_1, \theta_2, \theta_3) \]

### A Spherical Function Representation of SO(3)

- Polar coordinates of a point on \( S^2 \)
- Associated Gegenbauer function
- Associated Legendre function
- Complex numbers on the unit circle

\[ Y_{l1l2l3}(\theta_1, \theta_2, \theta_3) = \sum_{l1} \sum_{l2} \sum_{l3} \cos(\theta_1) C_{l1}^{l3}(\cos \theta_1) P_{l2}^{l3}(\cos \theta_2) e^{-il3\theta_3} \]

### Spherical Functions on SO(3)

- Even functions of spherical functions on \( S^2 \)
- Spherical functions on \( S^2 \)
- Spherical functions on \( S^3 \)
- \( S^3 \) is a double-covering of \( SO(3) \)

### Applications

- Analysis of a function \( f(R) \) of a rotation matrix
- \( f(R) = |x(Rx+t)|^2 \)

### Illustrations

- Rotation matrix and Spherical representation
- Are suit for pose representation
- Can be used to expanding an image as a function on \( SO(3) \)
- Polar coordinates:
- Associated Gegenbauer function
- Associated Legendre function
- Complex numbers on the unit circle

- Proposed method

- Spherical functions on \( SO(3) \)
- \( S^3 \) in \( R^4 \)
- \( SO(3) \) is a double-covering of \( S^3 \)

- Even functions of spherical functions on \( S^2 \)

- A sphere \( S^1 \) in \( R^2 \)