Comparison of Klinkenberg-corrected gas permeability and water permeability in sedimentary rocks

Authors

Wataru Tanikawa¹*
Toshihiko Shimamoto²

Addresses

1. Kochi Institute for Core Sample Research, Japan Agency for Marine-Earth Science and Technology (JAMSTEC), 200 Monobe-otsu, Kochi, Japan.
2. Department of Earth and Planetary Systems Science, Graduate School of Science, Hiroshima University, Higashi-Hiroshima, Japan

*Corresponding author

Address: Kochi Institute for Core Sample Research, Japan Agency for Marine-Earth Science and Technology, Nankoku 783-8502, Japan
Tel: +81-88-878-2203; fax: +81-88-878-2192
E-mail: tanikawa@jamstec.go.jp
Abstract

Quick measurements of permeability can be made by using gas as the pore fluid. To apply gas permeability data to the evaluation of water permeability, the difference between gas and water permeabilities needs to be assessed. We measured intrinsic permeability of sedimentary rocks from the western foothills of Taiwan by using nitrogen gas and distilled water as pore fluids in effective-pressure cycling tests at room temperature. The observed difference in gas and water permeabilities was analyzed in view of the Klinkenberg effect. This effect is due to the slip flow of gases at pore walls, which enhances gas flow when pore sizes are very small. Our experimental results showed that (1) gas permeability was larger than water permeability by several times to one order of magnitude, (2) gas permeability increased with increasing pore pressure, and (3) water permeability increased slightly as the pore-pressure gradient across the specimen increased. Results (1) and (2) can be explained quantitatively by an empirical power law in relation to the Klinkenberg constant $b$ that is applicable in low permeable range. This correlation enables us to estimate water permeability from gas permeability. The Klinkenberg effect is important when permeability is lower than $10^{-18}$ m$^2$ and at low pore-pressure differentials, and correction for the effect is essential to estimate water permeability from gas permeability measurement data. A simplified Bingham flow
model for water can partially explain the trend of result (3), though non-Darcy flow behavior or inertial forces of water-rock interaction are needed to account for the observed deviation from Darcy’s law.

Keyword

Gas permeability; Water permeability; Klinkenberg effect; Sedimentary rock
1. Introduction

Permeability (or intrinsic permeability) is an important controlling parameter of fluid flow systems at depth. Permeability ranges widely from more than $10^{12}$ to less than $10^{-23} \text{ m}^2$ depending on the rock type and depth [1, 2]. Permeability measurements under high confining pressure in a laboratory are one possible way to predict permeability structure at depth. One of the simplest methods to measure permeability is the steady-state method. Moreover, the use of gas as a pore fluid instead of water for the permeability measurement has the following advantages: (1) commercial gas flow-meters that cover a wide range of flow rates for all gases are available, allowing the measurement of a wide range of permeabilities quickly and accurately; (2) nitrogen gas is chemically inert, allowing geochemical effects to be ignored, so only permeability variation caused by change in confining pressure need be considered; and (3) the compressibility and viscosity of gas are less sensitive to temperature changes than those of water, so permeability measurement error due to these factors is less likely to be introduced.

Ideally, permeability does not depend on the type of pore fluid; therefore, permeability measured by using gas should be the same as that measured by using water. Several fluid-flow problems have been analyzed using permeabilities measured by
using gas instead of water [3-5]. However, it has been found that (1) permeability of a medium to gas changes in relation to pore-pressure changes at the same effective pressure [6], and (2) permeability of a medium to gas is higher than the permeability of the same medium to water [7]. These problems might be related to the pore-pressure dependence of gas permeability known as the Klinkenberg effect [8], which we define and discuss in detail in Section 2. There are few examples, however, in which permeabilities to different pore fluids are compared in the same samples [7,9-10] under high confining pressure. Therefore, in this study, permeability of the same specimens measured by using nitrogen gas as the pore fluid was compared with that measured by using water as the fluid in effective-pressure cycling tests. Then, the Klinkenberg effect on gas permeability and the pore-pressure dependence of permeability are discussed in relation to the experimental data. Furthermore, the relationship between the Klinkenberg slip factor and water permeability in low permeability range is discussed as the relationship in low permeable rocks is not well studied. Sedimentary rocks from the western foothills of Taiwan that have permeability variations were used for the permeability measurements.

2. Intrinsic permeability and the Klinkenberg effect
Intrinsic permeability is a measure of the mobility of fluid within a porous material that is related solely to the pore geometry of the rock (porosity, pore shape, pore-size distribution, etc.), and it is independent of the properties of the fluid. Therefore, intrinsic permeability measured by using any gas as the pore fluid in a laboratory test should be the same as that measured by using any other kind of fluid. Hydraulic conductivity is a function of both the medium and the properties of the fluid, and its relationship to intrinsic permeability is given by

\[ K = \frac{k}{\eta \rho g}, \]  

(1)

where \( K \) is hydraulic conductivity (m/s), \( k \) is intrinsic permeability (m\(^2\)), \( \eta \) is the viscosity of the fluid (Pa·s), \( \rho \) is the fluid density (kg/m\(^3\)), and \( g \) is gravitational acceleration (m/s\(^2\)). Equation (1) indicates that fluid mobility depends on fluid viscosity, density and gravity. Klinkenberg [8] discovered that the permeability of a medium to gas is relatively higher than that as water, and he interpreted this phenomenon to “slip flow” between gas molecules and solid walls. In Darcy flow, molecular collisions dominate and collisions between the gas molecules and the pore wall, which are described as adherence at the fluid–solid boundary, are neglected. As the pore radius approaches the mean free path of the gas molecules, the frequency of collisions between gas molecules and the solid walls increases. Therefore, this additional flux due to gas flow at the wall surface, which is called “slip flow,” begins to effectively enhance the
flow rate. This phenomenon is called the *Klinkenberg effect* and is expressed as follows:

\[
k_g = k_l \left( 1 + \frac{4cl}{r} \right) = k_l \left( 1 + \frac{c\kappa}{\pi \sqrt{2r} P} \right) = k_l \left( 1 + \frac{b}{P} \right),
\]

where \( k_g \) is the permeability of a medium to gas (m\(^2\)), \( k_l \) is the permeability of a medium to liquid (m\(^2\)), \( l \) is the mean free path of the gas molecules (m), \( r \) is the pore radius (m), \( \kappa \) is Boltzmann’s constant (JK\(^{-1}\)), \( T \) is temperature (K), \( c \) is a constant, \( P \) is pore pressure (Pa), and \( b \) is the Klinkenberg slip factor (Pa). The value of the parameter \( l \) in equation (2) is given by the following relationship [11]:

\[
l = \frac{\kappa T}{4\pi \sqrt{2r} P^3}
\]

Equation (2) indicates that if the pore radius and the gas pore pressure are small and the temperature of the gas is high, then \( k_g \) becomes much larger than \( k_l \); in contrast, \( k_g \) approaches \( k_l \) as pore pressure approaches infinity. For this reason, Brace et al. [9] and Zoback and Byerlee [12] performed gas permeability tests under high pore pressure conditions (\( P = 5 \) to 15 MPa) by the transient flow method to avoid the Klinkenberg effect. Brace et al. (1968) showed that the permeability of Westerly granite to water, ranging from \( 10^{-16} \) to \( 10^{-18} \), was similar to that as argon gas; however, it was not mentioned that how high a pore pressure was required to avoid the Klinkenberg effect.
in gas permeability tests.

The Klinkenberg factor \( b \) depends on the rock properties, and to a lesser degree on the physical properties of the gas. Previous researches showed that \( b \) took a wide range of values from \( 10^{-2} \) [8] to 18 [6]. Jones [13] investigated the relationship between the Klinkenberg slip factor \( b \) and estimated water permeability \( k_l \) in about 100 core samples, and found that \( \log b \) increased linearly with \( \log k_l \) within a permeability range from \( 10^{-12} \) to \( 10^{-17} \) m\(^2\). Persoff and Hulen [14] estimated the relationship between \( b \) and \( k_l \) in less permeable (\( 10^{-18} \) to \( 10^{-22} \) m\(^2\)) metagraywacke samples; their results were generally consistent with those of Jones, except that the slope of the regression line was apparently steeper than that obtained by Jones [13], although they used too few data points (8 points) to be sure.

### 3. Sample information and method

Outcropping sandstones from the west-central part of the western foothills of Taiwan (Pleistocene to Miocene rocks) were used for our permeability tests. The western foothills are known to be in a fold and thrust belt that developed during the Miocene as a result of the collision of the Luzon arc with the Asian continent [15]. The western foothills are well known for oil and natural gas fields [16], and permeability at depth in
the western foothills is a key parameter for understanding oil and natural gas storage and migration mechanisms.

All samples were cored to a cylindrical shape. All specimens were 5 to 40 mm long and approximately 20 mm in diameter. 4 samples were used for the comparison between gas and water permeabilities, and gas permeability were measured in 30 samples to show the relationship between $b$ and permeability. Specimens were dried at 80 °C in an oven to eliminate pore water before the gas permeability measurement. All experiments were performed at room temperature using the intravessel oil pressure apparatus at Kyoto University. In this experimental system, the oil apparatus can attain a confining pressure of up to 400 MPa in conjunction with a mini-pressure generator system. To confine the pressure, avoid leakage, and eliminate bypass flow between the side of the sample and the jacket during permeability tests, samples were jacketed in three to four layers of polyolefin, which contracts during heating. Fluid flows from the bottom to the top of a specimen in this apparatus.

When an incompressible fluid (liquid) is used for permeability tests, intrinsic permeability $k$ is determined from the linear form of Darcy’s law:

$$Q = \frac{kA}{\eta} \frac{dP}{ds},$$

(4)

where $Q$ is the volume of fluid measured per unit time, $A$ is the cross-sectional area of
the sample, $\eta$ is viscosity of the pore fluid, and $dP/dx$ is the pressure gradient.

One-dimensional gas flow is assumed in equation (4) as the enforced pressure gradient during permeability tests is very large. If a constant pressure gradient through the sample is assumed, then equation (4) can be expressed as

$$Q = \frac{kA}{\eta L} (P_{up} - P_{down})$$

(5)

$L$ is sample length and $P_{up}$ and $P_{down}$ are pore pressure at the upstream end ($x = 0$) and downstream end ($x = L$) of the specimen, respectively. However, when a compressible gas ($PV =$ constant) is used as the pore fluid, the density of the pore fluid differs within the specimen. In this case, the average gas permeability $k_{gas}$ is expressed as [6]

$$Q(x) = \frac{A k_{gas}}{\eta L} \frac{1}{\sqrt{P_{up}^2 - \frac{P_{up}^2 - P_{down}^2}{L}}}$$

(6)

When $x = L$, equation (6) becomes

$$Q(x = L) = \frac{A k_{gas}}{2\eta L} \left( \frac{P_{up}^2 - P_{down}^2}{P_{down}} \right)$$

(7) [17].

In our apparatus, $P_{up}$ was kept constant at a value between 0.2 and 2 MPa using the
gas regulator, and the gas flow rate was monitored at the downstream of the samples (Figure 1). Fluid flowing out of the specimen at the downstream end was released to atmospheric pressure, and $P_{down}$ was assumed to have a constant value of 0.1 MPa. Thus, the pore-pressure differential, $P_{up} - P_{down}$, can be assumed to be constant, and equation (7) can be applied to evaluate gas permeability. The volumetric gas flow rate at the downstream end of the samples was measured with a digital gas flow-meter (ADM2000; Agilent Technologies, USA) and a high-precision bubble gas flow meter (VP-1U; HoribaSTEC Ltd, Japan). Continuous, real-time digital measurements are made by the ADM2000, enabling fast measurement of the gas permeability. Several seconds to more than 10 minutes can be required to measure a volumetric flow rate with the VP-1U, though a very low flow rate can be achieved. The ADM2000 can measure a volumetric flow rate within the range 0.0005–1.0 L/min (accuracy, ±3% of reading, or ±0.2 mL/min, whichever is greater), and the VP-1U can measure a flow rate within the range 0.024–10 mL/min (accuracy, ±0.5% for 0.2–2 mL/min; ±1% for 2–10 mL/min). Therefore, a wide range of gas flow rates from 1 L/min to 0.024 mL/min, corresponding to gas permeabilities from $10^{-13}$ m$^2$ down to $10^{-19}$ m$^2$ in our experiments, was easily measured by using these gas flow-meters.

Water permeability was evaluated by the steady-state flow method, by using equation (5). Upstream pore pressure was controlled in the same way as before.
Distilled water was stored in the water tank and directly pressurized by nitrogen gas from the gas bottle. Pore water at the downstream end was released to atmospheric pressure. The water flow rate at the downstream end was determined by using a digital balance (GF400; A&D Company, Ltd., JAPAN) to continuously monitor the real-time weight increment of water flowing out from the specimen. Therefore, there was no lower limit for the water permeability measurement if time permitted. We observed no gas bubbles in the water flowing out from the downstream line during the water permeability measurements, indicating that two-phase (water and nitrogen gas) flow did not occur, even though we forced the water from the water tank with nitrogen gas under high pressure. The viscosities of the nitrogen gas and distilled water at room temperature of $17.4 \times 10^{-6} \text{ Pa}\cdot\text{s}$ and $1.0 \times 10^{-3} \text{ Pa}\cdot\text{s}$ were used for the permeability evaluation.

Permeability was measured at confining pressures that stepped up from 5 MPa to the maximum of 160 MPa. To compare permeability of a medium to nitrogen gas with that of same medium to water, nitrogen gas was used as the pore fluid for the first and second, or first, second, and third, pressure cycles, and then distilled water was used as the pore fluid for the last two pressure cycles. Before the pore fluid was switched from nitrogen gas to water, the pore flow line was vacuumed once for an hour, and then filled with CO$_2$. After we vacuumed for an hour again, distilled water was poured into the
specimens. By filling the pore flow lines once with CO$_2$, we could minimize the influence of gas remaining within the pores during the water permeability measurement, because the solubility of CO$_2$ in water is quite large under high pressure so that any CO$_2$ gas would dissolve in the water during tests. The pore-pressure dependence of permeability was investigated by changing $P_{up}$ while keeping the confining pressure constant.

4. Experimental result

General information on the test samples used to compare the difference between water and gas permeability is given in Table 1. Porosity was measured by the standard method, where porosity is calculated from the weight difference between the water-saturated sample and the dry sample, and by the mercury intrusion porosimetry technique (MIPT) with a commercial porosimeter (Porosimeter 2000, Carlo Erba Ltd.). The MIPT yielded lower porosity than that determined by the standard method. Pore-size distributions were also measured by MIPT (Figure 1). The radius of most micropores was in the range from 0.01 to 10 $\mu$m. The peak volumetric ratio was near the maximum end of the pore-size range in each case. Sample IVA407 (Tungkeng Fm), which had the highest porosity among the specimens, also had the largest average and modal pore radii.
The results of the pressure cycling tests on the sandstone specimens are shown in Figure 2. Permeability was plotted against the effective pressure, defined as the difference between the confining pressure and the pore pressure. In our tests, however, the effective pressure was assumed to be equivalent to the confining pressure because the pore pressure was extremely small compared with the confining pressure. The error bars in the figure mainly reflect differences caused by pore-pressure variation, because permeability varies significantly with changes in pore pressure (this pore-pressure dependence of permeability is described in the following section). The accuracy of the flow rate was less than 1%, and the error of each data point is within the symbol.

In sample IVA407, five effective-pressure cycles were conducted. During the first three pressure cycles, nitrogen gas was used as the pore fluid, and distilled water was used during the other two pressure cycles. The initial permeability at the lowest effective pressure was around $10^{-13} \text{ m}^2$, and permeability decreased as the effective pressure increased, although permeability was reduced by less than one order of magnitude from the initial value even at 100 MPa of effective pressure. The permeability during the first cycle was relatively larger than that during the second or third pressure cycles, and a similar cyclic path was described during the second and third cycles. Soon after the pore fluid was changed from nitrogen gas to water, the permeability decreased sharply; the permeability of the medium to nitrogen gas was 2 to
4 times that of the same medium to water at the same effective and differential pressures.

Permeability during the fifth cycle was relatively smaller than that during the fourth cycle. The influence of the pore fluid on the permeability change was larger than that caused by the change in effective pressure or the increase in the pressure cycle number (shown by error bars) for the same fluid. The other specimens (IVA418, IVA419, and IVA478) showed similar permeability behavior as the pressure cycle tests with IVA407, although the difference between the gas and water permeability differed among specimens. In the case of IVA419, with smaller permeability than IVA407, permeability to gas was 5 to 7 times that to water. In IVA418 and IVA478, which had the lowest permeabilities, permeability of a medium to gas was more than 10 times that of the same medium to water. These results imply that the permeability difference between nitrogen gas and water is larger in specimens with low intrinsic permeability. The permeability was also correlated with porosity and pore-size distribution; in our test samples, permeability of porous sandstones with larger pore size (IVA407, IVA419) was higher than that of the less porous sandstones (IVA418, IVA478). In IVA418, a sudden irregular drop of permeability was recognized at 40 MPa during the third cycle, but this drop was not observed during the other cycles or in other specimens. Wetting may enhance deformation by mechanical collapse, because it is generally true that rocks are weaker when wet than when dry. Therefore, an unexpected mechanical collapse might
have occurred at the time of wetting, but we do not have direct evidence of this from microstructural observations.

Figure 3 illustrates examples of the relationship between the permeability to nitrogen gas and the pore-pressure differential. Generally, permeability decreases as the pore-pressure differential increases at all effective pressures. In IVA419, the maximum permeability differed by a factor of 1.6 for pore-pressure differentials between 0.2 and 0.8 MPa.

The relationship between the permeability to water and the pore-pressure differential is shown in Figure 4. In IVA419, permeability increased as the pore-pressure differential increased, a trend opposite to that for permeability to gas. The maximum permeability differed by a factor of 1.5 in relation to pore-pressure differential changes.

In IVA478, however, a dependence of permeability on the pore-pressure differential was not clear.

5. Discussion

5-1. Klinkenberg effect

We plotted the permeability to nitrogen gas against the inverse of the average pore
pressure, $1/P_{av}$, \( P_{av} = (P_{up} + P_{down})/2 \) to verify the Klinkenberg effect (Figure 5). In most cases, the permeability to gas increased linearly as $1/P_{av}$ increased, a result that is consistent with the Klinkenberg equation, if it is assumed that average pore pressure is equivalent to the pore pressure $P$ in equation (2). Therefore, the permeability to water $k_l$ can be estimated from the slopes of the lines in Figure 6. Water permeability estimated by using the Klinkenberg equation, equation (2), showed almost the same value as the observed permeability to water in IVA478, but in IVA407, IVA418, and IVA419, the estimated water permeability was smaller than the observed permeability, though except in IVA407 the difference was much smaller than the difference between water and gas permeabilities (Figure 6). These results suggest that the difference between gas and water permeabilities is strongly affected by the Klinkenberg effect. In IVA407, the estimated water permeability did not show a linear relationship with average pore pressure.

The pressure dependence of nitrogen gas viscosity can also cause a pore-pressure dependence of permeability to gas. Nitrogen gas viscosity exhibits a positive dependence on pressure, which can increase the permeability to gas, though this sensitivity is much smaller than the temperature dependence. However, in our experiment, the permeability change due to differences in pore pressure is so large that the small pressure dependence of nitrogen gas viscosity (17.87 $\mu$Pa·s for 0.1 MPa and
17.94 for 2 MPa at 300 K; [18]) cannot explain the observed pore pressure dependence of permeability to gas.

5-2. Klinkenberg slip factor

Not only the water permeability but also the Klinkenberg slip factor $b$ can be approximated from the fitted slopes (Figure 5). When parameter $b$ was plotted against water permeability, both estimated from equation (2), for 30 sedimentary rocks from the western foothills (330 points) (Figure 7), we found that $b$ decreased with increasing permeability to water, although the data show considerable scatter. The relationship between water permeability and parameter $b$ is described by the formula

$$b = (0.15 \pm 0.06) \times k_l^{(-0.37 \pm 0.038)},$$  \hspace{1cm} (8)

when the units of $k_l$ and $b$ are selected as m$^2$ and Pa, respectively. This correlation was previously studied by Heid et al. [19], Jones [13], and Jones and Owens [20] for different permeability ranges. Heid et al. [19] showed the correlation in air at 25 °C as

$$b = 0.11k_l^{-0.39}$$  \hspace{1cm} (9)

for oil-field cores with permeability values of about $10^{-12}$ and $10^{-17}$ m$^2$. Jones and Owens [20] measured permeabilities of between $10^{-14}$ and $10^{-19}$ m$^2$ for low-permeable
sands, and the correlation with $b$ was expressed as

$$b = k_i^{-0.33}. \quad (10)$$

Both correlations are quite similar to our results, though in our tests the $b$ values were slightly smaller for all permeability ranges. Moreover, for permeabilities less than $10^{-19}$ m$^2$, the slope of the relationship seems to become steeper, a trend also shown by Persoff and Hulen [14].

When the correlation described by equation (8) is assumed to be applicable to any rocks, and when equation (8) substitutes into equation (2), the relationship between permeability to gas and that to water is

$$k_g = k_i \left(1 + \frac{0.15k_i^{-0.37}}{P_{av}} \right). \quad (11)$$

Therefore, the relationship between the ratio of permeability to gas and permeability to water, $k_g/k_i$, and the pore-pressure differential, $P_{up} - P_{down}$, in our experimental settings that $P_{down} = 0.1$ MPa (Figure 8) is such that when permeability of a medium to water is higher than $10^{-16}$ m$^2$, $k_g/k_i$ is less than 2 for all values of the pore-pressure differential. However, when permeability of a medium to water is less than $10^{-17}$ m$^2$, the discrepancy between $k_g/k_i$ values for different values of the pore-pressure differential becomes
larger: in the extreme case, $k_g/k_l$ is 30 when $k_l$ is $10^{20}$ m$^2$ and the pore-pressure differential is 0.05 MPa. Therefore, we cannot neglect the difference between gas and water permeability when both permeability and the pore-pressure differential applied for the permeability measurements are smaller than $10^{-17}$ m$^2$ and 0.2 MPa respectively.

5-3. Pore-pressure dependence of water permeability

Several specimens showed a dependence of permeability of a medium to water on the pore-pressure differential (Figure 4). Sasaki et al. [21] suggested that this positive dependence is caused by Bingham plastic flow within small pores. Byerlee [22] also asserted that fluid that flows within a relatively impermeable fault zone may exhibit Bingham flow, which may increase the fluid pressure in the fault zone, thus reducing fault strength. Generally, water exhibits Newtonian flow, in which the permeability does not depend on the pore-pressure differential. However, when pore spaces are small, adhesion to pore walls causes resistance to water flow, leading to a smaller flow rate and smaller permeability. The Buckingham-Reiner equation describing Bingham flow in a one-capillary-tube model is [23]

$$Q = \frac{\pi \Delta P R^4}{8 L \eta_p} \left[ 1 - \frac{4}{3} \left( \frac{2 L \tau_o}{R \Delta P} \right) + \frac{1}{3} \left( \frac{2 L \tau_o}{R \Delta P} \right)^4 \right]$$
where $Q$ is the volumetric flow discharge, $\Delta P$ is the pressure difference, $R$ is the radius of the capillary tube, $\eta_p$ is the slope of the relationship between shear stress and the rate of shear or dynamic viscosity when the shear stress $\tau > \tau_0$, $L$ is the length of the tube, and $\tau_0$ is the critical yield strength. Under Newtonian flow, $\tau_0$ becomes 0 and equation (12) becomes the Hagen-Poiseuille law:

$$Q = \frac{\pi \Delta P R^4}{8 L \eta_p}. \quad (13)$$

If we apply the Buckingham-Reiner equation to porous media, assuming that a capillary tube is analogous to a pore with radius $r = R$, permeability to a Bingham plastic fluid can be expressed by combining Darcy’s law (equation 4) with equation (12):

$$k_I = \frac{Q \eta_w L}{A \Delta P} = \frac{nr^2}{8} \left[1 - \frac{4}{3} \left(\frac{2L \tau_0}{r \Delta P}\right) + \frac{1}{3} \left(\frac{2L \tau_0}{r \Delta P}\right)^4\right], \quad (14)$$

where $n$ is porosity, $\eta_w$ is the viscosity of water, and $\eta_p = \eta_w$ is assumed, and the relationship between porosity of the medium, $n$, and pore radius ($nr^2 = nA$) that contains, on average, one capillary for a cross-sectional area $A$ is applied. This correlation can be employed if it is assumed that pore size can be represented by a single pore size, implying that water permeability reaches the stable value $nR^2/8$ at
larger pressure differentials. Two examples of the dependence of permeability on the pore-pressure differential are shown in Figure 10. The experimental result for IVA389 (the same specimen as IVA418 but a different experimental run) of 30 MPa of effective pressure was compared to the result calculated using equation (14), which assumes that all pores are the same size. Parameters $\tau_0$ and $R$ were evaluated by the least-squares method as 0.25 MPa and 0.13 $\mu$m, respectively. The prediction curve reaches a permeability of $2.9 \times 10^{-16}$ m$^2$ at a higher pressure differential. This curve is partly consistent with our experimental data, showing that permeability increases as the pore-pressure differential increases and that the pressure sensitivity decreases gradually as the pressure differential increases, but it does not reproduce them perfectly. The result for IVA478 shows similar behavior.

As shown by the pore-size distribution of IVA418 (Figure 1), the pore radius of the test specimens ranged widely from 0.01 to 10 $\mu$m, and the modal pore size was several times larger than the pore size approximated by using equation (14). The discrepancy between the two might be caused by the application of a simplified equation that assumes that the pore size of a sample can be characterized by a single representative pore size. When the pore-size distribution is taken into account, which is the more realistic case, equation (14) is rewritten as
where \( f(r_i) \) is the volumetric ratio of a pore of radius \( r_i \). It is also realistic that \( \tau_0 \) is related to pore geometry and can change with pore radius. Equation (15) assumes that the radius of each pore does not change in the flow direction through the specimen. The other simplification is that the pore structure is partitioned into several parts perpendicular to the flow direction. In this case, the equivalent permeability is given by the harmonic mean of the individual permeabilities,

\[
k_i = \frac{1}{\sum \frac{f(r_i)}{k_i}}.
\]

Using the pore-size distribution measured by the MIPT, permeability can be evaluated by these capillary tube models. The gas and water permeabilities determined by laboratory experiment were compared to the results of the three capillary models described by equations (14), (15), and (16) (Figure 10). Permeabilities calculated assuming an effective pressure of 10 MPa were plotted against the experimental values. The average pore radius (Table 1) was used in equation (14), and the pore-size distributions shown in Figure 2 were used in equations (15) and (16). To avoid the unknown parameters, \( \tau_0 = 0 \) was selected. The data suggest that permeability is influenced by the average pore radius, though the estimated values are large for all
specimens. The experimental values are between the harmonic and arithmetic means of the capillary model. The simplest model, equation (14), is the most suitable for estimating gas permeability, and the harmonic mean model yields more reasonable results in low permeability specimens. Therefore, it can be said that permeability of relatively porous rocks is controlled by the larger pore sizes (arithmetic mean model domination), whereas the permeability of impermeable samples is more influenced by the smaller pores (harmonic mean model domination).

It is not clear whether the chosen value of $\tau_0$ is realistic in nature. Furthermore, several data points do not show clearly the dependence of the water permeability on the pore-pressure differential. Even though the positive pore-pressure dependence in our water permeability test can be partially explained by the Buckingham-Reiner equation, water interactions with clays or inertial effects [24, 25] and the non-Darcy flow that is described as the Forchheimer equation [26] must have considered explaining our data thoroughly.

Our laboratory experiments were focused on porous sedimentary rocks, though it is unknown whether other rocks (incohesive rocks, granite, fractured rocks, etc.) would exhibit the same permeability characteristics. Faulkner and Rutter [7] compared the permeability of clay-rich fault rocks to argon and water. Their data suggest that permeability of a medium to argon gas was 10 times that of the same medium to water,
which in general is consistent with our results. However, they concluded that the
difference was caused by a reduction of the effective pore diameter due to the adhesion
of water molecules to crystal surfaces rather than to the Klinkenberg effect or Bingham
plastic flow behavior. It is possible that several mechanisms might concurrently affect
water and gas flow and permeability to each in a specimen. Therefore, understanding
the dominant mechanism accounting for permeability differences under various
conditions in relation to specimen characteristics such as pore shape and mineral
composition is important for our future study.

6. Conclusions

The permeability of sandstone specimens from the Taiwan western foothills to nitrogen
gas and water was measured on the same specimens in effective pressure cycling tests,
yielding the following significant results. (1) The permeability of a given specimen to
nitrogen gas was 2 to 10 times that to water. (2) The permeability to nitrogen gas
decreased as pore pressure increased, and this pore-pressure dependence on the
permeability of a medium to gas could be explained by the Klinkenberg equation for
most experimental data. The water permeability estimated by using the Klinkenberg
equation was consistent with the observed water permeability. (3) The relationship
between the Klinkenberg slip factor \( b \) and water permeability was consistent with that determined empirically by a previous study, namely, that \( \log b \) decreases linearly as \( \log \) water permeability increases over a wide range from \( 10^{-14} \) to \( 10^{-20} \) m\(^2\). (4) The permeability to water was positively related to the pore-pressure differential and pore-size distribution, which can be partially explained by the Buckingham-Reiner equation, though the calculated curve does not fully reproduce the experimental data because of the complex pore geometry of the specimens, inertial effects nor non-Darcy flow behavior that are.

Our experimental results suggest that the difference between the permeability of a medium to gas and that of the same medium to water can be partly explained by the Klinkenberg effect. Therefore, gas permeability values should not be corrected by using the Klinkenberg equation when the Klinkenberg effect is not relevant. This phenomena is only significant for low permeability media. The pore-pressure dependence of both water and gas permeabilities should also be taken into account in fluid flow problems.

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References


[17] Scheidegger AE. The physics of flow through porous media, 3rd Edition. Toronto:

Heid JG, McMahon JJ, Nielson RF, Yuster ST. Study of the permeability of rocks to homogeneous fluids. API Drilling Prod Pract 1950;230-244.


[26] Wu YS. Numerical simulation of single-phase and multiphase non-Darcy flow in
<table>
<thead>
<tr>
<th>Specimen symbol</th>
<th>Formation</th>
<th>Maximum burial depth (m)</th>
<th>Age (Ma)</th>
<th>Sample length (mm)</th>
<th>Sample diameter (mm)</th>
<th>Porosity (%)</th>
<th>Average pore radius (μm)</th>
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<td>Tungkeng Fm</td>
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<td>Kuanyinshan Ss</td>
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Table 1 Information on specimens

Figure 1. Pore-size distributions of the test samples.
Figure 2. Relationship between permeability and effective pressure from the pressure cycling tests. Nitrogen gas was used as the pore fluid in the first two or three pressure cycles, and then distilled water was used in the last two pressure cycles. Error bars mainly reflect the pore-pressure dependence of permeability.
Figure 3. The dependence of permeability of a medium to nitrogen gas on the pore-pressure differential.

Figure 4. The dependence of permeability of a medium to water on the pore-pressure differential.
Figure 5. Relationship between permeability of a medium to gas and the inverse of the average pore pressure, verifying the existence of the Klinkenberg effect described by equation (2).

Figure 6. Comparison of water permeability estimated by using the Klinkenberg equation and experimental data.
Figure 7. The Klinkenberg slip factor $b$ and estimated water permeability power law relationship, obtained from the results of gas permeability tests of 30 sedimentary rocks from the Taiwan western foothills. The experimental data obtained by this study are compared with data from previous studies.
Figure 8. The relationship between the ratio of permeability of a medium to gas to that to water and the pore-pressure differential, evaluated by using the empirical relationship described by equation (8).

Figure 9. Dependence of water permeability on the pore-pressure differential. The
trends obtained in two specimens are compared to those predicted by the Buckingham-Reiner model (equation 14), which is based on Bingham plastic flow into an assembly of capillary tubes of the same pore radius.

Figure 10. Comparison of observed permeabilities and the theoretical models of equations (14), (15), and (16).