Properties of non-neutral electron plasmas confined with a magnetic mirror field

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A low energy non-neutral electron plasma was confined with a magnetic mirror field and an electrostatic potential to investigate the basic confinement properties of a simple magnetic mirror trap. The mirror ratio of the magnetic field was increased up to 5. As expected the confinement time became longer as a function of the mirror ratio. The axially integrated radial density profiles in equilibrium were measured and compared with a theoretical model. The axial electrostatic oscillations of a confined electron plasma were also observed.

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I. INTRODUCTION

A magnetic mirror configuration can be observed in many magnetized plasmas, including large devices for studying the thermonuclear fusion [1], electron cyclotron resonance ion sources, the Earth’s magnet sphere, etc. Also, a special kind of magnetic mirror field has been studied for the production and confinement of antihydrogen atoms [2,3]. In many practical applications, mirror ratios of magnetic fields are less than 5.

So far, low energy non-neutral plasmas have been confined with a uniform magnetic field, a toroidal magnetic field, and a linear Paul trap. Various properties of them have been investigated both theoretically and experimentally [4]. Although there were some theoretical works on the magnetic mirror confinement of non-neutral plasmas [5,6], most of the previous experimental works were dedicated to high energy plasma confinement [7], where the application for thermonuclear fusion reactors or particle accelerators were studied, and few experiments were conducted for the confinement of a low energy non-neutral plasma with a magnetic mirror field. However, the confinement of charged particles and/or plasmas with a magnetic mirror (including a magnetic cusp) is getting more attention recently due to its potential application for the production and confinement of antihydrogen atoms and also of electron-positron plasmas [8,9]. Thus it might be helpful and important to understand the basic properties of non-neutral plasmas in a magnetic mirror field with various plasma parameters.

A unique feature of the present experiment is that a low energy non-neutral electron plasma is confined by an axis symmetric magnetic mirror field on one side and by an electrostatic field on the other side. Since the radial particle diffusion and the axial particle loss caused around the electrostatic potential are negligible compared with those around the magnetic mirror field, basic confinement properties of a simple magnetic mirror field can be investigated.

II. EXPERIMENTAL SETUP

Shown in Fig. 1 are examples of a measured magnetic field along the axis of symmetry and an electrostatic potential applied to ring electrodes. A magnetic mirror field was produced by two series of magnetic coils. For example, keeping a current of coils on the low field side $I_L$ fixed to 30 A, which produced the field of about 100 G, a current of the other coils on the high field side $I_H$ could be increased from 30 to 120 A. In the present experiment, it corresponded to an increase of the mirror ratio $R$ from 1 to 3. Here, the mirror ratio is defined as $R=B_{max}/B_{min}$ where $B_{min}$ is the minimum magnetic field at $z \approx 0$ cm and $B_{max}$ is the maximum field at $z \approx 70$ cm in the confinement region. It is seen that the magnetic field inside the confinement region could be approximated by the following equation with $L=140$ cm and $B_0=218$ G as shown by the thick solid line in Fig. 1.

$$B_z = B_0 \left(1 - \frac{R-1}{R+1} \cos \frac{2 \pi z}{L} \right).$$

For applying the electrostatic potential, a trap named multitiring electrode (MRE) was aligned along the axis of symmetry [10,11]. The trap contained 45 ring electrodes with an inner diameter of 7 cm and thickness of 1.2 cm. They were separated by 1.6 cm each in the axial direction from -6.7 to 63.7 cm. With these electrodes, the MRE trap was very flexible in applying the electrostatic potential. Four consecutive electrodes at low field side from -6.7 to -1.9 cm were used to provide the axial confinement of electrons with an electrostatic potential $V_L=-20$ V, as denoted by the thin solid line in Fig. 1. Another set of electrodes at the high field side from 58.9 to 63.7 cm were used to supply electrostatic voltage $V_H=-10$ V (the dotted line in Fig. 1) to confine a large number of electrons at the beginning of an experiment.

FIG. 1. The basic configuration of an applied magnetic field and electrostatic potential. Open circles denote the measured magnetic field strength on the axis of symmetry when $I_L=30$ A and $I_H=120$ A. The thin solid line and dotted line correspond to the applied electrostatic potential. The thick solid line is a fitting curve Eq. (1).
The vacuum pressure in a series of experiments reported here was about \( 2 \times 10^{-8} \) torr.

Electrons were provided by an electron gun installed on the axis of symmetry at \( z \sim 98 \) cm. Therefore electrons were injected from the high field side. A Faraday cup was placed on the low field side at \( z \sim -16 \) cm so that the number of confined electrons could be measured easily. The Faraday cup also had a movable hole with a diameter of 2 mm, so that the axially integrated radial density profile could be measured.

It is thought that the confinement properties inside a magnetic mirror field depend on the initial condition of trapped charged particles. The following procedure was employed in the present experiments. First, electrons were injected and confined axially with electrostatic potentials on both ends. The number of electrons was \( 10^9 \) at most. After 100 ms, the confined axially with electrostatic potentials on both ends. The number of electrons was \( 10^9 \) at most. After 100 ms, the confinement time \( \tau_c \) of electrons was measured with various magnetic mirror ratios. Here the total number of electrons \( N_e \) was kept constant at about \( 7 \times 10^8 \) at the beginning of the confinement. Shown in Fig. 2(a) is the number of confined electrons as a function of time. It is seen that the number of electrons decreased drastically at the beginning. This is because the space potential of the electrostatically confined non-neutral electron plasma was about several volts, which modified the loss cone distribution of a single charged particle and caused a higher loss rate at the magnetic mirror. When \( N_e \) was reduced to a certain level where the space potential of trapped electrons had little effect, electrons were in a steady state of the magnetic mirror confinement. Since the confinement with the electrostatic potential provided \( \tau_c \) longer than 450 ms in which the radial particle diffusion dominated the particle loss, the effect of the radial diffusion was ignored for the measurement time less than 100 ms. It is thought that the exponential decay of \( N_e \) after 5 ms was the confinement time \( \tau_c \) provided by the magnetic mirror field. It is clear that the larger \( R \) resulted in the confinement of more electrons and a longer confinement time.

Not only changing the mirror ratio \( R \), but also the strength of the magnetic field was changed for the same \( R \). The results are shown in Fig. 2(b) where \( \tau_c \) is plotted as a function of \( R \) for three different magnetic field strengths. Open circles, triangles, and diamonds correspond to \( \tau_c \) with \( I_L = 10, 20, \) and 30 A, respectively. It is seen that \( \tau_c \) is almost independent of the strength of the magnetic field, which also suggests that the measured \( \tau_c \) was dominated by the magnetic mirror confinement and was determined by \( R \).

According to a theory of a magnetic mirror confinement for neutral plasmas, it was expected that the ion confinement time inside a magnetic mirror was on the order of \( \tau \sim 0.8 \tau_i \ln R \) with \( \tau_i \) the ion-ion collision time [13]. Since scattering of ions by electrons was neglected in the theory, a similar equation of the confinement time for a non-neutral electron plasma is expected by replacing \( \tau_i \) with the electron-electron collision time \( \tau_{ee} \). If the collisions between electrons and residual neutral particles are dominant, \( \tau_{ee} \) can be replaced by the electron-neutral collision time \( \tau_{en} \) for a simple estimation. The shorter one should be employed.

With the electron density \( n_e \sim 1 \times 10^7 \) cm\(^{-3}\), which is an estimation by the measured radial density profile in the next section, \( \tau_{ee} \) is in the order of \( 10^{-1} \) s for \( T_e \sim 1.0 \) eV. When the vacuum pressure is \( 2 \times 10^{-8} \) torr, the residual neutral gas density \( n_a \) is about \( 7 \times 10^8 \) cm\(^{-3}\). Assuming that the residual gas is dominated by H\(_2\)O, the total cross section of 1.0 eV electrons is about \( 5 \times 10^{-15} \) cm\(^2\). Therefore \( \tau_{en} \) is on the order of \( 10^{-2} \) s, which is shorter than \( \tau_{ee} \). In fact, experimentally observed \( \tau_c \) remains constant during a magnetic mirror confinement, as seen in Fig. 2(a), which means that \( \tau_c \) is almost independent of \( n_e \). Thus it is thought that the collisions between electrons and residual neutral particles is dominant in the present experiments. As a result, the measured \( \tau_c \) is almost proportional to \( \ln R \) as seen by the dotted
fitting line in Fig. 2(b). A longer confinement time is expected with an improved vacuum pressure.

**B. Radial density profiles**

Although the radial density profile of a high energy non-neutral electron cloud was measured qualitatively [7], a low energy non-neutral electron plasma is more suitable for a quantitative measurement. It is known that a low density annular electron layer can be an equilibrium state when the magnetic mirror field is approximated by Eq. (1) [5,6]. In the theoretical model, the following distribution function is assumed:

\[ f = \frac{n_{e0}r^2}{2 \pi m} \delta(H - H_0) \delta(P - P_0). \]  

(2)

Here, \( n_{e0} \) is the electron density at the smallest plasma inner-radius \( r^e_0 \). The mass, total energy, and canonical angular momentum of an electron are denoted by \( m, H, \) and \( P \), respectively. Also, \( H_0 \) and \( P_0 \) are positive constants. Then, the electron density inside the plasma is given by \( n_e(r) = n_{e0}r^2/r \). Under the conditions that \( r \ll L \) and that space charge effects of the plasma are sufficiently weak, it is deduced that the boundary of the electron layer is given by

\[ z = \frac{L}{2 \pi} \cos^{-1} \left[ \frac{R - 1}{R + 1} \left( 1 - \frac{c_1}{r} + \frac{c_2}{r^2} \right) \right]. \]

(3)

where \( c_1 = (8H_0/m_0e)^{1/2} \) and \( c_2 = 2P_0/m_0e \) with the electron cyclotron frequency \( f_c = \omega_c/2\pi = eB_0/2mc \).

In experiments, all the measured radial profiles with \( R \) up to 5 had hollow profiles as expected and the peak of the profile shifted outward for larger \( R \). Shown in Fig. 3 are examples of measured radial density profiles integrated along the magnetic field lines when (a) \( R = 1.5 \) and (b) \( R = 4 \). Since the measured radial density profiles are integrated along curved magnetic field lines, they cannot be compared directly with Eq. (3) by multiplying the electron density \( n_e(r) \). However, the straight magnetic field line approximation still provides a useful zeroth order approximation and insights into the properties of trapped electrons when \( R \) is small. The dashed line in Fig. 3(a) is the fitting curve obtained with \( c_1 \sim 2.8, c_2 \sim 1.7 \), and \( n_0 \sim 1.3 \times 10^5 \text{ cm}^{-3} \). With these parameters, the axial extent of the trapped electrons can be estimated to be \( \sim 55 \text{ cm} \). It is thought that the electron plasma had a long axial profile inside the magnetic mirror field in the case of Fig. 3(a). With \( T_e \sim 1.0 \text{ eV} \), the estimated Debye length becomes \( \lambda_D \sim 2.4 \text{ cm} \). Thus the trapped electrons behave as a plasma in the axial direction. However, it is ambiguous if they are called plasma in the radial direction. It should be noted that there were some electrons measured near the axis. This is partly because the long Debye length leads to a diffuse boundary.

On the other hand, it was more difficult to fit the experimental data with Eq. (2) when \( R \) became larger as shown in Fig. 3(b). The dotted line in Fig. 3(b) is a calculation where the magnetic field curvature was taken into account for \( R = 4 \) with the parameters \( c_1 \sim 1.8, c_2 \sim 1.73 \), and \( n_0 \sim 5.7 \times 10^5 \text{ cm}^{-3} \). Although the calculated peak shifts outward due to the field curvature, it still underestimates the peak position and the calculated radial profile deviates from the measured one. Furthermore, some electrons were observed around the axis. It is thought that the low density annular electron layer model may not be applicable to explain the measured radial density profile when \( R \) is large in the present experiments.

The discrepancy between the theoretical model and the experiment is probably attributed to the assumed distribution function. In the theoretical model the distribution function was assumed by Eq. (2), while in the experiment, it is difficult to reproduce it precisely. In addition, the finite temperature of trapped electrons could induce an electrostatic potential and a density gradient along a magnetic field line [12]. Another possible reason is the space potential of trapped electrons. Since more electrons were confined with a larger \( R \), it is natural that the measured radial profile deviates from the theoretical model for the larger \( R \). The details should be investigated further because the larger \( R \) leads to the longer confinement time which is favorable for various applications.

**C. Electrostatic oscillations**

The electrostatic oscillations of non-neutral plasmas in Malmberg traps and Penning traps are employed as a very useful nondestructive diagnostic tool for confined plasmas. Dispersion relations of Trivelpiece-Gould modes [14] and Dubin modes [15] are applicable to these plasmas. For ex-
The signal was picked up through a ring electrode used to identify the number of axial nodes experimentally detected and the phase differences between induced signals are on these ring electrodes by a plasma oscillation can be determined. Small fluctuations of the electric potentials induced comes 0° or 180° because the axial electrostatic oscillations of a spheroidal non-neutral electron plasma have been monitored during the electron cooling of high energy antiprotons [16]. In the case of non-neutral plasmas in a magnetic mirror field, the situation is more complicated and there is no applicable model. However, the observation of axial electrostatic oscillations is important because they are functions of the plasma density and temperature. In general, the oscillation frequency becomes higher as the density or temperature becomes higher.

Here, the first observation of axial electrostatic oscillations in a non-neutral electron plasma confined with a magnetic mirror field is described. Unfortunately, the oscillations were observed only for larger $R \geq 3$. It is probably due to the radial extent of the plasma. As discussed in the previous section, the larger $R$ resulted in the larger plasma radius at $z \approx 0$, which induced the larger signal at an electrode. Shown in Fig. 4(a) is an example of an axial electrostatic oscillation observed for $2.6 \times 10^5$ electrons confined with $R=5$. It is seen that the fast Fourier transformed (FFT) spectrum has a sharp peak ($\Delta f \approx 0.05$ MHz) at the excitation frequency of 1.1 MHz. The signal was picked up through an ring electrode near $z=0$ after the excitation drive was ceased.

Many ring electrodes in the MRE trap also work as fixed probes. Small fluctuations of the electric potentials induced on these ring electrodes by a plasma oscillation can be detected and the phase differences between induced signals are used to identify the number of axial nodes experimentally [17,18]. Here, the phase difference between electrodes becomes $0^\circ$ or $180^\circ$ because the axial electrostatic oscillations are standing waves under the present experimental conditions. It was confirmed that the lower frequency mode with the peak around 1.1 MHz was $\ell = 1$ mode and there was only one node at $z \approx 15$ cm. The oscillation signal was detectable up to $z \approx 43$ cm. Also, the mode with the peak frequency around 2.2 MHz was $\ell = 2$ mode which had two nodes at $z \approx 10$ and 27 cm. When a non-neutral plasma is confined by magnetic mirror fields on both ends, these modes correspond to $\ell = 2$ and 4, respectively.

When the excitation frequency was changed within the range from 0.7 to 2.9 MHz, the peak of the observed frequency moved in proportion to the excitation frequency. The power of the observed peak was plotted as a function of excitation frequency in Fig. 4(b). These broad spectra are quite different from those of non-neutral plasmas confined with a uniform magnetic field where the resonance frequencies have little dependence on the excitation frequency as far as the excitation voltage is small enough to avoid plasma heating and nonlinear effects. The obtained results might mean that the plasma density is not uniform along the magnetic field line.

The observed confinement time of about tens of milliseconds with a simple magnetic mirror trap is quite short compared with a Penning-type trap. However, as seen above, it is long enough compared with the plasma oscillation period. Applying both positive and negative electrostatic potentials outside the magnetic mirror field may extend the confinement time. In any case, the observation of the electrostatic oscillations in a magnetic mirror trap is possible and will be useful for a nondestructive measurement of a confined plasma. A theoretical dispersion relation will be helpful for a better understanding of mirror confined plasmas.

### IV. SUMMARY

Low energy non-neutral electron plasmas were confined with a magnetic mirror field of the mirror ratio $R$ up to 5 to investigate the basic properties inside the magnetic mirror field. It was confirmed that the confinement time was proportional to $\ln R$ under the condition that $\tau_{en}$ is shorter than $\tau_{pe}$. The radial density profiles integrated along the magnetic field lines were found to be hollow ones. A theoretical model was applied to evaluate the density and axial extent of a confined electron plasma for the smaller $R \approx 1.5$. The observation of axial electrostatic oscillations of a mirror confined non-neutral electron plasma was demonstrated. The measured confinement time and plasma density suggest that a kind of magnetic mirror traps will be the easiest way to produce low energy electron-positron plasmas. Then, the electrostatic oscillations will be utilized for the diagnosis of confined plasmas.

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