Anisotropy and internal-field distribution of MgB$_2$ in the mixed state at low temperatures


1Physik-Institut der Universität Zürich, CH-8057 Zürich, Switzerland
2Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA
3Paul Scherrer Institute, CH-5232 Villigen PSI, Switzerland
4DPMC, Université de Genève, 24 Quai Ernest-Ansermet, CH-1211 Genève, Switzerland
5Faculty of Physics, University of Belgrade, 11001 Belgrade, Yugoslavia
6Solid State Physics Laboratory, ETH, CH-8093 Zürich, Switzerland

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Magnetization and muon spin relaxation on MgB$_2$ were measured as a function of the applied magnetic field at 2 K. Both indicate an inverse-squared penetration depth strongly decreasing with increasing field $H$ below about 1 T. Magnetization also suggests the anisotropy of the penetration depth increases with increasing $H$, interpolating between a low $H_{c1}$ and a high $H_{c2}$ anisotropy. Measurements of the torque as a function of the angle between the field and the c axis of the crystal are in agreement with this finding, while also ruling out drastic differences between the mixed state anisotropies of the two basic length scales penetration depth and coherence length.

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The understanding of the physical properties of the recently discovered 39 K superconductor MgB$_2$ has made rapid progress in the last 3 years. A central issue of research has been the involvement in superconductivity of two sets of bands with different dimensionality and pairing strength. This “two-band superconductivity” leads to an array of unusual superconducting properties such as specific heat, particularly also to a very unusual behavior of the superconducting anisotropies.

For example, a pronounced temperature $T$ dependence of the anisotropy $\gamma_H$ of the upper critical field $H_{c2}$, directly related to the coherence length $\xi$, has been involved in superconductivity of two sets of bands with different dimensionality and pairing strength. This “two-band superconductivity” leads to an array of unusual superconducting properties such as specific heat, particularly also to a very unusual behavior of the superconducting anisotropies.

For example, a pronounced temperature $T$ dependence of the anisotropy $\gamma_H$ of the upper critical field $H_{c2}$, directly related to the coherence length $\xi$, has been observed$^7,8$ and calculated based on the two-band model.$^9,10$ Strikingly, calculations of the low field penetration depth anisotropy $\gamma_s$ predicted a much lower anisotropy of this quantity, with a $T$ dependence opposite to the one of $H_{c2}$. This was experimentally confirmed as well, based on measurements, e.g., of $H_{c1,2}$ by small angle neutron scattering (SANS)$^{14,15}$ and scanning tunneling spectroscopy (STS).$^{16}$ However, the experiment of Ref. 15 indicates that whereas in the limit of very low fields $H \ll \gamma_s$, it is indeed close to 1, it is rising with increasing $H$, as deduced earlier more indirectly.$^7$

The behavior of the anisotropies of the length scales in the mixed state $H_{c1} < H < H_{c2}$ still needs to be clarified. One point of view$^{17}$ surmises constant (with respect to $H$) anisotropies of the penetration depth $\gamma_s$ and the coherence length $\gamma_H$, which are, however, different from each other. This difference was predicted to lead to a sign reversal in the angle dependent torque.$^{17}$ Another point of view is that these anisotropies are not drastically different from each other, but both increase with increasing field, interpolating from the $H_{c1}$ anisotropy in low fields to the $H_{c2}$ anisotropy in high fields.$^{18}$

Here, we support the latter point of view by analyzing SQUID (superconducting quantum interference device) and torque magnetization data measured on a MgB$_2$ single crystal with very low pinning, and muon spin relaxation ($\mu$SR) data measured on randomly aligned MgB$_2$ powder. In the absence of a more elaborate model of the mixed state of a two-band superconductor,$^{18}$ we base the analysis on the London model, allowing however for a $H$ dependent penetration depth, which is obtained from the $H$ or angle $\theta$ dependence of the bulk magnetization (SQUID/torque) as well as from the average variation of the internal field ($\mu$SR). From SQUID and $\mu$SR we find a rapid decrease of the inverse-squared penetration depth $1/\lambda^2$ (“superfluid density”) with $\mu_0 H$ increasing below about 1 T, and SQUID and torque data agree on the anisotropy $\gamma_s$ increasing strongly with increasing field. The analysis of the torque data further suggests that $\gamma_H$ is not very different from $\gamma_s$.

Single crystals of MgB$_2$ were grown with a high pressure cubic anvil technique, as described in Ref. 19, and a crystal with particularly low pinning was selected for measurements with a Quantum Design MPMS-XL SQUID magnetometer and a noncommercial torque magnetometer.$^{20}$ The crystal has a wedge shape, with one of the faces parallel to the $ab$ planes. The $\mu$SR experiment on polycrystalline MgB$_2$ was similar to the one of Ref. 21.

The magnetization of the single crystal was measured as a function of $H$, as can be seen in Fig. 1, the irreversibility is very low above about 0.15 T. Larger irreversibility in lower fields may be due to geometrical barriers, which is why we did not attempt to directly extract $H_{c1}$. The curves shown are not corrected for demagnetizing effects (the exact demagnetizing factor is difficult to estimate due to the sample shape), but we verified that any reasonable demagnetization correction does not noticeably affect above 0.2 T the results discussed below.

Within the London model of a standard superconductor, the magnetization is proportional to the logarithm of the applied field, when the field is not too close to either $H_{c1}$ or $H_{c2}$. Keeping to an analysis within the London approach, but dropping the requirement of a constant penetration depth $\lambda$,
we have $1/\lambda^2 \propto dM_{\text{rev}}/d (\ln H)$. The so obtained $1/\lambda^2$ is plotted in Fig. 2. To avoid overloading the graph, only curves assuming $M_{\text{rev}}=(M_{H^+}+M_{H^-})/2$ are shown; except in the lowest $H$ using instead $M_{H^+}$ or $M_{H^-}$ leads to very similar results. The curves for both field directions were normalized by the same constant factor. The shaded box indicates the vicinity of $H_{c1}$ (see below).

Also plotted in Fig. 2 is $1/\lambda^2$ obtained from the muon spin depolarization rate $\sigma$ measured on randomly aligned powder at the same temperature. The depolarization rate $\sigma$ is a measure of the average variation of the internal field within a superconductor, and in the mixed state (again for the external field not too close to either of the critical fields) is directly proportional to $1/\lambda^2$, since $\lambda$ is the fundamental length scale of the variation of the field in a superconductor (cf. Ref. 21). An issue to be aware of when deducing $\lambda$ in this way is the possible influence of pinning, which can lead to an extrinsic increase in $\sigma$. In a previous $\mu$SR experiment on MgB$_2$, the whole $H$ dependence of $\sigma$ was indeed ascribed to pinning. To check for the possible influence of pinning on $\sigma$ (as opposed to the magnetization), we performed time-dependent measurements of $\sigma$ in several fields: After reaching 2 K (field cooled), statistics was gathered for 10 min, then stopped and restarted (repeated 5 to 10 times). Except for 0.1 T, changes of $\sigma$ with time are well below error bars, and no clear trend discernible. This suggests that for higher $H$ even at 2 K, pinning is not influencing $\sigma$ much, and the $H$ dependence of $\sigma$ is indeed intrinsic. The observation of a very similar $H$ dependence of $\sigma$ in samples from two sources and synthesized using slightly different techniques supports this assertion; an intrinsic $\sigma(H)$ dependence was also proposed in Ref. 23.

Concerning the $dM/d(\ln H)$ curves, it may be argued that the low field behavior is not unexpected even for a normal superconductor, since in the limit $H \to H_{c1}$ it is expected$^{24}$ that $dM/d(\ln H) \propto H/(H-H_{c2})$. However, $dM/d(\ln H)$ should reach values close to the normal London slope rather quickly (within 2–3 $H_{c1}$) and the variation presented in Fig. 2 is spread over a considerably larger field range. Furthermore, the depolarization rate $\sigma$ is decreased rather than increased in the vicinity of $H_{c1}$, contrary to our observation (Fig. 2). The close similarity of the $H$ dependence of the penetration depth obtained from rather different quantities (bulk magnetization from SQUID and internal field variation from $\mu$SR) strongly suggests that all curves in Fig. 2 indeed show the $\lambda(H)$ dependence, outside of the shaded box indicating the vicinity of $H_{c1}$. A similar, strong depression of $1/\lambda^2$ with $H$ was deduced previously (for $H \parallel c$) from an analysis of SANS form factors.$^{15}$ The SQUID curves additionally indicate that above 10 kOe $1/\lambda^2$ no longer varies strongly.

The SQUID measurements in the two field configurations also yield the anisotropy of $\lambda$. For $H \parallel c$ the screening currents flow within the $ab$ plane, giving $1/\lambda_{ab}^2$. For $H \parallel ab$ the currents flow also perpendicular to the planes, giving $1/(\lambda_{c}^2 \lambda_x)$. The ratio of the $dM/d(\ln H)$ curves for the two field configurations thus corresponds to $\gamma_{c}$. Considering the curves in Fig. 2, we can see that (i) in low $H$ the anisotropy is very small, (ii) in high $H$ $\gamma_{c}$ is of the order of about 6 or even 7, and (iii) the variation with $H$ of $\gamma_{c}$ is most pronounced in low $H$. We stress the fact that when considering the high field region along the standard London model with constant $\lambda$ and $\xi$ and a constant common anisotropy $\gamma$ describes the data reasonably well. This indicates that at low $T$ in high $H$, MgB$_2$ is close to a “standard superconductor” with high anisotropy. Corresponding to this is the absence of an unusual $H_{c2}(\theta)$ dependence at low $T$, in contrast to the situation closer to $T_c$ (Refs. 10, 25, and 26) [of course the $\gamma_{c}$ analysis breaks down as $\mu_0 H \to \mu_0 H_{c2}^0 (=2.8$ T for this crystal)].

An alternative method to determine the penetration depth anisotropy is to analyze the angular $\theta$ dependence of the torque $\tau$ in fixed $H$. We previously used this method at much higher $T$, finding also indications of an anisotropy increasing with $H$. However, thermal fluctuations and additional inter-mixture of the two sets of bands by thermally excited quasiparticles, complicate the analysis there. To provide a direct comparison with the SQUID results and give a quantitative estimate of $\gamma_{c}$ we measured $\tau(\theta)$ at low temperature.

Due to increased irreversibility at lower $T$, it is important not only to use a crystal with low pinning, but also employ...
the “shaking technique” developed by Willemi et al.\textsuperscript{28} Since the magnetometer equipped with this technique cannot reach 2 K measurements were conducted at 8, 11, and 15 K. These temperatures should be low enough to avoid too strong of an influence of thermal fluctuations/excitations, as well as to probe the low temperature limit of the calculated\textsuperscript{9,11} anisotropies.

The data were analyzed with a formula derived by Kogan, Eq. (18) of Ref. 17,

\[
\tau = \frac{V d H (1 - \gamma^2) \sin 2 \theta}{64 \pi^2 V_s \varepsilon_{\lambda}} \times \left[ \ln \left( \frac{\eta H_{\text{c}2}}{H (\varepsilon_{\lambda} + \varepsilon_H)^2} \right) - \frac{2 \varepsilon_{\lambda}}{\varepsilon_{\lambda} + \varepsilon_H} \left( 1 + \frac{\varepsilon_H}{\varepsilon_{\lambda}} \right) \right],
\]

where \( \varepsilon_{\lambda,H}(\theta) = (\cos^2 \theta + \sin^2 \theta / \gamma_{\lambda,H}^2)^{1/2} \) denotes differentiation with respect to the angle \( \theta \). \( V \) is the sample volume, and \( \varepsilon = 2.718 \ldots \). Equation (1) explicitly allows for different \( \gamma_{\lambda} \) and \( \gamma_H \). Such a difference was, however, not found in any of the curves analyzed, and a sign reversal of the torque, a key prediction of Ref. 17 for \( \gamma_{\lambda} \approx \gamma_{\mu} \). The large number of parameters involved and the numerical condition of the fit formula result in extended error bars though, so that small differences between \( \gamma_{\lambda} \) and \( \gamma_{\mu} \) cannot be completely ruled out (but large differences\textsuperscript{17} as calculated with \( H \) independent anisotropies can).

The resulting field dependence of the anisotropy \( \gamma = \gamma_{\lambda} = \gamma_{\mu} \) is shown in Fig. 4.\textsuperscript{29} The anisotropy is monotonically increasing with increasing field, up to 1.5 T (the maximum attainable by the magnetometer used). In the lowest fields, this rise seems to be much steeper than above 0.5 T. However, as a cautionary note, even with “shaking,” the irreversibility cannot be said to be negligible below 0.1 T and it should also be kept in mind that we are approaching \( H_{c1} \) (see Fig. 1).

A report by another group of torque vs angle measurements performed at 10 K claimed a field-independent anisotropy of the order of \( \gamma = 4.3 \).\textsuperscript{30} However, analyzing the same data, a different conclusion of an anisotropy that does increase with \( H \), in not too large fields, may also be reached.\textsuperscript{31} The results of Ref. 30, as well as Ref. 32 (not finding a field dependence as well) may be reconciled with the ones of Ref. 7 and the present results by assuming a tendency of \( \gamma \) to Saturn in higher fields.

For comparison, a rough estimate of \( \gamma(H) \) from SQUID magnetometry in fields along the principal axes (Fig. 2) is plotted in Fig. 4 as well. SQUID and torque results are in qualitative agreement: the anisotropy increases with increasing field. We attribute the numerical discrepancy to the large scattering as visible in Fig. 2 and the corresponding uncertainty in the estimation of \( \gamma \). The low \( T \) behavior of \( \gamma(H) \) is in strong contrast to the one very close to \( T_c \), where between \( H_{c1} \) and \( H_{c2} \) \( \gamma = 2 \) is constant,\textsuperscript{33} as indicated by the dotted line in Fig. 4.

A field dependent anisotropy at low \( T \) (2 K) had been deduced based on different experiments as well. Bouquet et al.\textsuperscript{34} reported a \( H \) dependent effective anisotropy based on specific heat measurements. Since these are sensitive mainly to the coherence length, the experiment suggests the anisotropy \( \gamma_{\mu} \) to be \( H \) dependent. Cubitt et al.\textsuperscript{15} observed the anisotropy of the vortex lattice \( \gamma_{VL} \), to increase strongly with increasing \( H \), from less than 1.5 in 0.2 T to about 3.8 in 0.5 T, \( \gamma_{VL}(H) \) being more steep in higher \( H \). Keeping to the London model, the anisotropy of the vortex lattice should be equal to the penetration depth anisotropy \( \gamma_c \).\textsuperscript{35} Our results extend to higher \( H \) and agree qualitatively with those of Cubitt et al., but we do not find a particularly steep \( \gamma(H) \) around 0.5 T, but rather a slower field dependence. Very recently, Lyard et al.\textsuperscript{12} proposed a similar \( H \) dependence of anisotropies, based on a London analysis of magnetization data measured at much higher \( T \).

The strong field dependence at low \( T \) of all anisotropies obtained from the measurements presented here, as well as by other groups, are readily explained qualitatively by a faster suppression with \( H \) of superconductivity in the more isotropic \( \pi \) bands, increasing the relative contribution of the highly anisotropic \( \sigma \) bands. Such a suppression, consistent with the overall decrease of \( 1/\lambda^2 \) (Fig. 2), was also observed.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{(Color online) Angle \( \theta \) dependence of torque \( \tau \) in 0.2 T at 8 K (symbols) (Ref. 27). Dashed line, theoretical description (Ref. 17) assuming \( \gamma_{\lambda} \ll \gamma_{\mu} \); full line, description with \( \gamma_{\lambda} = \gamma_{\mu} \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4.png}
\caption{(Color online) Anisotropy determined from an analysis of the torque data (Fig. 3) with Eq. (1) (Ref. 17), at various low temperatures, as a function of field \( H \) (Ref. 29). Also shown is the anisotropy determined from Fig. 2 (open circle) and the anisotropy very close to \( T_c \) (dotted line, see text).}
\end{figure}
In conclusion, μSR and magnetization data show the “superfluid density” $1/\lambda^2$ in MgB$_2$ at 2 K to strongly decrease with increasing field below about 1 T. In parallel, the penetration depth anisotropy increases, and is not drastically smaller than the coherence length anisotropy (in the same field). This behavior is due to the fast suppression of the contribution to superconductivity of the more isotropic $\pi$ bands with weaker superconductivity.

After submission of this manuscript we became aware of a torque study finding, assuming $\gamma_s = \gamma_H$, a similar $H$ dependence of the anisotropy at 10 K.

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intrinsic $\sigma$ and $\pi$ length scales was recently proposed in Ref 23.


27 The asymmetry in the small remaining hysteresis is most likely due to the wedge shape of the crystal.


29 After it was established that allowing for different $\gamma_{H}$ and $\gamma_{H}$ yields identical anisotropies for different $H, T$ conditions, the rest of the data were analyzed assuming $\gamma_{H} = \gamma_{H}$, in which case Eq. (1) reduces to a simpler and numerically more amiable formula derived years earlier, also by Kogan [V. G. Kogan, Phys. Rev. B 38, 7049 (1988)].


33 S. Kohout (unpublished). See also Ref. 25.


