Distortion Analysis of A Casting Mold in Dental Casting
(2-D Finite Element Method)

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ABSTRACT
This study was to investigate the distortions of a casting investment mold with an MOD-type resin pattern in mold by 2-D finite element method (2-D FEM). The prediction agreed fairly well with the measurement results. The result showed that a proximal side of the resin pattern was opening because of setting expansion of investment inside of the pattern. Setting expansion of investment was greatly restricted and reduced to 10 percent of a free setting expansion value of the investment, 0.009, near inside corner of MOD-type resin pattern and setting expansion outside of the pattern was almost free. From these results, we summarize that resin pattern has enough strength to restrict the setting expansion value of investment and thus this restriction may give the distortion of metal or alloys during dental casting.

INTRODUCTION
In dental metal casting, gypsum-bonded investment needs to expand to compensate for thermal shrinkage of metal to achieve enough casting accuracy for successful restorations. However, many studies on this problem have shown that setting expansion of investment is partially restricted by a pattern material and a steel investing ring. This means that the investment mold can not expand uniformly and a distortion of metal casted will ultimately occur. This phenomena should be explained on the basis of effects of restrictive stress on setting expansion behavior of investment.

Fusayama measured directly setting expansions of investments with an MOD-type pattern inside, and a obtained distortion which showed that expansion inside the pattern was restricted. Results obtained by direct measurements of expansions, however, cannot be applied to general problems such as setting expansions under practical investing conditions.

Earnshaw studied effects of restrictive stress on the setting expansion values by measuring expansions under various loading conditions. However, there was no description on a distortion of mold that should be introduced by the restriction of setting expansion in his report. Thus, the result is not available to estimate expansions of investment in a casting ring with a pattern invested inside. Effect of restrictive stress on a setting expansion of investment was also studied by analyzing stress distributions in the pattern and distortion of the pattern. However, this kind analysis contributes indirectly because the pattern and the mold need not exhibit the same distortion. Our main interest is distortions of investment mold rather than that of pattern.

Numerical method is a useful approach to predict setting expansion values under restrictive stresses. Nakatsuka developed a computer program for FEM analysis based on the relationship between stresses and strains shown in previous reports.

In practical casting procedure, wax has been used as a pattern material. One of the potential problems in use of wax pattern is that it could be deformed by external forces before investing process because of small mechanical strength. Thus, resin pattern has come to be used as a pattern material for some cases, such as for adhesion bridge and telescopic system restorations where large strength and smoothness at surface of patterns are required. Resin pattern could restrict more setting expansion of investment than wax pattern because of its high mechanical strength, and finally
result in distortion of metal casted.

The objective of this study is to apply FEM analysis to predict setting expansion behavior of investment in a casting ring with a MOD-type resin pattern made of acrylic resin invested inside. These calculated expansions were compared with measured ones, in order to deduce the internal behaviours after setting of investment as a calculation model.

MATERIALS AND METHODS

1. FEM analysis for expansion of investment

FEM analysis was conducted using the method introduced in a previous report\(^6\). Comparison between constitutive equations for investment and that for elastic materials shows that a setting expansion value during a small time interval, as indicated by \( \Delta t \), can be obtained as a strain of elastic body with a thermal expansion of \( a_0 \exp (-kt \Delta t) \) and Young's modulus of \( a_0 \exp (-kt \Delta t) \Delta t/E' \), where \( a_0 = 0.009, k = 0.0032 (1/min) \), and \( E' = 0.49 \) (MPa).

Fig. 1 is a finite element model showing that the investment part was divided into 48 triangular elements, and the resin part into 18 elements. This analysis needs contact-related boundary conditions between the pattern and the investment in addition to usual displacement and force conditions. In this study, two contact-related boundary conditions were employed as follows:

![Finite element model](image)

**Figure 1** Finite element model for a mold with a MOD-type resin pattern

(Case 1) Number of contact points between the investment and the pattern, NIE, is 5. Nodal points I, J, K, L, and O of the investment contact nodal points 3, 4, 5, 6, and 9 of the pattern, respectively.

Displacements of nodes J, K, L, and O for the investment equal that of nodes 4, 5, 6, and 9 for the pattern respectively in the X-axis. Displacements of node I of the investment and that of node 3 for the pattern are the same in the y-axis direction.

(Case 2) NIE is 3. Nodal points J, K, and L of the investment contact nodal points 4, 5, and 6 of the pattern, respectively. At each of these contact points, the displacement of mode for the investment and for the pattern is same only in the x-axis direction.

The following conditions were the same for both cases 1 and 2: 1) Considering a symmetry of the model, displacements of nodes are zero in the x-axis direction and free in the y-axis direction at A, B, C, E, F, G, and H for the investment, and at node I for the pattern. 2) Displacements are zero in the both x-axis and y-axis directions at node D for the investment and at 2 for the pattern.

No frictional force between the investment and the pattern was considered in this study. Details of calculation process were the same that was described in the previous report\(^6\). Time was divided into 20 finite steps in this study. All calculations were made on a personal computer (PC-9801VM, NEC Ltd., Tokyo, Japan).

2. Measurements of expansion of investment

An MOD-type acrylic resin pattern having dimensions shown in Fig. 2 was processed by the usual method which is utilized to make denture prostheses\(^2\).

Investment (Cristobalite Investment, G-C, Tokyo, Japan) was mixed with water at w/p ratio of 0.32 following the manufacture's recommendation. Then, the investment slurry was poured into a wax-made ring having a internal radius of 30 mm, and a height of 10 mm. The MOD-type resin pattern was placed at the center of the wax-made ring as shown in Fig. 2. After that, two L-shaped wires 0.7 mm in diameter were placed at five different positions, a-e, on the investment, and wires were fixed when the investment set. Displacements of these two wire's edges were measured by a microscopic micrometer with a magnitude of 200 times. Measurements were made from 18 minutes through 2 hours after the start of mixing.
RESULTS AND DISCUSSION

Fig. 3 shows distortions of the pattern and that of the investment mold by FEM analysis. Table 1 shows calculated equivalent forces which act on the pattern at the contact points between the investment and the pattern 2 hours after the start of mixing.

These results suggest that case 1 is reasonable because no overlap between the investment and the pattern was found. Furthermore, forces are acting in
Table I  Force acting on the resin pattern

<table>
<thead>
<tr>
<th>Force (N)</th>
<th>Nodal point</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fx</td>
<td>8.4 × 10^{-6}</td>
<td>-5.0</td>
<td>-11.9</td>
<td>-9.4</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>Fy</td>
<td>0.17</td>
<td>-1.9 × 10^{-5}</td>
<td>5.6 × 10^{-5}</td>
<td>-1.3 × 10^{-4}</td>
<td>-2.1 × 10^{-4}</td>
<td></td>
</tr>
</tbody>
</table>


the minus direction of the x-axis at nodes 4, 5, and 6 for the pattern, and in the plus direction of the x-axis at node 9. At node 3, force is acting in the minus direction of the y-axis. These results well correspond to the deformation of the pattern, i.e., the proximal sides of the MOD-type pattern were open because of the setting expansion of investment inside the pattern, and this deformation was restricted by the investment at the contact points I and O at the same time. Fig. 4 shows expansions of nodes J, K, L, M, and N of the investment by FEM for case 1. Setting expansion was largely restricted near the inside corner of the pattern. On the other hand, expansion outside of the pattern was almost free.

Measured setting expansions of the investment are shown in Fig. 5. Fig. 6 shows the final measured and calculated setting expansion values (2 hours after the start of mixing). Setting expansion 0.1 mm from the occlusal side of the MOD-type pattern was reduced to 10% of the free expansion value of the investment, 0.0997. Results of the FEM analysis agreed fairly well with measured expansion values.

Force distribution along the side wall of the resin pattern was analyzed from the results of the FEM analysis. Ohno et al. have reported the same kind of pattern deformation that was obtained in this study, and also have reported stress distribution in the resin pattern by photo-elastic analysis which shows that force is concentrated at the inside corner of MOD-type pattern6. Fig. 7 shows the equivalent force values F which act on the side wall of the pattern 2 hours after the start of mixing. These forces were calculated from KU = F for case 1, where K is a total stiffness matrix and U is a displacement matrix of nodes of the pattern. As shown in Fig. 7, distributed force along the side wall of the pattern were calculated from the equivalent forces as follows:

1) Force acting at each node was allotted to the element of the pattern which includes that node according to a ratio between the side lengths of elements. For example, force acting at node 5, 11.9 N, was divided into 4.0 N for side 4–5, and 7.9 N for side 5–6 because side

![Figure 4](image_url)  Setting expansion of the mold by the FEM analysis
lengths are 0.2 cm for side 4–5, and 0.4 cm for side 5–6. i.e. force acting at side 4-5 is $11.9 \times 0.2/(0.4 + 0.2) = 7.9$ (N), and that acting on side 5–6 is $11.9 \times 0.2/(0.4 + 0.2) = 4.0$ (N).

2) Distributed forces were calculated from the total amount of force acting on each side, assuming that force were uniformly distributed along the side. For example, the total force acting on side 4–5 is $5.0 + 4.0 = 9.0$ (N), then uniformly distributed force on side 4–5 became $9.0/0.2 = 45$ (N/cm).

3) Finally, this uniform force distribution was modified so that force value at node 3 became zero, and force values were continuous at each node at the same time as shown in Fig. 7. The total force for each element was kept unchanged in this process. This force distribution was similar to that obtained by photo-elastic analysis, though quantitative comparison between them was not performed.

This FEM model predicted the distortion of the investment mold fairly well. The results suggest that resin pattern has enough strength to restrict setting expansion.
value of investment and cause distortion of metal casted.

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REFERENCES


