Non-perturbative renormalization factors of bilinear quark operators for Kogut-Susskind fermions and light quark masses in quenched QCD


Light quark masses are computed for Kogut-Susskind fermions by evaluating non-perturbatively the renormalization factor for bilinear quark operators. Calculations are carried out in the quenched approximation at $\beta = 6.0, 6.2, 6.4$. For the average up and down quark mass we find $m_\pi/(2\mathrm{GeV}) = 4.15(27)\text{MeV}$ in the continuum limit, which is significantly larger than $3.51(20)\text{MeV}$ ($q^* = 1/a$) or $3.40(21)\text{MeV}$ ($q^* = \pi/a$) obtained with the one-loop perturbative renormalization factor.

1. Introduction

Light quark masses are important unknown parameters of the standard model, and a number of lattice QCD calculations have been carried out to evaluate quark masses employing the Wilson, clover or Kogut-Susskind (KS) fermion action. Among the results, those with the KS action appear more accurate than others because of small lattice discretization errors and small statistical errors.

A worry with the KS result, however, has been that the employed one-loop renormalization factor takes a large value of $\approx 2$ in the range of $\beta$ studied, calling into question the viability of perturbation theory. In this article we report a study to circumvent this problem: we calculate the renormalization factor of bilinear quark operators for the KS action non-perturbatively using the method of Ref. [2] developed for the Wilson/clover actions. This calculation is carried out in quenched QCD at $\beta = 6.0, 6.2, 6.4$ on a $32^4$ lattice. The results, combined with our previous calculation of bare quark masses [1], lead to a non-perturbative determination of the light quark mass.

2. Method

The renormalization factor of a bilinear operator $\mathcal{O}$ is obtained from the amputated Green function,

$$\Gamma_\mathcal{O}(p) = S(p)^{-1}\langle 0|\phi(p)\mathcal{O}\bar{\phi}(p)|0\rangle S(p)^{-1}$$

where the quark two-point function is defined by $S(p) = \langle 0|\phi(p)\bar{\phi}(p)|0\rangle$. The quark field $\phi(p)$ with momentum $p$ is defined from the original one-component field $\chi(x)$ by $\phi_A(p) = \sum_y \exp(-ip\cdot y)\chi(y+aA)\chi(y+aA)^*$, where $y_\mu = 2an_\mu$, $p_\mu = 2\pi/(aL)n_\mu$ ($n_\mu = [-L/4,L/4]$) and $A_\mu = [0,1]$.

The renormalization condition imposed on $\Gamma_\mathcal{O}(p)$ is given by

$$\Gamma_\mathcal{O}(p) = Z_\phi(p)Z_\mathcal{O}(p)\Gamma_\mathcal{O}^{(0)}$$

where $\Gamma_\mathcal{O}^{(0)}$ is the amputated Green function at tree level and $Z_\phi(p)$ is the wave function renormalization factor which can be calculated by the

*presented by N. Ishizuka
condition $Z_V(p) = 1$ for the conserved vector current.

The relation between the bare operator on the lattice and the renormalized operator in the continuum takes the form,

$$C_{\text{MS}}(\mu) = U_{\text{MS}}(\mu, p)Z_{\text{MS}}(p)/Z_{\text{O}}(p)C_{\text{lat}}(a)$$ (3)

where $U_{\text{MS}}(\mu, p)$ is the renormalization-group running factor, and $Z_{\text{MS}}(p)$ is the matching factor from the RI scheme defined by (2) to the MS scheme, calculated perturbatively in the continuum.

For the light quark mass we apply relation (3) in the scalar channel in the chiral limit.

We use a source in momentum eigenstate to evaluate quark propagators. This results in very small statistical errors of $O(0.1\%)$ in the Green functions.

The external momentum $p$ should be taken in the range $A_{\text{QCD}} \ll p \ll O(1/a)$ in order to keep under control higher order effects in continuum perturbation theory, non-perturbative hadronization effect on the lattice, and discretization errors on the lattice. In this work we choose 15 momenta in the range $0.038553 < (ap)^2 < 1.9277$ for all values of $\beta$.

3. Result

In Fig. 1 we compare the scalar renormalization factor $Z_S(p)$ with that for pseudoscalar $Z_P(p)$ for three values of bare quark mass $am$ at $\beta = 6.0$. From chiral symmetry of KS fermions, we expect naively $Z_S(p) = Z_P(p)$ for all momenta $p$ in the chiral limit. Clearly this relation does not hold with our result toward small momenta, where $Z_P(p)$ rapidly increases as $m \to 0$, while $Z_S(p)$ does not show such a trend.

To understand this result, we note that chiral symmetry of KS fermion leads to the following identities:

$$Z_S(p) \cdot Z_\phi(p) = \partial M(p)/\partial m$$
$$Z_P(p) \cdot Z_\phi(p) = M(p)/m$$ (4)

with $M(p) = \text{Tr}[S(p)^{-1}]$. In Fig. 2 $M(p)$ in the chiral limit obtained by a linear extrapolation in $m$ is plotted. It rapidly dumps for large momenta, but takes large values in the small momentum region. Combined with (4) this implies that $Z_P(p)$ diverges in the chiral limit for small momenta, which is consistent with the result in Fig. 1.

The function $M(p)$ is related to chiral condensate as follows:

$$\langle \phi \bar{\phi} \rangle = \sum_p \text{Tr}[S(p)] = \sum_p \frac{M(p)}{C_\mu(p)^2 + M(p)^2}$$ (5)

where $C_\mu(p) = -i\text{Tr}[(\gamma_\mu \otimes I)S(p)^{-1}]/\cos(p_\mu a)$. A non-vanishing value of $M(p)$ for small momenta would lead to a non-zero value of the condensate. Therefore the divergence of $Z_P(p)$ near the chiral limit is a manifestation of spontaneous symmetry breakdown of chiral symmetry; it is a non-perturbative effect arising from the presence of massless Goldstone boson.

We expect this non-perturbative effect to affect the scalar renormalization factor $Z_S(p)$ much less,
since the scalar operator can not interact directly with the pseudoscalar meson. Indeed the quark mass dependence is quite small as we have seen in Fig. 1.

In Fig. 3 we show the momentum dependence of the ratio \( m^{MS}(\mu) / m = U^{MS}(\mu, p) Z^{MS}(p) Z_S(p) \) calculated in the chiral limit where we set \( \mu = 2\text{GeV} \) and use the three-loop formula for \( U^{MS} \) and \( Z^{MS} \). While the ratio should be independent of the quark momentum \( p \), our results show a large momentum dependence which is almost linear in \((ap)^2\) for \( 0.6 < (ap)^2 \).

A natural origin of the linear dependence on \((ap)^2\) is the lattice discretization error of the scalar operator, which differs by terms of \( O(a^2) \) from that of the continuum for the KS fermion. We then remove this error from the renormalization factor by a linear extrapolation in \((ap)^2\) to \((ap)^2 = 0\). In Fig. 4 the fitting lines are plotted, where filled data points are used for the linear extrapolation. For comparison, the ratio calculated with the one-loop value equals 1.867, 1.877, and 1.871 for \( \beta = 6.0, 6.2 \) and 6.4 at \( q^* = 1/a \). Hence one-loop perturbation theory underestimates the ratio by 40% to 20%.

Our final results for the averaged up and down quark mass at \( \mu = 2\text{GeV} \) are shown in Fig. 4 by filled symbols. Here we use the JLQCD results for bare quark mass \( \beta \). The values are substantially larger than those obtained with one-loop perturbation theory (open circles for \( q^* = 1/a \) and squares for \( q^* = \pi/a \)). Furthermore they exhibit a significant \( a^2 \) dependence, which we ascribe to the discretization error of the quark mass itself. Making a linear extrapolation in \( a^2 \), our final result in the continuum limit is given by

\[
m^{MS}(2\text{GeV}) = 4.15(27)\text{MeV}.
\]

This value is 20% larger than the perturbative estimates : 3.51(20)MeV for \( q^* = 1/a \) and 3.40(21)MeV for \( q^* = \pi/a \).

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