Heavy-light spectrum and decay constant from NRQCD with two flavors of dynamical quarks

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We report on a study of $B$ mesons on $N_f = 2$ full QCD configurations using an RG-improved gauge action, NRQCD heavy quark action and tadpole-improved clover light quark action. Results on the heavy-light spectrum and the decay constants from $16^3 \times 32$ lattices at $\kappa^{-1} \approx 1.5$ GeV are presented, and compared with quenched results obtained with the same action combination at matching lattice spacings.

1. Introduction

The decay constant $f_B$ is being studied extensively on the lattice because of its importance for the determination of CKM matrix elements. The spectrum of excited $B$ mesons and $b$ baryons is being measured in present experiments, whereas there exist only few lattice results on this subject.

In this article we report on our study of $B$ mesons in two-flavor full QCD employing the NRQCD action for heavy quark and a tadpole-improved clover action for light quark. The dynamical configurations have been generated using the same light quark action and an RG-improved gauge action with a plaquette and a rectangular term. Details on our full QCD configurations can be found in Refs. [1]. A parallel study of $B$ mesons using the clover action for heavy quark is presented in Ref. [5].

2. Simulation Details

We present results for two sets of dynamical lattices corresponding to the heaviest and the lightest sea quark in our configuration set at $\beta = 1.95$. The results are compared to those from quenched lattices generated with the same RG-improved gauge action at $\beta = 2.187$, the lattice spacing from the string tension matched to the dynamical lattice with $\kappa_{sea} = 0.1375$. Some details on these runs are given in Table [1].

We take 5 $\kappa$ values for the light valence quark corresponding to $m_{PS}/m_{V} \approx 0.8 - 0.5$. The strange quark mass $m_s$ is fixed using the $K$ and the $\phi$ meson. Our results for the $B_s$ meson are obtained with $m_s$ from the $K$, and the $\phi$ is used to estimate the systematic error.

### Table 1
Parameters of lattices. The statistics for the dynamical lattices has been increased since Lattice’99. The scale is fixed by $\sqrt{\sigma} = 427$ MeV (for each sea quark for dynamical configurations)

<table>
<thead>
<tr>
<th>$\kappa_{sea}$</th>
<th>0.1375</th>
<th>0.1410</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{PS}/m_{V}$</td>
<td>0.8048(9)</td>
<td>0.586(3)</td>
<td></td>
</tr>
<tr>
<td>$a^{-1}_{\sigma}$ [GeV]</td>
<td>0.937(6)</td>
<td>1.127(10)</td>
<td>0.919(7)</td>
</tr>
<tr>
<td>#conf.</td>
<td>648</td>
<td>490</td>
<td>195</td>
</tr>
</tbody>
</table>
Table 2
Results for decay constants. Errors given in this table are statistical (including the statistical uncertainty in $M_b$), and, where applicable, the uncertainty in fixing the strange quark mass. Other systematic errors are discussed in the text.

<table>
<thead>
<tr>
<th>$\kappa_{\text{sea}}$</th>
<th>$f_B$ [MeV]</th>
<th>$f_{B_s}$ [MeV]</th>
<th>$f_{B_s}/f_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>193(4)</td>
<td>221(4)(+7)</td>
<td>1.147(10)(35)</td>
</tr>
<tr>
<td>0.1375</td>
<td>216(4)</td>
<td>250(4)(+8)</td>
<td>1.157(9)(+35)</td>
</tr>
<tr>
<td>0.1410</td>
<td>215(6)</td>
<td>251(6)(+6)</td>
<td>1.166(14)(+31)</td>
</tr>
</tbody>
</table>

For the heavy quarks, we use NRQCD at $O(1/M)$ with a symmetric evolution equation as defined in [2]. We employ 5 bare heavy quark masses, covering a range of roughly 2.5–4.5 GeV.

The heavy-light meson mass $M$ is determined from the difference of the meson energy at finite momentum and at rest, assuming the dispersion relation, $E(p) - E(0) = \sqrt{p^2 + M^2} - M$. As a consistency check, we use both the $B_d$ and the $B_s$ meson to determine the $b$ quark mass.

In our calculation of decay constants, the heavy-light current is corrected through $O(\alpha/M)$. The mixing coefficients between the lattice operators [2] contributing at this order to the time component of the axial vector current $J_4$, and the matching factor to the continuum current has been calculated [3] in one-loop perturbation theory,

$$J_4 = (1 + \alpha \rho_0)J_{4,\text{lat}}^{(0)} + (1 + \alpha \rho_1)J_{4,\text{lat}}^{(1)} + \alpha \rho_2 J_{4,\text{lat}}^{(2)}.$$  \hspace{1cm} (1)

For the RG-improved gluon action, $\alpha_V$ has not been calculated, and we use a tadpole-improved one-loop expression for the $\overline{MS}$ coupling, $\alpha_{\overline{MS}}(1/\alpha)$.

3. Decay Constants

Our preliminary results for $f_B, f_{B_s}$, and $f_{B_s}/f_B$ are given in Table 2, along with the statistical error and, where applicable, the uncertainty in the determination of $m_s$. Additional systematic errors are estimated as follows: $O(\alpha^2)$ corrections, taken to be $\alpha^2 \times O(1)$, are 5%. A previous NRQCD calculation using the plaquette gluon action at $a^{-1} \sim 1$ GeV finds the tree level $O(1/M^2)$ corrections to be $\sim 2\%$; we estimate our error from the truncation of the $1/M$ expansion to be $\sim 4\%$. The leading discretization effects from the light quarks and gluons of $O(\alpha a \Lambda_{\text{QCD}})$ and $O(\alpha^2 \Lambda_{\text{QCD}}^2)$ are 5%. Added in quadrature, these estimates give 7%.

Our two-flavor results for $f_B$ and $f_{B_s}$ given in Table 2 show a 10% increase compared to the quenched values (see also Fig. 1). We do not resolve any sea quark mass dependence. The dependence on the value of $\alpha_s$ is weaker than for the plaquette gauge action, and the difference between renormalized and bare decay constants is only about 5%.

In Fig. 1 we show the one-loop corrections to

Figure 1. $\Phi \equiv (\alpha_s(M)/\alpha_s(M_B))^{(2/\alpha_0)} f_\sqrt{M}$ (top), and one-loop corrections to $f_\sqrt{M}$ (bottom) as a function of the inverse pseudoscalar meson mass. In the upper plot, squares stand for $\kappa_{\text{sea}} = 0.1375$, diamonds for $\kappa_{\text{sea}} = 0.1410$ and fancy squares for quenched. In the lower plot, circles denote $\alpha \rho_0 J_{4,\text{lat}}^{(0)}/J_4$, squares, $\alpha \rho_1 J_{4,\text{lat}}^{(1)}/J_4$, and diamonds, $\alpha \rho_2 J_{4,\text{lat}}^{(2)}/J_4$. 


the current $J_4$ as a function of the heavy-light meson mass. In the $B$ region, $1/M \sim 0.2$, we find the correction to $J_{4, \text{lat}}^{(1)}$ to be very small and the two other terms to contribute about the same amount. The $J_{4, \text{lat}}^{(2)}$ contribution also contains a discretization correction to the current first pointed out in [2]. We note that this discretization correction is considerably smaller for the RG gauge action than for the plaquette gauge action [3].

For $f_{B}/f_{B}$, we cannot resolve a difference between the three lattices.

In a parallel study of B mesons using clover heavy quarks [5], we have obtained $f_{B}$ and $f_{B}$, taking the chiral limit for sea quark at $\beta = 1.8, 1.95$ and $2.1$. The results from that study at $\beta = 1.95$ agree within the estimated errors with the present results from NRQCD.

4. Spectrum

In Fig. 2, we give our results for several $B$ splittings from the lattices with $n_f = 0$ and $n_f = 2, \kappa_{\text{sea}} = 0.1375$. The top part of the figure shows the $B^* - B$ splitting. At present, we cannot resolve any unquenching effects. For quarkonia, on the same lattices, the hyperfine splitting is found to increase from the quenched value only by a few MeV [6]. We find the $B^* - B$ splitting to be $\sim 30\%$ smaller than the experimental value. Possible sources of systematic error are the finiteness of the sea quark mass, the $O(\alpha)$ correction to the coefficient of the $\sigma \cdot B$ operator, and higher order relativistic corrections.

In the middle part of Figure 2, we show results for the $B^*_2 - B$ splitting, and in the lower part, the spin-averaged $\Lambda_b - \bar{B}$ splitting. We do not find significant unquenching effects. However, for definite conclusions, we need to study several sea quark masses and lattice spacings, which is in progress.

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