Hadron matrix elements for nucleon decay with the Wilson quark action

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We report preliminary results of our study of matrix elements of baryon number violating operators which appear in the low-energy effective Lagrangian of (SUSY-)Grand Unified Theories. The calculation is performed on a $32^3 \times 80$ lattice at $\beta = 6.1$ using Wilson fermions in the quenched approximation. Our calculation is independent of details of (SUSY-)GUT models and covers all interesting decay modes.

1. Introduction

Proton decay is one of the most exciting predictions of Grand Unified Theories (GUTs). Experimental effort over the years has pushed the lower limit on the partial lifetimes to $\tau / B_{p \rightarrow \pi^0 e^+} > 5.5 \times 10^{32}$ years and $\tau / B_{p \rightarrow K^+ \tau} > 1.0 \times 10^{32}$ years at the 90% confidence level\cite{1}, and further improvement is expected from the SuperKamiokande experiment.

A crucial link to relate these numbers to constraints on (SUSY-)GUT models is the values of hadron matrix elements relevant for proton decay. Model calculations suffer from high degree of uncertainty, various estimations easily differing by a factor ten.

Pioneering lattice QCD studies to remove this source of uncertainty were made about ten years ago, first combining calculations of the matrix element $\langle 0 | \mathcal{O}^B | p \rangle$ and soft-pion theorems to estimate $\langle \pi^0 | \mathcal{O}^B | p \rangle$\cite{2,3}, and subsequently directly evaluating the matrix element $\langle \pi^0 | \mathcal{O}^B | p \rangle$ itself\cite{4}.

In this article we report preliminary results of our renewed effort to determine the matrix elements from first principles of QCD. In addition to the use of larger lattice sizes and higher statistics to achieve a much better precision, which is made possible through the increase of computing power, we aim to advance the calculation on two fronts: (i) calculation of $p \rightarrow K$ matrix elements relevant for SUSY-GUT as well as those for the $p \rightarrow \pi$ mode, for physical values of $u$-$d$ and $s$ quark masses and for physical momenta, and (ii) evaluation of matrix elements of all dimension-6 baryon number violating operators classified according to $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariance\cite{5,6}, so as to cover various GUT models and decay processes.

2. Calculational procedure

Our calculation is carried out in quenched QCD at $\beta = 6.1$ with the Wilson quark action on a $32^3 \times 80$ lattice. We analyze 100 configurations for the hopping parameter $K = 0.15428, 0.15381, 0.15333, 0.15287$. The lattice scale fixed by $m_\rho = 769$MeV in the chiral limit ($K_c = 0.15499(2)$) equals $a^{-1} = 2.56(4)$GeV, and the point for strange quark estimated from $m_K/m_\rho = 0.648$ is given by $K = 0.15304(5)$. All errors are estimated by the single elimination jackknife procedure.

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at the scale \( \mu \rightarrow 0 \) of pseudo scalar meson as a function of \( u-d \) and \( s \) quark mass in lattice units. \( u-d \) quark mass is taken to the chiral limit for \( A_K \).

To calculate the nucleon \( (N) \) to pseudo scalar (PS) meson matrix element of a baryon number violating operator \( O^B \), we form the ratio,

\[
\langle PS|O^B|N \rangle = \frac{\langle 0|J_{PS}O^B J_N|0 \rangle}{\langle 0|J_N J_N|0 \rangle \langle 0|J_{PS} J_{PS}|0 \rangle} \times \langle 0|J_{PS}|PS \rangle \langle N|J_N|0 \rangle ,
\]

where \( \langle 0|J_{PS}|PS \rangle \) and \( \langle N|J_N|0 \rangle \) are extracted from local-local hadron propagators. We fix the nucleon source at \( t=0 \), PS sink at \( t=32 \) and move the operator between them. Matrix elements are evaluated for four spatial momenta \( \vec{p}_a=(0,0,0), (1,0,0), (0,1,0), (0,0,1) \) in units of minimum momentum \( a_{min} = \pi/16 \) injected in the PS meson.

We distinguish \( u-d \) and \( s \) quark masses; the former is taken to the chiral limit, and the latter interpolated to the physical \( s \) quark mass in our calculations. After this procedure, we interpolate the spatial momentum to the physical value.

Matrix elements are renormalized, with mixing included, by tadpole-improved one-loop renormalization factors to the \( \overline{MS} \) scheme\cite{7} calculated at the scale \( \mu = 1/a \).

Figure 1. Matrix element relevant for \( p \rightarrow \pi^0 \) and \( p \rightarrow K^0 \) decay normalized by measured proton mass at \( \vec{p} = 0 \) of pseudo scalar meson as a function of \( u-d \) and \( s \) quark mass in lattice units.

3. Results

Let us define the \( p \rightarrow \pi^0 \) and \( p \rightarrow K^0 \) matrix elements \( A_\pi \) and \( A_K \) by

\[
\langle \pi^0(K^0)|\epsilon_{ijk}(u_j^* C d_i^* |s_L^i) u_R^k|p \rangle \equiv A_{\pi(K)} N_R. \tag{2}
\]

In Fig. 1 we show our results for these matrix elements at zero momentum \( \vec{p} = 0 \), normalized by proton mass measured for relevant values of \( u-d \) quark mass. For \( A_\pi \), abscissa represents the \( u-d \) quark mass, while it represents the \( s \) quark mass for \( A_K \), the \( u-d \) quark mass having been taken to the chiral limit.

Compared to a first calculation of the \( A_\pi \) matrix element\cite{4}, whose results are consistent with ours, our statistical errors of \( 4-8\% \) are improved by about a factor five. This allows us to observe that the amplitude exhibits a clear decrease of about \( 40\% \) from the region of \( s \) quark \( \langle m_u, d |a \rangle \approx 0.04 \) to the chiral limit, which was not apparent in results of Ref. \cite{4}.

For the \( p \rightarrow K^0 \) matrix elements \( A_K \), after chiral extrapolation of \( u-d \) quark mass, the dependence on the \( s \) quark mass is small. The point plotted with a cross shows the value interpolated to the physical \( s \) quark mass.

In Fig. 2 we plot \( A_\pi \) and \( A_K \) as a function of squared momentum after quark masses are taken to the physical values. We observe a quite signifi-
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