Study of the effective field theory for the model with light and heavy scalars

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2022

Abstract

The standard model (SM) for the elementary particles is known as a very successful theory. However the extention of the SM is necessary to give the non-zero mass for the neutrino. By introducing the right-handed neutrino, the small mass of the neutrino could be explained with the tiny Yukawa couplings between the neutrino and Higgs.

In this work, we propose a model in which the tiny Yukawa coupling is naturally generated. The model consists of a light scalar, a heavy scalar and a neutrino. A light scalar corresponds to the SM-like Higgs and it does not directly couple to the neutrino. While, the heavy scalar couples to the neutrino and mixes with the light scalar. The tiny Yukawa coupling to the light scalar is generated by integrating the heavy scalar out. The Yukawa coupling is suppressed by the ratio of the mixing mass to the heavy scalar mass.

We derive the low energy effective potential for this model including the one-loop corrections by performing the path integration for both heavy and light scalars. The heavy scalar generates the correction to the light scalar mass squared and it is proportional to the heavy scalar mass squared. The light scalar generates the large logarithm of the ratio of the two scalar masses and it is resummed with the renormalization group (RG). Then the RG improved effective potential is derived.

We study the RG improved potential and its stationary point, i.e., the vacuum expectation value (VEV). We numerically study how the potential and the VEV depend on the heavy scalar mass and the mixing mass. By requiring the correction to the VEV remains within 20 % compared to its tree level value, the upper limit for the heavy scalar mass is obtained. Combined with the suppression factor of the Yukawa coupling for the neutrino, one can also obtain the limit for the mixing mass.

Contents

Chapter 1

Introduction

1.1 Motivation and background

Since the observation of neutrino oscillations in Kamiokande, it has become clear that neutrinos have a small mass[1]. According to the ground experiments [2, 3] and the cosmological experiments [4, 5], the mass of neutrinos must be less than the eV scale. Therefore if the same Higgs mechanism gives mass to the top quark and neutrino, the difference of the Yukawa couplings for these particles becomes twelve orders of the magnitude. This means that the standard model (SM) is not perfect, and we need a beyond standard model (BSM) to explain the small mass of neutrinos. One of the simple SM extensions is two Higgs doublet model (2HDM) which adds one Higgs boson. There are the models in the 2HDM which can explain the smallness of the mass of neutrino. In these models, the second Higgs doublet has the much smaller vacuum expectation value (VEV). The scalar corresponding to the SM-like Higgs generates masses to charged fermions, while the second Higgs has a very small VEV and gives small mass to the neutrino with O(1) Yukawa couplings. A model with the small Dirac nuetrino mass is proposed by S. M. Davidson and H.E. Logan in Ref.[6]. In this model, they impose the grobal $U(1)$ symmetry to prevent the SM-like Higgs boson from coupling with neutrinos directly. In the other models, the Z_2 symmetry is adopted for the same purpose. Such models are proposed by S.Gabriel and S.Nandi in Ref.[7], N.Haba and K.Tsumura in Ref.[8]. The constraint on these models from electroweak precision data is studied in Ref.[9].

If the second Higgs in the above theories is discovered, it will be an important contribution to the explanation of neutrinos properties and verification of BSM. There are two ways to observe the second Higgs (the new particle). One is the direct search and the other is the indirect search. In the direct searches, if the energy of the accelerator reaches the threshold for the new particle production, the new particles will be directly produced and discovered. Though the energy of the accelerator has reached several TeV, there is no report of the observation of the second Higgs. On the other hand, the indirect search looks for the second Higgs by verifying the deviation from the SM. With this method, the second Higgs may be verifiable in current and near future experiments. The effective field theory is known as a method of studying from this point of view. In the effective field theory, we construct the theory without the new particles which exisit in the BSM. The effect of the new particles such as the second Higgs is included by integrating them out. The remnants of the new particles appear as deviation in the coefficients of the interactions. It can be investigated indirectly by verifying the deviation.

In this work, we propose a simple toy model with a heavy and a light scalars. We derive the low energy effective potential with the method of the effective field theory. Then we analyse the effective potential, its stationary point and the VEV.

This thesis is based on Ref.[10] and is organized as follows. In the rest of Chapter 1, we explain the idea and the example of the effective field theory. We also explain the Dirac neutrino mass model in Ref.[6]. In Chapter 2, we present the action for the model with the heavy and light scalars. In Chapter 3, the definition of the effective action for the light scalar is given. In Chapter 4, the heavy scalar and the fluctuation of the light scalar are integrated out. In Chapter 5, the counter terms and the renormalized effective potential are derived. In Chapter 6, the RG improved effective potential is obtained. In Chapter 7, the numerical analysis is discussed and Chapter 8 is devoted to summary and discussion.

1.2 Effective field theory

1.2.1 The idea of the effective field theory

In this subsection, we introduce the outline of the effective field theory (EFT). There is the wide range of the mass of particle in the nature. For example, the mass of up quark and top quark are about 2.16 MeV and 172.76 GeV $\fallingdotseq 1.73 \times 10^5$ MeV, respectively [11]. These particles have very similar properties such as having the same electric charge but their mass scales have a 5 digit difference. The theory often includes the multi-mass scales. When studying the low-energy physics of the multi-scale model, proper handling of the heavy particle field can simplify the theory and extract the essential conclusion. The EFT is often used when studying physics at low energy (energy regions below a certain energy scale Λ). In EFT, the field operator for the particle which has the higher mass than energy scale Λ is integrated out. The field of the heavy particle is removed from the theory and the theory is described by only the field of the particle with the mass below the energy scale Λ. Note that the effect of heavy particle is not ignored. By integrating out, the effect of the heavy particle appears as the radiative corrections to the mass and the coupling constant. The EFT is defined by the effective action.

Next, we explain the points when making the EFT. The main elements of making an EFT for a theory are shown below. The first step is to accurately select the dominant field in the low energy region. The dominant field in the low energy is called the relevant operator, the other field is called the irrelevant operator. Other fields are integrated, so it is the most important step in deciding what field is included in the effective theory. The second step is integrating the field of the irrelevant operator. By integrating irrelevant operators, their effect appears in the coupling constant of the relevant operator by a series expansion with terms inversely proportional to the square of the mass of the irrelevant operator. This series expansion generally contains an infinite number of terms, and handling them properly is the third step. Since these terms are suppressed by the heavy mass, they can be terminated with a finite expansion, and it is necessary to consider what order the expansion is maintained.

We only wrote the outline of the the EFT in this subcection. In next subsection, we show the weak EFT for β decay as the example. There are a lot of lectures for the EFT, the more concrete examples and explanations of the EFT are found in Refs.[12, 13].

1.2.2 Example of EFT

As for example of the EFT, we explain the weak EFT for β decay. In Fermi's theory, which describes the weak interaction, the Lagrangian of the weak interaction of the quark of the standard model is written as,

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} V_{ij} \bar{u}_i \gamma^\mu L d_j W_\mu^+ + h.c., \tag{1.1}$$

where V_{ij} is the CKM matrix elements and L is the left-handed projection operator. u_i and d_i denote the up type quark and the down type quark, respectively. W_μ denotes the W boson, g is the coupling of the interaction. The process of the β decay is given by $n \to p + e + \bar{\nu}_e$. It is also represented as $d \to u + e + \bar{\nu}_e$ at the quark level. In the left panel of Fig.1.1, we show the Feynman daiagram for beta decay at quark level. The amplitude of the β decay in the SM is given by,

$$\begin{aligned} i\mathcal{A}_{SM} &= \left[\bar{q}_u \left(-\frac{ig}{\sqrt{2}} V_{ud} \gamma^\mu L\right) q_d\right] \left(\frac{-ig_{\mu\nu}}{p^2 - M_W^2}\right) \left[\bar{l}_e \left(-\frac{ig}{\sqrt{2}} \gamma^\nu L\right) l_{\nu_e}\right] \\ &= i\frac{g^2}{2} V_{ud} \frac{1}{p^2 - M_W^2} \left[\bar{q}_u \gamma^\mu L q_d\right] \left[\bar{l}_e \gamma_\mu L l_{\nu_e}\right], \end{aligned} \tag{1.2}$$

where q_α with $\alpha = u, d$ and l_β with $\beta = e, \nu_e$ denote the Dirac spinors correspond to the quarks and leptons, respectively. Then M_W is the mass of the W boson, p is the momentum of the internal W boson. The typical scale of the momentum transfer is $p^2 \sim (m_N - m_p)^2 \sim 1.3^2$ (MeV)^2. On the other hand, the mass of the W boson M_W is the 80.376 GeV [14]. So the relation $p^2 \ll M_W^2$ is satisfied, we can expand the propagator with the Taylor expansion as,

$$\frac{1}{p^2 - M_W^2} = \frac{1}{-M_W^2}\left(1 + \frac{p^2}{M_W^2} + \cdots\right). \tag{1.3}$$

Therefore the amplitude of Eq.(1.2) can be expanded as,

$$\mathcal{A}_{SM} \simeq -\frac{1}{M_W^2}\frac{g^2}{2} V_{ud} \left[\bar{q}_u \gamma^\mu L q_d\right] \left[\bar{l}_e \gamma_\mu L l_{\nu_e}\right] + \mathcal{O}\left(\frac{p^2}{M_W^4}\right). \tag{1.4}$$

On the other hand, the amplitude in the weak EFT can be obtained by introducing the effective operator $O(\mu)$ as,

$$\mathcal{A}_{\text{eff}} = -C(\mu) O(\mu) \equiv -C(\mu) \left[\bar{q}_u \gamma^\mu L q_d\right] \left[\bar{l}_e \gamma_\mu L l_{\nu_e}\right], \tag{1.5}$$

where $C(\mu)$ is the coupling constant of the operator $O(\mu)$ at the energy scale μ, which is called Wilson coefficient. In order to derive the effective amplitide of the β decay, the operator $O(\mu)$ is defined by $\left[\bar{q}_u \gamma^\mu L q_d\right] \left[\bar{l}_e \gamma_\mu L l_{\nu_e}\right]$. The operator has the mass dimension 6, so the coefficient

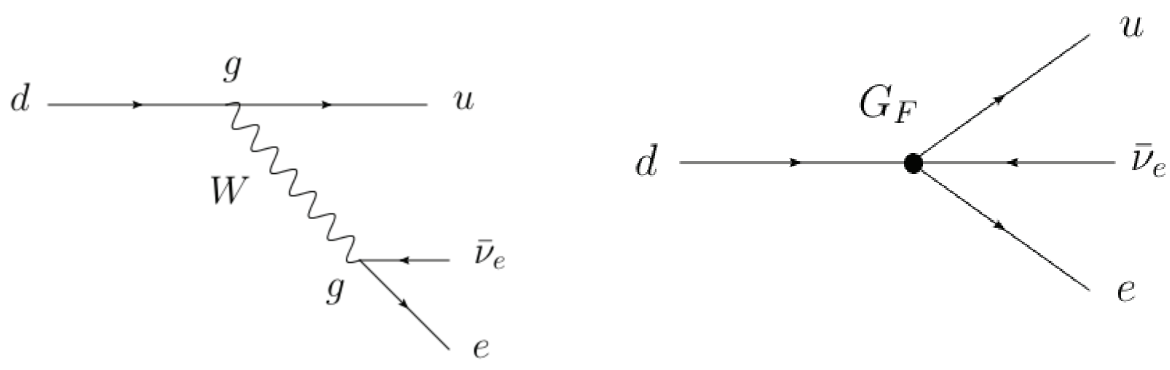

Figure 1.1: In the left panel, the Feynman daiagram for beta decay at quark level is shown. The right panel is the Feynman in the weak EFT in which the W boson is integrated out.

has the mass dimmension -2. The Wilson coefficient can be determined by considering the condition of the amplitude in the weak EFT of Eq.(1.5) equals to that in the SM of Eq.(1.4). We impose the matching condition $\mathcal{A}_{\text{eff}} = \mathcal{A}_{SM}$ at the scale of integrating W boson field out. This scale is the $\mu_{EW} \simeq M_W$. Therefore, the Wilson coefficient is obtained as,

$$C(\mu_{EW}) = \frac{g^2}{2M_W^2} V_{ud} = \frac{4G_F}{\sqrt{2}} V_{ud}, \tag{1.6}$$

where $G_F = \frac{g^2}{4\sqrt{2}M_W^2} = \frac{1}{\sqrt{2}v^2}$ is the Fermi constant. The Wilson coefficient at any scale μ can be obtained by solving the renormalization group (RG) equation, but for the sake of simplicity we ignore the RG effect. In the right panel of Fig.1.1, we show the Feynman diagram in the weak EFT in which the W boson is integrated out.

From Eq.(1.5), we can find out that the amplitude contains only the degrees of freedom which appear in the initial and final state of the β decay. The amplitude do not include W bosons as dynamical degrees of freedom, but it includes the information of W boson's mass M_W and the weak coupling constant g in the form of the Wilson coefficient. The measurement of the β decay constrains the Wilson coefficient. The value of $V_{ud}G_F$ can be determined from the constraints of the Wilson coefficient, taking into account the relationship in Eq.(1.6).

1.3 Dirac neutrino mass model of Davidson and Lorgan

We explain the Dirac neutrino mass model which is proposed by Davidson and Lorgan [6]. The most straightforward way to incorporate the Dirac neutrino mass into the SM is to introduce three right-handed neutrinos ν_R. They couple with SM Higgs doublet Φ_1 as well as the SM quarks and cgarged leptons. The Yukawa Lagrangian is given by,

$$\mathcal{L}_{\text{Yuk}} = -y^d_{ij}\bar{d}_{R_i}\Phi_1^\dagger Q_{L_j} - y^u_{ij}\bar{u}_{R_i}\tilde{\Phi}_1^\dagger Q_{L_j} - y^l_{ij}\bar{e}_{R_i}\Phi_1^\dagger L_{L_j} - y^\nu_{ij}\bar{\nu}_{R_i}\tilde{\Phi}_1^\dagger L_{L_j} + h.c., \tag{1.7}$$

where $Q_L \equiv (u_L, d_L)^T$ and $L_L \equiv (\nu_L, e_L)^T$correspond to the left-handed quark and lepton doublets, respectively. The indices i, j represent the generations. $y^\alpha_{ij}(\alpha = u, d, l, \nu)$ represents each Yukawa coupling which is the 3×3 matrix. In this framework, the Dirac neutrino mass m^ν_{ij} is given as follows,

$$m^\nu_{ij} = \frac{y^\nu_{ij} v}{\sqrt{2}}, \tag{1.8}$$

where v is the vacuum expectation value (VEV) of the SM Higgs boson. The realistic neutrino mass is below eV order [11]. This fact requires the Yukawa couplings $|y^\nu_{ij}| \lesssim 10^{-11}$.

In the model in Ref.[6], the new scalar doublet Φ_2 is introduced. This doublet has the same quantum number as the SM Higgs doublet. They also impose a global $U(1)$ symmetry. Under this symmetry, the new scalar Φ_2 and ν_{Ri} carry charge +1, while other SM fields are uncharged. Therefore, only the Φ_2 couples to the right-handed neutrinos ν_{Ri}. Then the neutrino term in Eq.(1.7) is replaced as,

$$-y^\nu_{ij}\bar{\nu}_{R_i}\tilde{\Phi}_1^\dagger L_{L_j} \to -y^\nu_{ij}\bar{\nu}_{R_i}\tilde{\Phi}_2^\dagger L_{L_j}. \tag{1.9}$$

If the $U(1)$ symmetry is unbroken, the VEV of Φ_2 becomes zero and the neutrinos are strictly massless [15]. In order to generate the small Dirac neutrino mass without supposing tiny Yukawa coupling y^ν_{ij}, the Φ_2 must have a small VEV. To get the small VEV of Φ_2, they explicitly break the global $U(1)$ symmetry by a term with mass dimension two. This term has the form of $m^2_{12}\Phi_1^\dagger\Phi_2$. The terms with mass dimension four are invariant under the symmetry. The symmetry that is preserved for the terms with the mass dimension four and is broken for the terms with the mass dimension less than four is called the softly broken symmetry. The Higgs potential with the softly broken term is obtained as,

$$\begin{aligned} V &= m^2_{11}\Phi_1^\dagger\Phi_1 + m^2_{22}\Phi_2^\dagger\Phi_2 - \left[m^2_{12}\Phi_1^\dagger\Phi_2 + h.c.\right] + \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 \\ &\quad +\lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right). \end{aligned} \tag{1.10}$$

$\lambda_1, \lambda_2 > 0$, $\lambda_3 > -\sqrt{\lambda_1\lambda_2}$ and $\lambda_4 > -\sqrt{\lambda_1\lambda_2} - \lambda_3$ are required for the stability of the potential at large field values. Non-zero VEV for Φ_1 is generated with the usual spontaneous symmetry breaking mechanism achieved when $m^2_{11} < 0$. In order to avoid a very light pseudo-Nambu-Goldstone boson, we consider the case that the mass term m^2_{22} for Φ_2 is positive. Substituting the VEV v_1 and v_2 for Φ_1 and Φ_2, we find the values of the VEVs with Higgs potential parameters by applying the stationary condition as follows,

$$\left.\frac{\partial V}{\partial|\Phi_1|}\right|_{\min} = m_{11}^2 v_1 - m_{12}^2 v_2 + \frac{1}{2}\lambda_1 v_1^3 + \frac{1}{2}(\lambda_3+\lambda_4)v_1 v_2^2 = 0, \tag{1.11}$$

$$\left.\frac{\partial V}{\partial|\Phi_2|}\right|_{\min} = m_{22}^2 v_2 - m_{12}^2 v_1 + \frac{1}{2}\lambda_2 v_2^3 + \frac{1}{2}(\lambda_3+\lambda_4)v_1^2 v_2 = 0. \tag{1.12}$$

In this discussion, we require the $m_{12}^2 \ll v_1^2$, $v_2 \ll v_1$. Therefore, we can ignore m_{12}^2 and v_2 when solving Eq.(1.11), then we obtain v_1 as,

$$v_1^2 = \frac{-2m_{11}^2}{\lambda_1}. \tag{1.13}$$

Concerning v_2, we have to keep m_{12}^2. Then v_2 is obtain from Eq.(1.12) as follows,

$$v_2 = \frac{m_{12}^2 v_1}{m_{22}^2 + \frac{1}{2}(\lambda_3+\lambda_4)v_1^2}, \tag{1.14}$$

where the term proportional to v_2^3 is ignored compared to terms proportional to $v_1^2 v_2$. The ratio v_2/v_1 becomes small naturally. As the result, the neutrino mass is explained without tiny Yukawa coupling.

Chapter 2

Model

2.1 The small neutrino mass from the EFT

In this subsection, we will explain the mechanism for the small neutrino mass using the EFT. In subsection 1.3, we explained the model which leads to the small Dirac neutrino mass by introducing the additional second Higgs. Currently, there are no reports on the discovery of the second Higgs. This indicates that the second Higgs is heavier than the energy scale to which the current collider experiments can reach. Therefore, in this research, we will study a model in the veiw of the EFT. Our model is not built on gauge theories. We consider a model with one light scalar, one heavy scalar and the neutrino. The light scalar correponds to the SM like Higgs, while the heavy scalar corresponds to the second Higgs in Ref.[6].

In the view of the EFT, we will discuss how the small mass of Dirac neutrinos is explained. Similar to Eq.(1.9), the neutrino mass term is given in the following form : $y\overline{n_R}n_L\rho_2$. ρ_2 represents the heavy scalar, n_R (n_L) represents the right-handed (left-handed) neutrino. y is the Yukawa coupling of the neutrino. By introducing the mixing mass term which has the form as $m_{12}^2\rho_1\rho_2$, the neutrino can couple to ρ_1 through the mixing mass term as follows,

$$y\overline{n_R}n_L\rho_2 \to y\overline{n_R}n_L\left(\frac{m_{12}^2}{m_2^2-k_2^2}\right)\rho_1, \tag{2.1}$$

where ρ_1 represents the light scalar. m_2 and k_2 are the mass and momentum of the heavy scalar, respectively. In Fig.2.1, we draw the Feynman diagram of the right-side of Eq.(2.1). We consider the heavy scalar such that $m_2^2 \gg k_2^2, m_{12}^2$ is satisfied. In the limit, we obtain the following result.

$$\left(y\frac{m_{12}^2}{m_2^2}\right)\overline{n_R}n_L\rho_1 \tag{2.2}$$

In Fig.2.2, we draw the Feynman diagram of Eq.(2.2). From Eq.(2.2), the effective Yukawa coupling of the neutrino is suppressed as $y\frac{m_{12}^2}{m_2^2}$. It naturally leads to the small neutrino mass.

In this thesis, we will build a model which includes the above mechanism. Then we integrate the heavy scalar out to derive the EFT. We will also study the relation between the parameters of the full theory and those of the EFT.

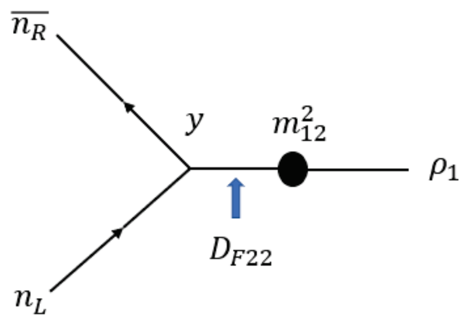

Figure 2.1: The Feynman diagram of the right-side of Eq.(2.1). D_{F22} represents the propagator of the second Higgs.

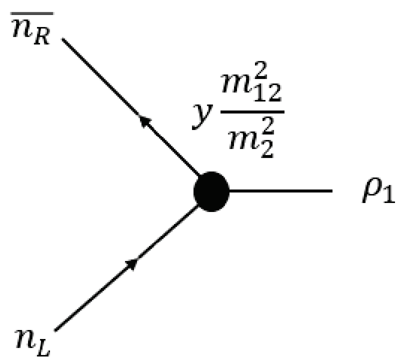

Figure 2.2: The Feynman diagram of Eq.(2.2).

2.2 Action for the model

As mentioned in the previous subsection, we study a simple toy model which leads to a tiny Yukawa coupling to neutrinos with the light scalar in the view of the effective theory. The model consists of two scalars, neutrino and cosmological constants. We denote the light scalar as ρ_1, the heavy scalar as ρ_2 and the neutrino as n. The action is written in terms of bare fields ρ_{0i}, bare masses m_{0i} and bare couplings λ_{0i} as,

$$\begin{aligned} S &= \int d^dx \Big[-\frac{1}{2}\sum_{i=1}^{2} \rho_{0i}\Big(\Box + m_{0i}^2 + \frac{\lambda_{0i}}{2}\rho_{0i}^2\Big)\rho_{0i} - \frac{\lambda_{03}}{4}\rho_{01}^2\rho_{02}^2 - (y_0\bar{n}_0 n_0 + m_{012}^2\rho_{01})\rho_{02} \\ &\quad + \sum_{i=1}^{2} h_{0i}m_{0i}^4 + h_{012}m_{012}^4 + 2h_{03}m_{01}^2m_{02}^2\Big], \end{aligned} \tag{2.3}$$

where m_{012}^2 is the bare mixing mass. y_0 is the Yukawa coupling between the neutrino and the heavy scalar. The cosmological constants of this model are written in terms of the mass parameters of the model [16]. In this work, we do not consider the kinetic term of the neutrino. The quantum correction of the neutrino is also ignored.

This model has the following features.

- The model is renormalizable.
- The neutrino n does not couple to the light scalar ρ_{01} directly, while it couples to the second scalar ρ_{02} directly.
- The neutrino n couples to the light scalar indirectly because the second scalar and the light scalar have the mass mixing term.
- The mass of the second scalar is large.

2.3 Symmetry for the model

To realize the above features, we impose two Z_2 symmetries. One is the exact symmetry called as Z_2 and another is the softly broken symmetry called as Z_2'. The charge assignment is shown in Table2.1. Under the Z_2 symmetry, two scalar fields ρ_1, ρ_2 and right-handed neutrino n_R have the odd parity and the left-handed neutrino n_L has the even parity. Under the softly broken Z_2', only the light scalar field ρ_1 has odd parity. Only the heavy scalar ρ_2 has the Dirac type Yukawa coupling to the neutrino with this assignment.

In the scalar potentials, the cubic interactions of scalars are prohibited by Z_2 symmetry. For the quartic couplings of the scalar field, the number of each scalar field must be even due to Z_2' symmetry. For the quadratic part, the mixing term proportional to ρ_1 and ρ_2 is allowed. This term softly breaks Z_2' symmetry. About the Yukawa interaction between neutrinos and scalar fields, only the Dirac type Yukawa coupling of the form $\rho_2\overline{n_L}n_R$ are allowed. All the other Yukawa couplings, such as $\rho_1\overline{n_L}n_R$, $\rho_i(n_L)^c n_L$ and $\rho_i(n_R)^c n_R$ are forbidden by imposing both Z_2 and Z_2' symmetries. Concerning the neutrino mass term with the dimension three, Dirac mass term $m_\nu\overline{n_L}n_R$ is forbidden due to Z_2 symmetry. The symmetry under the transformation (n_L, n_R) into $e^{i\frac{\pi}{2}}(n_L, n_R)$ prohibits Majorana mass term such as $\overline{(n_R)^c}n_R$ and $\overline{(n_L)^c}n_L$.

Symmetry	ρ_1	ρ_2	n_L	n_R
Z_2	$-$	$-$	$+$	$-$
Z_2'	$-$	$+$	$+$	$+$

Table 2.1: The charge assignment under Z_2 and Z_2' symmetries.

2.4 Renormalized quantities and fields

In this subsection, we rewrite the action of Eq.(2.3) in terms of the renormalized fields, renormalized couplings and masses. The relations between the bare quantities and the renormalized ones are given as follows,

$$\rho_{0i} = \sqrt{Z_i}\rho_i, \tag{2.4}$$

$$n_0 = \sqrt{Z_n}n, \tag{2.5}$$

$$m_{0i}^2 Z_i = \sum_{j=1}^{2} Z_{mij} m_j^2, \tag{2.6}$$

$$m_{012}^2\sqrt{Z_1 Z_2} = m_{12}^2 Z_{12}, \tag{2.7}$$

$$\lambda_{0i} Z_i^2 = \sum_{I=1}^{3} Z_{\lambda_{iI}} \lambda_I \mu^{2\eta}, \tag{2.8}$$

$$\lambda_{03} Z_1 Z_2 = \sum_{I=1}^{3} Z_{\lambda_{3I}} \lambda_I \mu^{2\eta}, \tag{2.9}$$

$$y_0 Z_n \sqrt{Z_2} = Z_y y \mu^{\eta}, \tag{2.10}$$

where the index i $(i = 1, 2)$ is not summed. μ denotes the renormalization scale and η is $2 - \frac{d}{2}$. Concerning the cosmological constants, the relations between the renormalized parameters and the bare ones are given as follows,

$$\sum_{i=1}^{2} h_{0i} m_{0i}^4 + 2h_{03} m_{01}^2 m_{02}^2 = \mu^{-2\eta}\left(\sum_{i=1}^{2} Z_{hi} h_i {m_i}^4 + 2Z_{h3} h_3 m_1^2 m_2^2\right), \tag{2.11}$$

$$h_{012} m_{012}^4 = \mu^{-2\eta} Z_{h12} h_{12} m_{12}^4. \tag{2.12}$$

Using Eqs.(2.4)-(2.12), the action of Eq.(2.3) can be written in terms of the renormalized quantities as follows,

$$
\begin{aligned}
S[\rho_1, \rho_2, n] \quad = \quad & -\frac{1}{2}\int d^d x \sum_{i=1}^{2}\left(Z_i \rho_i \Box \rho_i + \rho_i^2 Z_{mij} m_j^2 + \frac{\mu^{2\eta}}{2}\sum_{I=1}^{3}\left(\rho_i^4 Z_{\lambda_{iI}} \lambda_I + \rho_1^2 \rho_2^2 Z_{\lambda_3 I} \lambda_I\right)\right) \\
& - \int d^d x \left(Z_y y \mu^{\eta} \bar{n} n + Z_{12} m_{12}^2 \rho_1 \right) \rho_2 \\
& + \int d^d x \mu^{-2\eta} \left(\sum_{i=1}^{2} Z_{hi} h_i {m_i}^4 + 2 Z_{h3} h_3 m_1^2 m_2^2 + Z_{h12} h_{12} m_{12}^4 \right). \qquad (2.13)
\end{aligned}
$$

Chapter 3

The definition of the effective action

In this section, we give the definition of the effective action for the light scalar ρ_1. For this purpose, we integrate the heavy scalar ρ_2 and quantum fluctuation of the light scalar. To begin with, we define the generating functional $W[J_1, n]$ by

$$e^{iW[J_1,n]} = \int d\rho_1 \int d\Delta_2 e^{iS[\rho_1,\Delta_2,n]+i\int \rho_1 J_1 d^4x}, \tag{3.1}$$

where we have introduced the source term J_1 for the light scalar field ρ_1. We do not introduce any source term for the heavy scalar. Instead, we integrate it by setting $\rho_2 = \Delta_2$ in the action S of Eq.(3.1). In other words, we expand the heavy scalar field around the vanishing VEV. $\bar{\rho}_1$ which is the expectation value of ρ_1 is defined by,

$$\bar{\rho}_1|_{J_1} = \frac{\delta W[J_1, n]}{\delta J_1} = \frac{\int d\rho_1 \int d\Delta_2 \rho_1 e^{iS[\rho_1,\Delta_2,n]+i\int \rho_1 J_1 d^4x}}{\int d\rho_1 \int d\Delta_2 e^{iS[\rho_1,\Delta_2]+i\int \rho_1 J_1 d^4x}}. \tag{3.2}$$

Then we can define the effective action $\Gamma_{\text{eff}}[\bar{\rho}_1, n]$ which is a functional of $\bar{\rho}_1$ with Legendre transformation of $W[J_1, n]$ as,

$$\begin{aligned} \Gamma_{\text{eff}}[\bar{\rho}_1, n] &= W[J_1, n] - \int J_1 \bar{\rho}_1 d^4x \\ &= -i \log \int d\Delta_1 \int d\Delta_2 e^{iS[\bar{\rho}_1+\Delta_1,\Delta_2,n]-i\int \Delta_1 \frac{\delta\Gamma_{\text{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)} d^4x}, \end{aligned} \tag{3.3}$$

$$\Delta_1 \equiv \rho_1 - \bar{\rho}_1, \tag{3.4}$$

where the relation $J_1(x) = -\frac{\delta\Gamma_{\text{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}$ is used in the transformation from the first line to the second line. In Eq.(3.3), the variable of the path integral was changed from ρ_1 to Δ_1 defined by Eq.(3.4). Δ_1 is the quantum fluctuation from the expectation value $\bar{\rho}_1$.

The following equation is obtained by functional derivative of Eq.(3.3) with J_1.

$$\frac{\int d\Delta_1 \Delta_1 \int d\Delta_2 e^{iS[\bar{\rho}_1+\Delta_1,\Delta_2,n]-i\int \Delta_1 \frac{\delta\Gamma_{\text{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}d^4x}}{\int d\Delta_1 \int d\Delta_2 e^{iS[\bar{\rho}_1+\Delta_1,\Delta_2,n]-i\int \Delta_1 \frac{\delta\Gamma_{\text{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}d^4x}} = 0. \tag{3.5}$$

From this equation, we can show the tadpole vanishing condition for Δ_1 as follows.

$$\int d\Delta_1 \Delta_1 \int d\Delta_2 e^{iS[\bar{\rho}_1+\Delta_1,\Delta_2,n]-i\int \Delta_1 \frac{\delta\Gamma_{\text{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}d^4x} = 0. \tag{3.6}$$

This condition leads to the one particle irreducibility of the effective action $\Gamma_{\text{eff}}[\bar{\rho}_1, n]$. On the other hand, the one particle reducible diagrams for Δ_2 are included.

Chapter 4

Integrating the scalar fields

In this section, we will integrate the light and heavy scalar field out. We assume the following hierarchy for the mass parameters.

$$m_2^2 \gg -m_1^2 \geq m_{12}^2 > 0, \quad \epsilon = \frac{m_{12}^2}{m_2^2} \ll 1 \tag{4.1}$$

The integration is performed in two steps:

- As the first step, we integrate the heavy scalar field Δ_2 in Eq.(3.3).
- As the second step, we integrate the lighter scalar field Δ_1 which correspond to the quantum fluctuation around the background field $\bar{\rho}_1$.

For the quantum corrections, we keep the terms up to the second order of the coupling constants within the one loop approximation. In the following subsection, we will integrate the light scalar field and the heavy scalar field step by step.

4.1 Integrating the heavy scalar field

In this subsection, we integrate the heavy scalar field Δ_2. Using Eq.(2.13) and Eq.(3.4), the action in Eq.(3.3) is written as follows,

$$\begin{aligned} S[\bar{\rho}_1 + \Delta_1, \Delta_2, n] &= S[\bar{\rho}_1, 0, 0] + \int d^d x \Delta_i(x) \frac{\delta S[\rho_1, \rho_2, n]}{\delta \rho_i(x)}|_{\rho_1=\bar{\rho}_1, \rho_2=0} \\ &+ \frac{1}{2} \int d^d x \int d^d y \Delta_i(x) \frac{\delta S[\rho_1, \rho_2, n]}{\delta \rho_i(x) \delta \rho_j(y)}|_{\rho_1=\bar{\rho}_1, \rho_2=0} \Delta_j(y) \\ &+ S_{\text{int}}(\Delta_i, \bar{\rho}_1). \end{aligned} \tag{4.2}$$

Each term in Eq.(4.2) is given as follows,

$$\begin{aligned} S[\bar\rho_1,0,0] &= \int d^dx\left(\frac{Z_1}{2}\partial^\mu\bar\rho_1\partial_\mu\bar\rho_1-\frac{Z_{m11}}{2}m_1^2\bar\rho_1^2-\frac{\lambda_{01}Z_1^2}{4}\bar\rho_1^4\right)\\ &+\int d^dx\mu^{-2\eta}\left(\sum_{i=1}^{2}Z_{hi}h_i{m_i}^4+2Z_{h3}h_3m_1^2m_2^2+Z_{h12}h_{12}m_{12}^4\right), \end{aligned} \tag{4.3}$$

$$\frac{\delta S[\rho_1,\rho_2,n]}{\delta\rho_i(x)}|_{\rho_1=\bar\rho_1,\rho_2=0} = -\begin{pmatrix} \{Z_1(\Box+m_{01}^2)+\lambda_{01}Z_1^2\bar\rho_1^2\}\bar\rho_1 \\ \mu^\eta y Z_y\bar n n+Z_{12}m_{12}^2\bar\rho_1\end{pmatrix}, \tag{4.4}$$

$$\begin{aligned}\frac{\delta S[\rho_1,\rho_2,n]}{\delta\rho_i(x)\delta\rho_j(y)}|_{\rho_1=\bar\rho_1,\rho_2=0} &= -\begin{pmatrix} Z_1(\Box+m_{01}^2)+3\lambda_{01}Z_1^2\bar\rho_1^2 & Z_{12}m_{12}^2 \\ Z_{12}m_{12}^2 & Z_2(\Box_x+m_{02}^2)+\frac{\lambda_{03}}{2}Z_1Z_2\bar\rho_1^2\end{pmatrix}\\ &\times\delta^d(x-y), \end{aligned} \tag{4.5}$$

$$S_{\rm int}(\Delta_i,\bar\rho_1) = S_{1\,\rm int}(\Delta_1,\bar\rho_1)+S_{2\,\rm int}(\Delta_2)+S_{12\,\rm int}(\Delta_1,\Delta_2,\bar\rho_1), \tag{4.6}$$

$$S_{1\,\rm int}(\Delta_1,\bar\rho_1) = -\int d^dx\lambda_{01}Z_1^2\Delta_1^3\bar\rho_1-\int d^dx\frac{1}{4}\lambda_{01}Z_1^2\Delta_1^4, \tag{4.7}$$

$$S_{2\,\rm int}(\Delta_2) = -\int d^dx\frac{1}{4}\lambda_{02}Z_2^2\Delta_2^4, \tag{4.8}$$

$$S_{12\,\rm int}(\Delta_1,\Delta_2,\bar\rho_1) = -\int d^dx\frac{1}{2}\lambda_{03}Z_1Z_2\Delta_1\Delta_2^2\bar\rho_1-\int d^dx\frac{1}{4}\lambda_{03}Z_1Z_2\Delta_1^2\Delta_2^2. \tag{4.9}$$

We define the effective action $\tilde\Gamma_{\rm eff}[\bar\rho_1,n]$ by subtracting the classical action $S[\bar\rho_1,0,0]$ from $\Gamma_{\rm eff}[\bar\rho_1,n]$ in Eq.(3.3),

$$\tilde\Gamma_{\rm eff}[\bar\rho_1,n] = \Gamma_{\rm eff}[\bar\rho_1,n]-S[\bar\rho_1,0,0]. \tag{4.10}$$

Then Eq.(4.10) is rewritten as follows,

$$e^{i\tilde\Gamma_{\rm eff}[\bar\rho_1,n]}=\int d\Delta_1 e^{i\left\{\frac12\int d^dx\int d^dy\Delta_1(x)\frac{\delta^2 S[\bar\rho_1,0,n]}{\delta\bar\rho_1(x)\delta\bar\rho_1(y)}\Delta_1(y)+S_{1\,\rm int}(\Delta_1,\bar\rho_1)-\int d^dx\Delta_1(x)\left(\frac{\delta\tilde\Gamma_{\rm eff}[\bar\rho_1,n]}{\delta\bar\rho_1(x)}\right)\right\}}e^{iW_2[J_2,\Delta_1,n]}. \tag{4.11}$$

The last factor of Eq.(4.11) represents the contribution to the effective action from the path integral of the heavy scalar. This factor is written in terms of the path integral with respect to Δ_2 as follows,

$$\begin{aligned}&e^{iW_2[J_2,\bar\rho_1,\Delta_1]}\\ &=\int d\Delta_2 e^{\frac{i}{2}\int d^dx\int d^dy\Delta_2(x)\frac{\delta^2 S[\rho_1,\rho_2,n]}{\delta\rho_2(x)\delta\rho_2(y)}|_{\rho_1=\bar\rho_1,\rho_2=0}\Delta_2(y)+iS_{2\,\rm int}(\Delta_2)+iS_{12\,\rm int}(\Delta_1,\Delta_2,\bar\rho_1)+i\int d^dx\Delta_2(x)J_2(x)},\end{aligned} \tag{4.12}$$

where J_2 is given by,

$$J_2(x)=\left(\frac{\delta S[\rho_1,\rho_2,n]}{\delta\rho_2(x)}|_{\rho_1=\bar\rho_1,\rho_2=0}-Z_{12}m_{12}^2\Delta_1\right)=-Z_{12}m_{12}^2(\Delta_1+\bar\rho_1)-\mu^\eta yZ_y\bar n n. \tag{4.13}$$

J_2 is the collection of fields that linearly couple to the heavy scalar Δ_2. Note that W_2 in Eq.(4.12) is the generating function of the Green's function of the heavy scalar Δ_2. Next we calculate $W_2[J_2, \bar{\rho}_1, \Delta_1]$ by integrating the heavy scalar field Δ_2 out. To simplify the result, we introduce the exponentiated functional differentiation and the notation $\langle \cdots \rangle_0$ as,

$$\langle F[\Delta_2] \rangle_0 \equiv \exp\left[\frac{1}{2}\frac{\delta}{\delta\Delta_2} \cdot D_{F22}\frac{\delta}{\delta\Delta_2}\right] F[\Delta_2]\Big|_{\Delta_2=0}, \tag{4.14}$$

where

$$\frac{\delta}{\delta\Delta_2} \cdot D_{F22}\frac{\delta}{\delta\Delta_2} \equiv \int d^dx \int d^dy \frac{\delta}{\delta\Delta_2(x)} D_{F22}(x,y)\frac{\delta}{\delta\Delta_2(y)}, \tag{4.15}$$

$D_{Fii}(x,y)$ denotes the Feynman propagator of Δ_i. The propagators $D_{F11}(x,y)$ and $D_{F22}(x,y)$ are defined to satisfy the following equations,

$$\int d^dy i D_{ii}^{-1}(x,y) D_{Fii}(y,z) \;=\; \delta^d(x-z), \quad (i=1,2). \tag{4.16}$$

where the $D_{11}^{-1}(x,y)$ and $D_{22}^{-1}(x,y)$ are defined by,

$$-D_{11}^{-1}(x,y) \equiv -\{Z_1(\Box_x + m_{01}^2) + 3\lambda_{01} Z_1^2 \bar{\rho}_1^2(x)\}\delta^d(x-y), \tag{4.17}$$

$$-D_{22}^{-1}(x,y) \equiv -\left\{Z_2(\Box_x + m_{02}^2) + \frac{\lambda_{03}}{2} Z_1 Z_2 \bar{\rho}_1^2(x)\right\}\delta^d(x-y). \tag{4.18}$$

Note that $D_{11}^{-1}(x,y)$ and $D_{22}^{-1}(x,y)$ depend on the background field $\bar{\rho}_1$. Using Eqs.(4.16)-(4.18), the propagators are symbolically written as follows,

$$D_{F11}(x,y) \;=\; -\frac{i}{Z_1(\Box_x + m_{01}^2) + 3\lambda_{01} Z_1^2 \bar{\rho}_1^2(x)}, \tag{4.19}$$

$$D_{F22}(x,y) \;=\; -\frac{i}{Z_2(\Box_x + m_{02}^2) + \frac{\lambda_{03}}{2} Z_1 Z_2 \bar{\rho}_1^2(x)}. \tag{4.20}$$

Note that $D_{F11}(x,y)$ and $D_{F22}(x,y)$ also depend on the background field $\bar{\rho}_1$. Using the notation of Eq.(4.14), the generating functional $W_2[J_2, \bar{\rho}_1, \Delta_1]$ is calculated as follows,

$$e^{iW_2[J_2,\bar{\rho}_1,\Delta_1]} = \left(\det \frac{\delta^2 S[\rho_1,\rho_2,n]}{\delta\rho_2(x)\delta\rho_2(y)}\Big|_{\rho_1=\bar{\rho}_1,\rho_2=0}\right)^{-\frac{1}{2}} \langle e^{iS_{2\,\mathrm{int}}(\Delta_2)+iS_{12\,\mathrm{int}}(\Delta_1,\Delta_2,\bar{\rho}_1)}\rangle_0 \; e^{iW_2^c[J_2,\bar{\rho}_1,\Delta_1]}, \tag{4.21}$$

where $e^{iW_2^c[J_2,\bar{\rho}_1,\Delta_1]}$ is given as,

$$e^{iW_2^c[J_2,\bar{\rho}_1,\Delta_1]} \;=\; \frac{\left\langle e^{iS_{2\,\mathrm{int}}(\Delta_2)+iS_{12\,\mathrm{int}}(\Delta_1,\Delta_2,\bar{\rho}_1)+i\int d^dx \Delta_2(x)J_2(x)}\right\rangle_0}{\langle e^{iS_{2\,\mathrm{int}}(\Delta_2)+iS_{12\,\mathrm{int}}(\Delta_1,\Delta_2,\bar{\rho}_1)}\rangle_0}. \tag{4.22}$$

In Eq.(4.21), the first factor is the contribution of the vacuum graph from the quadratic part of Δ_2. The second part corresponds to the contribution from the interaction. The third factor is the connected Green function contribution of Δ_2 with the source term J_2 which is defined by Eq.(4.13). $W_2^c[J_2, \Delta_1]$ can be written as the sum of the contributions from the tree diagrams and that of one loop diagrams,

$$iW_2^c[J_2, \bar{\rho}_1, \Delta_1] \quad = \quad iW_2^{c(\text{tree})}[J_2, \bar{\rho}_1, \Delta_1] + iW_2^{c(1\,\text{loop})}[J_2, \bar{\rho}_1, \Delta_1]. \tag{4.23}$$

$iW_2^{c(\text{tree})}[J_2, \bar{\rho}_1, \Delta_1]$ and $iW_2^{c(1\,\text{loop})}[J_2, \bar{\rho}_1, \Delta_1]$ are given as follows,

$$\begin{aligned}
iW_2^{c(\text{tree})}[J_2, \bar{\rho}_1, \Delta_1] \quad = \quad & -\frac{1}{2}\int d^dx \int d^dy J_2(x) D_{F22}(x,y) J_2(y) - i\frac{\lambda_{02}}{4}\int d^dx D_{J_2}(x)^4 \\
& -i\frac{\lambda_3\mu^{2\eta}}{4}\int d^dx (\Delta_1^2(x) + 2\Delta_1(x)\bar{\rho}_1(x)) D_{J_2}(x)^2 - \frac{\lambda_{02}^2}{2}\int d^dx \int d^dy D_{F22}(x,y) D_{J_2}(x)^3 D_{J_2}(y)^3 \\
& -\frac{\lambda_3^2\mu^{4\eta}}{8}\int d^dx \int d^dy (\Delta_1^2(x) + 2\Delta_1(x)\bar{\rho}_1(x))(\Delta_1^2(y) + 2\Delta_1(y)\bar{\rho}_1(y)) D_{F22}(x,y) D_{J_2}(x) D_{J_2}(y) \\
& -\frac{\lambda_2\lambda_3\mu^{4\eta}}{2}\int d^dx \int d^dy (\Delta_1^2(x) + 2\Delta_1(x)\bar{\rho}_1(x)) D_{F22}(x,y) D_{J_2}(x) D_{J_2}(y)^3
\end{aligned} \tag{4.24}$$

$$\begin{aligned}
iW_2^{c(1\,\text{loop})}[J_2, \bar{\rho}_1, \Delta_1] \quad = \quad & -\frac{3i\lambda_2\mu^{2\eta}}{2}\int d^dx D_{F22}(x,x) D_{J_2}(x)^2 \\
& -\frac{\lambda_2^2\mu^{4\eta}}{2}\int d^dx \int d^dy \left\{\frac{9}{2} D_{F22}(x,y)^2 D_{J_2}(x)^2 D_{J_2}(y)^2 + 6 D_{F22}(x,x) D_{J_2}(x) D_{F22}(x,y) D_{J_2}(y)^3\right\} \\
& -\frac{3\lambda_2\lambda_3\mu^{4\eta}}{4}\int d^dx \int d^dy (\Delta_1^2(x) + 2\Delta_1(x)\bar{\rho}_1(x)) \\
& \times\{2D_{F22}(0) D_{F22}(x,y) D_{J_2}(x) D_{J_2}(y) + D_{F22}^2(x,y) D_{J_2}(y)^2\},
\end{aligned} \tag{4.25}$$

where we have used the following definition,

$$D_{J_2}(x)^n \quad = \quad \prod_{i=1}^{n}\int d^dx_i D_{F22}(x, x_i) iJ_2(x_i). \tag{4.26}$$

The tree contribution to $iW_2^c[J_2, \bar{\rho}_1, \Delta_1]$ by Δ_2 integration is obtained by setting Δ_1 to be zero in Eq.(4.24),

$$\begin{aligned}
i\bar{W}_2^{c(\text{tree})}[\bar{\rho}_1, n] \quad & \equiv \quad iW_2^{c(\text{tree})}[J_2(\Delta_1 = 0), \bar{\rho}_1, 0] \\
& = \quad -\frac{1}{2}\int d^dx \int d^dy O(x) D_{F22}(x,y) O(y) - i\frac{\lambda_{02}}{4}\int d^dx D_O(x)^4 \\
& \qquad -\frac{\lambda_{02}^2}{2}\int d^dx \int d^dy D_{F22}(x,y) D_O(x)^3 D_O(y)^3,
\end{aligned} \tag{4.27}$$

where we have defined $D_O(x)$ as follows,

$$D_O(x) = \int d^dx_i D_{F22}(x, x_i) iO(x_i), \tag{4.28}$$

where $-J_2(\Delta_1 = 0) = O(x) = Z_{12} m_{12}^2 \bar{\rho}_1 + \mu^\eta y Z_y \bar{n} n$. The diagrams of the tree contribution for Eq.(4.24) are shown in Fig.4.1. All the propagators in the diagrams are the heavy scalars and they are the one particle reducible diagrams. In Eq.(4.27), these tree level contributions contain the counter terms that subtract the divergences of the one loop graphs,

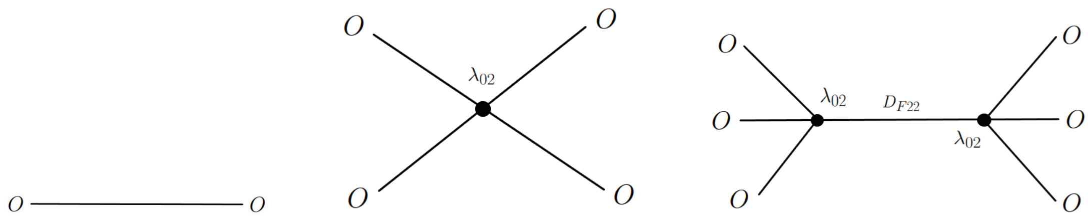

Figure 4.1: The tree level diagrams which contribute to $i\bar{W}_2^{c(\text{tree})}[\bar{\rho}_1, n]$. All the propagators are heavy scalars. The external source terms denoted by O correspond to the background field $\bar{\rho}_1$ and the neutrino bilinear $\overline{n}n$. From the figure of the left to that of the right, each Feynman diagram corresponds to the terms of Eq.(4.27) from the first term to the last term respectively.

so the bare coupling constant λ_{02} is substituted. The terms with Δ_1 in Eq.(4.24) contribute to the effective action beyond the tree level and the renormalized coupling constants are substituted for their interactions.

The diagrams of Fig.4.2 are the diagrams for the one-loop contribution of Δ_2 defined by $i\bar{W}_2^{c(1\,\text{loop})}$. The contribution is summarized as,

$$\begin{aligned}
&i\bar{W}_2^{c(1\,\text{loop})}[\bar{\rho}_1, n] \equiv iW_2^{c(1\,\text{loop})}[-O(x), \bar{\rho}_1, 0] \\
&= -\frac{3i\lambda_2\mu^{2\eta}}{2}\int d^dx D_{F22}(x,x) D_O(x)^2 \\
&- \frac{\lambda_2^2\mu^{4\eta}}{2}\int d^dx \int d^dy \left(\frac{9}{2}D_{F22}(x,y)^2 D_O(x)^2 D_O(y)^2 + 6D_{F22}(x,y)D_{F22}(x,x)D_O(x)D_O(y)^3\right).
\end{aligned} \tag{4.29}$$

In this contribution, Δ_1 is set to be zero because the contribution of Δ_1 generates either another loop effect or one-particle reducible contribution that is excluded from the effective action $\tilde{\Gamma}_{\text{eff}}[\bar{\rho}_1, n]$ in Eq.(4.11). With Eq.(4.24) and Eq.(4.29) , $iW_2^c[J_2, \bar{\rho}_1, \Delta_1]$ is finally obtained as follows,

$$iW_2^c[J_2, \bar{\rho}_1, \Delta_1] \;=\; iW_2^{c(\text{tree})}[J_2, \bar{\rho}_1, \Delta_1] + i\bar{W}_2^{c(1\,\text{loop})}[\bar{\rho}_1, n]. \tag{4.30}$$

Next we calculate the vacuum graph contribution which corresponds the second factor of Eq.(4.21). This contribution is computed as follows,

$$\begin{aligned}
&\langle e^{iS_{2\,\text{int}}(\Delta_2)+iS_{12\,\text{int}}(\Delta_1,\Delta_2,\bar{\rho}_1)}\rangle_0 \\
=\;& \exp\Big[-\frac{\lambda_3\mu^{2\eta}}{4}i\int d^dx(\Delta_1^2(x) + 2\Delta_1(x)\bar{\rho}_1(x))D_{F22}(x,x) \\
&-\left(\frac{\lambda_3}{4}\right)^2\mu^{4\eta}\int d^dx\int d^dy(\Delta_1^2(x) + 2\Delta_1(x)\bar{\rho}_1(x))(\Delta_1^2(y) + 2\Delta_1(y)\bar{\rho}_1(y))D_{F22}^2(x,y)\Big] \\
=\;& 1.
\end{aligned} \tag{4.31}$$

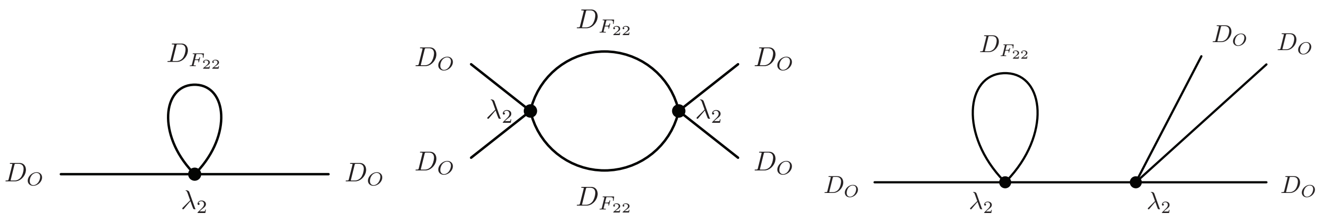

Figure 4.2: One loop diagrams of the heavy scalar Δ_2 which contribute to the $i\bar{W}_2^{c(1\,\mathrm{loop})}[\bar{\rho}_1, n]$. From the figure of the left to that of the right, each Feynman diagram corresponds to terms of Eq.(4.29) from the first term to the last term respectively. This figure was reproduced from Ref.[10].

In this contribution, the one loop contribution of the heavy scalar is included. Therefore this contribution leads to two loop contribution because another loop effect of the light scalar. We set Δ_1 to be zero in the last line of Eq.(4.31) because the same reason as that of one loop contribution. Combining Eq.(4.24) and Eq.(4.29), the expression for Eq.(4.12) can be summarized as follows,

$$e^{iW_2[J_2,\bar{\rho}_1,\Delta_1]} = e^{-\frac{1}{2}\,\mathrm{TrLn}\,D_{22}^{-1}(\bar{\rho}_1)+i\bar{W}_2^{c(\mathrm{tree})}[\bar{\rho}_1,n]+i\bar{W}_2^{c(1\,\mathrm{loop})}[\bar{\rho}_1,n]}e^{iW_2^{c(\mathrm{tree})}[J_2,\bar{\rho}_1,\Delta_1]|_{\mathrm{rest}}}, \quad (4.32)$$

where $iW_2^{c(\mathrm{tree})}|_{\mathrm{rest}}$ is defined as the difference of Eq.(4.24) and Eq.(4.27),

$$iW_2^{c(\mathrm{tree})}[J_2, \bar{\rho}_1, \Delta_1]|_{\mathrm{rest}} \equiv iW_2^{c(\mathrm{tree})}[J_2, \bar{\rho}_1, \Delta_1] - i\bar{W}_2^{c(\mathrm{tree})}[\bar{\rho}_1, n]. \quad (4.33)$$

4.2 Integrating the light scalar field

In this subsection, we integrate the light scalar Δ_1 out. We consider the corrections up to one loop level. Using the result of previous section, the effective action of Eq.(4.11) is written by,

$$e^{i\tilde{\Gamma}_{\mathrm{eff}}[\bar{\rho}_1,n]} = e^{-\frac{1}{2}\,\mathrm{Tr\,Ln}\,D_{11}^{-1}(\bar{\rho}_1)}e^{-\frac{1}{2}\,\mathrm{Tr\,Ln}\,D_{22}^{-1}(\bar{\rho}_1)}e^{i\bar{W}_2^{c(\mathrm{tree})}[\bar{\rho}_1,n]+i\bar{W}_2^{c(1\,\mathrm{loop})}[\bar{\rho}_1,n]}$$
$$\times\frac{\int d\Delta_1 e^{i\left\{\frac{1}{2}\int d^dx\int d^dy\Delta_1(x)\frac{\delta^2 S[\bar{\rho}_1,0,n]}{\delta\bar{\rho}_1(x)\delta\bar{\rho}_1(y)}\Delta_1(y)+S_{1\,\mathrm{int}}(\Delta_1,\bar{\rho}_1)+W_2^{c(\mathrm{tree})}[J_2,\bar{\rho}_1,\Delta_1]|_{\mathrm{rest}}-\int d^dx\Delta_1(x)\left(\frac{\delta\tilde{\Gamma}_{\mathrm{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}\right)\right\}}}{\int d\Delta_1 e^{i\left\{\frac{1}{2}\int d^dx\int d^dy\Delta_1(x)\frac{\delta^2 S[\bar{\rho}_1,0,n]}{\delta\bar{\rho}_1(x)\delta\bar{\rho}_1(y)}\Delta_1(y)\right\}}}. \quad (4.34)$$

From Eq.(4.34), the effective action is summarized as follows,

$$i\tilde{\Gamma}_{\mathrm{eff}}[\bar{\rho}_1, n] = -\frac{1}{2}\,\mathrm{Tr\,Ln}\,D_{22}^{-1}(\bar{\rho}_1) - \frac{1}{2}\,\mathrm{Tr\,Ln}\{(D_{11}^{-1})'(\bar{\rho}_1)\} + i\bar{W}_2^{c(\mathrm{tree})}[\bar{\rho}_1, n] + i\bar{W}_2^{c(1\,\mathrm{loop})}[\bar{\rho}_1, n]$$
$$+\log\left[\frac{\int d\Delta_1 e^{i\left\{-\frac{1}{2}\int d^dx\int d^dy\Delta_1(x)\{(D_{11}^{-1})'(\bar{\rho}_1)\}\Delta_1(y)-\int d^dx\Delta_1(x)\left(\frac{\delta\tilde{\Gamma}_{\mathrm{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}\right)\right\}}e^{\mathcal{L}_{\mathrm{int}}[J_2,\Delta_1,\bar{\rho}_1]}}{\int d\Delta_1 e^{i\left\{-\frac{1}{2}\int d^dx\int d^dy\Delta_1(x)\{(D_{11}^{-1})'(\bar{\rho}_1)\}\Delta_1(y)\right\}}}\right]. \quad (4.35)$$

In Eq.(4.35), we define $\mathcal{L}_{\rm int}[J_2, \Delta_1, \bar{\rho}_1]$ by,

$$\begin{aligned}\mathcal{L}_{\rm int}[J_2, \Delta_1, \bar{\rho}_1] &\equiv iS_{1\,\rm int}(\Delta_1, \bar{\rho}_1) + iW_2^{c\,\rm tree}[J_2, \Delta_1, \bar{\rho}_1]|_{\rm rest} + \frac{1}{2}m_{12}^4 \int d^dx \int d^dy \Delta_1(x) D_{F22}(x,y)\Delta_1(y) \\ &\simeq iW_2^{c\,\rm tree}[J_2, \Delta_1, \bar{\rho}_1]|_{\rm rest}^{\text{quadratic part of }\Delta_1} + \frac{1}{2}m_{12}^4 \int d^dx \int d^dy \Delta_1(x) D_{F22}(x,y)\Delta_1(y) \quad (4.36)\end{aligned}$$

In the second line from the first line of Eq.(4.36), the quartic interaction term with respect to Δ_1 is ignored because it contributes to beyond the one loop order. The contribution to the tadpole diagram from the cubic interaction of Δ_1 does not exist in the 1 PI effective action and the second order contributions from the cubic interaction which contributes beyond the one loop order are also ignored. In the derivation of Eq.(4.35) from Eq.(4.34), we absorb the mixing effect of the heavy scalar which comes from the first term of Eq.(4.24) into the propagator of the light scalar. The corresponding inverse propagator is given as,

$$-D_{11}^{-1\prime}(x,y) = -D_{11}^{-1}(x,y) + im_{12}^4 D_{F22}(x,y). \quad (4.37)$$

The mixing of the heavy scalar is the second term of Eq.(4.37) and the modified propagator for the light scalar is defined by the following equation,

$$i\int d^dz D_{11}^{-1\prime}(x,z) D'_{F11}(z,y) = \delta^d(x-y). \quad (4.38)$$

Due to this change, the quadratic term with respect to Δ_1;

$$-\frac{1}{2}m_{12}^4 \int\int d^dx d^dy \Delta_1(x) D_{F22}(x,y)\Delta_1(y) \quad (4.39)$$

must be subtracted from $iW_2^{c\,\rm tree}[J_2, \Delta_1, \bar{\rho}_1]|_{\rm rest}$. The second term of the last line of Eq.(4.36) is added for this purpose. Therefore within one loop approximation, we keep only the quadratic terms with respect to Δ_1 and the explicit expression of $\mathcal{L}_{\rm int}[O, \Delta_1, \bar{\rho}_1]$ is given as follows,

$$
\begin{aligned}
&\mathcal{L}_{\rm int}[O,\Delta_1,\bar{\rho}_1] \\
= \;& \frac{3im_{12}^4\lambda_2\mu^{2\eta}}{2}\int d^dx\left(\prod_{i=1}^{2}\int d^dx_iD_{F22}(x,x_i)\right)D_O(x)^2\Delta_1(x_1)\Delta_1(x_2) \\
&-\frac{i\lambda_3\mu^{2\eta}}{4}\int d^dx\Delta_1^2(x)D_O(x)^2 \\
&+m_{12}^2\lambda_3\mu^{2\eta}\left(\int d^dx\Delta_1(x)\bar{\rho}_1(x)D_O(x)\right)\left(\int d^dx_1D_{F22}(x,x_1)\Delta_1(x_1)\right) \\
&-\frac{\lambda_3^2\mu^{4\eta}}{2}\int d^dx\int d^dy\Delta_1(x)\bar{\rho}_1(x)\Delta_1(y)\bar{\rho}_1(y)D_{F22}(x,y)D_O(x)D_O(y) \\
&-\frac{\lambda_2\lambda_3\mu^{4\eta}}{4}\int d^dx\int d^dy\Delta_1^2(x)D_{F22}(x,y)D_O(x)D_O(y)^3 \\
&-\frac{im_{12}^2\lambda_2\lambda_3\mu^{4\eta}}{2}\int d^dx\int d^dy\Delta_1(x)\bar{\rho}_1(x)D_{F22}(x,y)D_O(y)^3\left(\int d^dx_1D_{F22}(x,x_1)\Delta_1(x_1)\right) \\
&-i\frac{3}{2}m_{12}^2\lambda_2\lambda_3\mu^{4\eta}\int d^dx\int d^dy\Delta_1(x)\bar{\rho}_1(x)D_{F22}(x,y)D_O(x)D_O(y)^2\left(\int d^dy_1D_{F22}(y,y_1)\Delta_1(y_1)\right) \\
&+3m_{12}^4\lambda_2^2\mu^{4\eta}\int d^dx\int d^dyD_{F22}(x,y)D_O(x)D_O(y)^3\left(\prod_{i=1}^{2}\int d^dx_iD_{F22}(x,x_i)\Delta_1(x_i)\right) \\
&+\frac{9m_{12}^4\lambda_2^2\mu^{4\eta}}{2}\int d^dx\int d^dyD_{F22}(x,y)D_O(x)^2D_O(y)^2\left(\int d^dx_1D_{F22}(x,x_1)\Delta_1(x_1)\right)\left(\int d^dy_1D_{F22}(y,y_1)\Delta_1(y_1)\right).
\end{aligned}
\tag{4.40}
$$

In Eq.(4.40), we replaced J_2 by O to subtract Δ_1 term from J_2. We have shown the diagrams for Eq.(4.40) in Fig.4.3.

Next we calculate the contribution from $\mathcal{L}_{\rm int}[O,\Delta_1,\bar{\rho}_1]$ in Eq.(4.40). For simplicity, we define the following function;

$$
F[\bar{\rho}_1,n] \equiv \frac{\int d\Delta_1 e^{i\left\{-\frac{1}{2}\int d^dx\int d^dy\Delta_1(x)(D_{11}^{-1})'(\bar{\rho}_1)\Delta_1(y)\right\}}e^{\left(\mathfrak{L}_{\rm int}[O,\Delta_1,\bar{\rho}_1]-i\int d^dx\Delta_1(x)\left(\frac{\delta\tilde{\Gamma}_{\rm eff}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}\right)\right)}}{\int d\Delta_1 e^{i\left\{\frac{1}{2}\int d^dx\int d^dy\Delta_1(x)(D_{11}^{-1})'(\bar{\rho}_1)\Delta_1(y)\right\}}}. \tag{4.41}
$$

This function coressponds to the expression inside the logarithmic function in the last term of Eq.(4.35). Note that the effect of $\int d^dx\Delta_1(x)\left(\frac{\delta\tilde{\Gamma}_{\rm eff}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}\right)$ removes the contribution from one particle reducible graphs. We calculate $F[\bar{\rho}_1,n]$ up to the second order of the couplings,

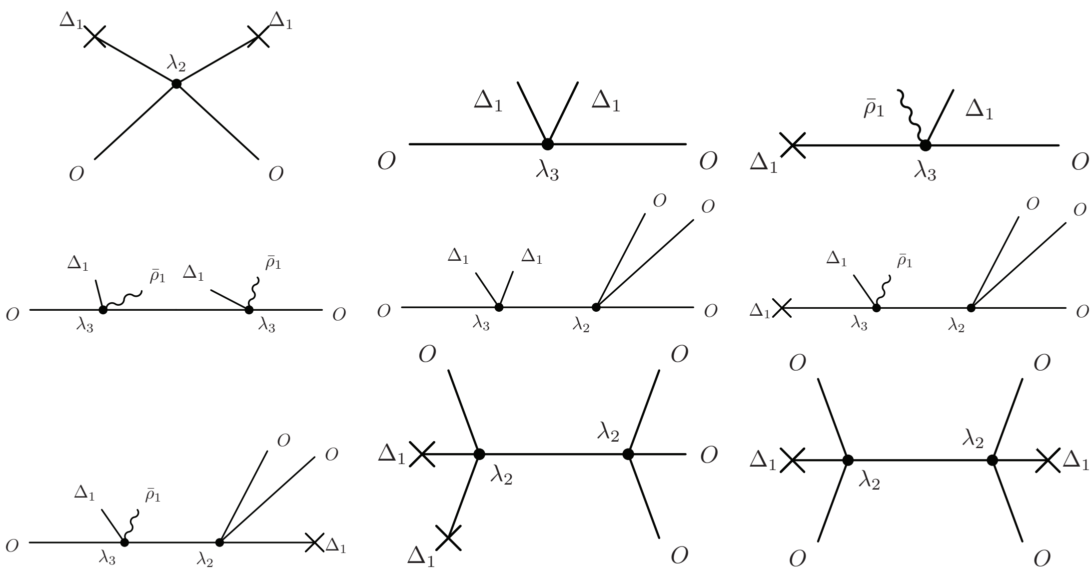

Figure 4.3: From the figure of the upper left to that of the lower right, each Feynman diagram corresponds to each line of Eq.(4.40) from the top line to the bottom. The cross mark denotes an insertion of the mass mixing term m_{12}^2. All the propagators are heavy scalars. This figure was reproduced from Ref.[10].

then it is given as,

$$
\begin{aligned}
F[\bar{\rho}_1, n] \approx\ & 1 + \frac{i}{4}\lambda_3 \mu^{2\eta} \int d^d x \left(\prod_{i=1}^{2} \int d^d x_i D^{(0)}_{F22}(x - x_i) O(x_i) \right) D'_{F11}(x, x) \\
& + i\lambda_3 m_{12}^2 \mu^{2\eta} \int d^d x \bar{\rho}_1(x) \left(\prod_{i=1}^{2} \int d^d x_i D^{(0)}_{F22}(x - x_i) \right) O(x_2) D'_{F11}(x, x_1) \\
& + i\lambda_3 m_{12}^2 \mu^{2\eta} \int d^d x \bar{\rho}_1(x) \int d^d x_1 \int d^d x_2 D^{(1)}_{F22}(x, x_1) D^{(0)}_{F22}(x - x_2) O(x_2) D'_{F11}(x, x_1) \\
& + i\lambda_3 m_{12}^2 \mu^{2\eta} \int d^d x \bar{\rho}_1(x) \int d^d x_1 \int d^d x_2 D^{(0)}_{F22}(x - x_1) D^{(1)}_{F22}(x, x_2) O(x_2) D'_{F11}(x, x_1) \\
& + \frac{\lambda_3^2 \mu^{4\eta}}{2} \int d^d x \int d^d y \bar{\rho}_1(x) \bar{\rho}_1(y) D^{(0)}_{F22}(x - y) \\
& \times \int d^d x_1 \int d^d y_1 D^{(0)}_{F22}(x - x_1) D^{(0)}_{F22}(y - y_1) O(x_1) O(y_1) D'_{F11}(x, y). \qquad (4.42)
\end{aligned}
$$

In the derivation of Eq.(4.42), we approximate the exact expression by expanding the result with the small parameter $\epsilon = \frac{m_{12}^2}{m_2^2} \ll 1$. For the quadratic terms of the background field $\bar{\rho}$, we keep the terms suppressed up to the order of $m_{12}^2 \epsilon \bar{\rho}_1^2$. For the quartic terms of $\bar{\rho}_1$, we keep the terms suppressed by $\epsilon^2 \bar{\rho}_1^4$. $D^{(n)}_{F22}(x, y)$ in Eq.(4.42) are obtained by solving the following integral equation with iteration.

$$
D_{F22}(x, y) = D^{(0)}_{F22}(x - y) - i \int d^d z D^{(0)}_{F22}(x - z) \frac{\lambda_3}{2} \bar{\rho}_1^2(z) D_{F22}(z, y), \qquad (4.43)
$$

where the leading contribution $D^{(0)}_{F22}(x-y)$ is given as follows,

$$D^{(0)}_{F22}(x-y) \;=\; \int \frac{d^dk}{(2\pi)^d i}\frac{1}{m_2^2-k^2}e^{-ik(x-y)}. \tag{4.44}$$

We need to find the first order correction and the second order correction to perform the calculation. These corrections are obtained by solving Eq.(4.43), it is given by,

$$D^{(1)}_{F22}(x,y) = -i\int d^dz D^{(0)}_{F22}(x-z)\frac{\lambda_3}{2}\bar{\rho}_1^2(z)D^{(0)}_{F22}(z-y), \tag{4.45}$$

$$D^{(2)}_{F22}(x,z) = -\int d^dz\int d^d\omega D^{(0)}_{F22}(x-z)\frac{\lambda_3}{2}\bar{\rho}_1^2(z)D^{(0)}_{F22}(z-\omega)\frac{\lambda_3}{2}\bar{\rho}_1^2(\omega)D^{(0)}_{F22}(\omega-y). \tag{4.46}$$

The same approximation is also used when we expand D_{F22} in Eq.(4.27) and Eq.(4.29). We expand the propagator $D'_{F11}(x,y)$ with respect to m_{12}. When $\bar{\rho}_1$ is independent of the space time, $D'_{F11}(x,y)$ up to the fourth power of m_{12} is given as follows.

$$\begin{aligned} D'_{F11}(x-y) \;=\; & \int \frac{d^dk}{(2\pi)^d}e^{-ik\cdot(x-y)}\frac{-i}{m_1^2-k^2+3\lambda_1\bar{\rho}_1^2} \\ & +\int \frac{d^dk}{(2\pi)^d}e^{-ik\cdot(x-y)}\frac{-im_{12}^4}{(m_1^2-k^2+3\lambda_1\bar{\rho}_1^2)^2\left(m_2^2-k^2+\frac{\lambda_3}{2}\bar{\rho}_1^2\right)}. \end{aligned} \tag{4.47}$$

We study the effective potential for the scalar and the bilinear in this paper. Therefore, we consider the case of the field $\bar{\rho}_1$ and the fermion bilinear of $\bar{n}n$ for the neutrino that do not depend on the space time. To study the scalar loop effect on the neutrino Yukawa coupling, it is sufficient to consider the space time independent mode of bilinear $\bar{n}n$. We summarize the tree level action and one loop contribution for the constant background fields.

$$\Gamma_{\rm eff}[\bar{\rho}_1,n] \;=\; S[\bar{\rho}_1,0,0]+\tilde{\Gamma}_{\rm eff}[\bar{\rho}_1,n], \tag{4.48}$$

$$\begin{aligned} S[\bar{\rho}_1,0,0] \;=\; & \int d^dx\left[-\frac{Z_{m11}}{2}m_1^2\bar{\rho}_1^2-\frac{\lambda_{01}{Z_1}^2}{4}\bar{\rho}_1^4\right] \\ + & \int d^dx\left[\mu^{-2\eta}\left(\sum_{i=1}^{2}Z_{hi}h_i{m_i}^4+2Z_{h3}h_3m_1^2m_2^2+Z_{h12}h_{12}m_{12}^4\right)\right], \end{aligned} \tag{4.49}$$

$$\tilde{\Gamma}_{\rm eff}[\bar{\rho}_1,n] \;=\; \tilde{\Gamma}_{\rm eff}[\bar{\rho}_1,n]^{\rm tree}+\tilde{\Gamma}_{\rm eff}[\bar{\rho}_1,n]^{1\,\rm loop}_{\Delta_2}+\tilde{\Gamma}_{\rm eff}[\bar{\rho}_1,n]^{1\,\rm loop}_{\Delta_1}+\tilde{\Gamma}_{\rm eff}[\bar{\rho}_1,n]^{\rm TrLn}, \tag{4.50}$$

$$\tilde{\Gamma}^{\rm tree}_{\rm eff}[\bar{\rho}_1,n] \;=\; \int d^dx\left[\frac{m_{012}^2}{2}\epsilon_0\bar{\rho}_{01}^2+y_0\epsilon_0\bar{n}_0n_0\bar{\rho}_{01}-\frac{\lambda_{03}}{4}\epsilon_0^2\bar{\rho}_{01}^4+\frac{1}{2m_2^2}\left(\frac{\lambda_3\mu^{2\eta}}{2}\epsilon\bar{\rho}_1^3-y\mu^\eta\bar{n}n\right)^2\right], \tag{4.51}$$

$$\begin{aligned} \tilde{\Gamma}^{1\,\rm loop(\Delta_2)}_{\rm eff}[\bar{\rho}_1,n] \;=\; & \int d^dx\left[\frac{3\lambda_2\mu^{2\eta}}{16\pi^2}(\Gamma(\eta)+1+\log 4\pi-\log m_2^2)\epsilon(y\mu^\eta\bar{n}n\bar{\rho}_1+\frac{m_{12}^2}{2}\bar{\rho}_1^2)\right. \\ + & \left.\frac{3\lambda_2\lambda_3\mu^{4\eta}}{64\pi^2}(-\Gamma(\eta)-2-\log 4\pi+\log m_2^2)\epsilon^2\bar{\rho}_1^4\right], \end{aligned} \tag{4.52}$$

$$\begin{aligned}
\tilde{\Gamma}_{\rm eff}^{1\,{\rm loop}(\Delta_1)}[\bar{\rho}_1, n] &= \int d^dx \left[-\frac{\lambda_3\mu^{2\eta}}{16\pi^2}(\Gamma(\eta) + \log 4\pi + 1 - \log m_2^2)\epsilon(y\mu^\eta \bar{n}n\bar{\rho}_1 + m_{12}^2\bar{\rho}_1^2) \right. \\
&+ \frac{\lambda_3\mu^{2\eta}}{16\pi^2}\left\{ \frac{3\lambda_1\mu^{2\eta}}{4}\left(\Gamma(\eta) + 1 + \log 4\pi - \log(m_1^2 + 3\lambda_1\mu^{2\eta}\bar{\rho}_1^2) - 4\log\left(\frac{m_1^2 + 3\lambda_1\mu^{2\eta}\bar{\rho}_1^2}{m_2^2}\right)\right)\right. \\
&+ \left.\left. \lambda_3\mu^{2\eta}\left(\Gamma(\eta) + \log 4\pi + \frac{3}{2} - \log m_2^2\right)\right\}\epsilon^2\bar{\rho}_1^4\right], \qquad (4.53) \\
\tilde{\Gamma}_{\rm eff}[\bar{\rho}_1, n]^{\rm TrLn} &= \int d^dx \left[\frac{m_{12}^4}{32\pi^2}(\Gamma[\eta] + \log 4\pi + 1 - \log m_2^2) + \frac{\epsilon^2 m_1^2 m_2^2}{32\pi^2}\log\frac{m_1^2 + 3\lambda_1\mu^{2\eta}\bar{\rho}_1^2}{m_2^2} \right. \\
&+ \frac{m_1^4}{64\pi^2}\left(\Gamma[\eta] + \log 4\pi + \frac{3}{2} - \log(m_1^2 + 3\lambda_1\bar{\rho}_1^2\mu^{2\eta})\right) + \frac{m_2^4}{64\pi^2}\left(\Gamma[\eta] + \log 4\pi + \frac{3}{2} - \log m_2^2\right) \\
&+ \frac{m_{12}^2}{2}\left(\frac{3\lambda_1\mu^{2\eta}}{16\pi^2}\log\frac{m_1^2 + 3\lambda_1\mu^{2\eta}\bar{\rho}_1^2}{m_2^2} - \frac{\lambda_3\mu^{2\eta}}{32\pi^2}\right)\epsilon\bar{\rho}_1^2 \\
&+ \left\{ \frac{3\lambda_1\mu^{2\eta}m_1^2}{32\pi^2}\left(\Gamma[\eta] + \frac{3}{2} + \log 4\pi - \log(m_1^2 + 3\lambda_1\bar{\rho}_1^2\mu^{2\eta})\right)\right. \\
&+ \left. \frac{\lambda_3\mu^{2\eta}m_2^2}{64\pi^2}\left(\Gamma[\eta] + 1 + \log 4\pi - \log m_2^2\right)\right\}\bar{\rho}_1^2 \\
&+ \left\{ \frac{\lambda_3^2\mu^{2\eta}}{256\pi^2}\left(\Gamma[\eta] + \log 4\pi - \log m_2^2\right)\right. \\
&+ \left. \frac{9\lambda_1^2\mu^{4\eta}}{64\pi^2}\left(\Gamma[\eta] + \frac{3}{2} + \log 4\pi - \log(m_1^2 + 3\lambda_1\bar{\rho}_1^2\mu^{2\eta})\right)\right\}\bar{\rho}_1^4 \\
&+ \left.\left\{ \frac{9\lambda_1^2\mu^{2\eta}}{32\pi^2}\log\frac{m_1^2 + 3\lambda_1\mu^{2\eta}\bar{\rho}_1^2}{m_2^2} - \frac{3\lambda_1\lambda_3\mu^{4\eta}}{64\pi^2}\left(1 + \log\frac{m_1^2 + 3\lambda_1\mu^{2\eta}\bar{\rho}_1^2}{m_2^2}\right) + \frac{\lambda_3^2\mu^{4\eta}}{256\pi^2}\right\}\epsilon^2\bar{\rho}_1^4\right], \qquad (4.54)
\end{aligned}$$

where we define the ϵ_0 by $\frac{m_{012}^2}{m_{02}^2}$. $\Gamma[\eta]$ is a gamma function that represents the divergence in the dimensional regularization, it is defined by $\frac{1}{\eta} - \gamma$.

Chapter 5

The counter terms and the effective potential

One loop contribution in Eqs.(4.52)-(4.54) contains the divergent terms. We show that the counter terms of $S[\bar{\rho}_1, 0, 0]$ in Eq.(4.49) and $\tilde{\Gamma}_{\rm eff}^{\rm tree}$ in Eq.(4.51) are determined to cancel the divergences. We replace the bare mass terms and the bare coupling constants with the renormalized ones using the relations from Eqs.(2.4)-(2.10). We also consider the fact that the wave function renormalization for scalars from their one-loop diagrams does not exsist. Since we do not consider the contribution of the fermion loop, we can set $Z_i = 1 (i = 1, 2)$. $m_1^2 m_2^2$ term does not contain divergence, so we can set $Z_{h_3} = 1$. The other counter terms are generated by splitting the Z factors as follows,

$$
\begin{aligned}
Z_{12} &= 1 + (Z_{12} - 1), &(5.1)\\
Z_{\lambda_{ii}} &= 1 + (Z_{\lambda_{ii}} - 1)(i = 1, 2, 3), &(5.2)\\
Z_{mii} &= 1 + (Z_{mii} - 1)(i = 1, 2), &(5.3)\\
Z_{h_i} &= 1 + (Z_{h_i} - 1)(i = 1, 2), \quad Z_{h_{12}} = 1 + (Z_{h_{12}} - 1). &(5.4)
\end{aligned}
$$

Using these Z factors, the tree part of the effective action and the counter terms are obtained as follows,

$$
\begin{aligned}
S_{\rm tree}[\bar{\rho}_1, n] &= S[\bar{\rho}_1, 0, 0] + \tilde{\Gamma}_{\rm eff}[\bar{\rho}_1, n]^{\rm tree} = S^{\rm 4dim}_{\rm tree}[\bar{\rho}_1, n] + S^{\rm 6dim}_{\rm tree}[\bar{\rho}_1, n] + S_{\rm C}[\bar{\rho}_1, n], &(5.5)\\
S^{\rm 4dim}_{\rm tree}[\bar{\rho}_1, n] &= \int d^d x \left[-\frac{1}{2} m_1^2 \bar{\rho}_1^2 - \frac{\lambda_1}{4} \bar{\rho}_1^4 + \frac{m_{12}^2}{2} \epsilon \bar{\rho}_1^2 + y\mu^\eta \epsilon \bar{n} n \bar{\rho}_1 - \frac{\lambda_3 \mu^{2\eta}}{4} \epsilon^2 \bar{\rho}_1^4 \right] \\
&\quad + \int d^d x \mu^{-2\eta} \left[\sum_{i=1}^{2} h_i {m_i}^4 + 2h_3 m_1^2 m_2^2 + h_{12} m_{12}^4 \right], &(5.6)\\
S^{\rm 6dim}_{\rm tree}[\bar{\rho}_1, n] &= \int d^d x \left[\frac{1}{2m_2^2} \left(\frac{\lambda_3 \mu^{2\eta}}{2} \epsilon \bar{\rho}_1^3 - y\mu^\eta \bar{n} n \right)^2 \right], &(5.7)
\end{aligned}
$$

$$
\begin{aligned}
S_{\mathrm{C}}[\bar{\rho}_1, n] \;=\; & \int d^d x \Bigg[\Bigg(-\frac{(Z_{m11}-1)m_1^2 + Z_{m12}m_2^2}{2}\bar{\rho}_1^2 - \frac{\{(Z_{\lambda_{11}}-1)\lambda_1 + Z_{\lambda_{12}}\lambda_2 + Z_{\lambda_{13}}\lambda_3\}\mu^{2\eta}}{4}\bar{\rho}_1^4 \Bigg) \\
& + \frac{1}{2} m_{12}^2 \left(2(Z_{12}-1) - \{(Z_{m22}-1)\}\right) \epsilon \bar{\rho}_1^2 + \left((Z_{12}-1) - \{(Z_{m22}-1)\}\right) y \mu^{\eta} \epsilon \bar{n} n \bar{\rho}_1 \\
& - \frac{\mu^{2\eta}}{4} \left[Z_{\lambda_{31}}\lambda_1 + Z_{\lambda_{32}}\lambda_2 + (Z_{\lambda_{33}}-1)\lambda_3 + 2\lambda_3 \{(Z_{12}-1) - ((Z_{m22}-1))\} \right] \epsilon^2 \bar{\rho}_1^4 \\
& + \mu^{-2\eta} \left(\sum_{i=1}^{2} (Z_{hi}-1) h_i {m_i}^4 + (Z_{h12}-1) h_{12} m_{12}^4 \right) \Bigg]. \qquad (5.8)
\end{aligned}
$$

We keep the suppressed terms such as $\epsilon\bar{\rho}_1^2$, $\epsilon^2\bar{\rho}_1^4$ and $\epsilon\bar{n}n\bar{\rho}_1$ and ignore the terms with the further suppression factor of $\frac{m_1^2}{m_2^2}$. Correspondingly, the counter terms with the same suppression factor, such as $Z_{m21}\frac{m_1^2}{m_2^2}$, are also ignored in Eq.(5.8). The Z factors in Eq.(5.8) are determined by full theory and its derivation is shown in appendix A. Substituting the Z coefficient of Eqs.(A10)-(A14), cancels all divergence of Eqs.(4.52)-(4.54). As the results, the tree level part and the one loop part of the effective action are given as,

$$
\begin{aligned}
S_{\mathrm{eff}}^{\mathrm{tree}} &= S_{\mathrm{tree}}^{4\mathrm{dim}}[\bar{\rho}_1, n] + S_{\mathrm{tree}}^{6\mathrm{dim}}[\bar{\rho}_1, n], \qquad (5.9)\\
S_{\mathrm{eff}}^{\mathrm{loop}} &= \tilde{\Gamma}_{\mathrm{eff}}[\bar{\rho}_1, n]_{\Delta_2}^{1\,\mathrm{loop}} + \tilde{\Gamma}_{\mathrm{eff}}[\bar{\rho}_1, n]_{\Delta_1}^{1\,\mathrm{loop}} + \tilde{\Gamma}_{\mathrm{eff}}[\bar{\rho}_1, n]^{\mathrm{TrLn}} + S_{\mathrm{C}}[\bar{\rho}_1, n], \qquad (5.10)\\
S_{\mathrm{eff}} &= S_{\mathrm{eff}}^{\mathrm{tree}} + S_{\mathrm{eff}}^{\mathrm{loop}} \\
&= \int d^4x \Bigg[m_1^4 \left\{ h_1 + \frac{1}{64\pi^2}\left(\frac{3}{2} - \log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{\mu^2}\right)\right\} + m_2^4 \left\{ h_2 + \frac{1}{64\pi^2}\left(\frac{3}{2} - \log\frac{m_2^2}{\mu^2}\right)\right\} \\
&+ \; m_{12}^4 \left\{ h_{12} + \frac{1}{32\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right)\right\} + 2m_1^2 m_2^2 \left\{ h_3 + \frac{\epsilon^2}{32\pi^2}\log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{\mu^2}\right\} \\
&- \; \frac{1}{2}\left\{ m_1^2 \left(1 - \frac{3\lambda_1}{16\pi^2}\left(\frac{3}{2} - \log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{\mu^2}\right)\right) - \frac{\lambda_3 m_2^2}{32\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right)\right\}\bar{\rho}_1^2 \\
&+ \; \frac{m_{12}^2}{2}\left\{ 1 + \frac{3\lambda_1}{16\pi^2}\log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{m_2^2} + \frac{3\lambda_2}{16\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right) - \frac{\lambda_3}{8\pi^2}\left(\frac{5}{4} - \log\frac{m_2^2}{\mu^2}\right)\right\}\epsilon\bar{\rho}_1^2 \\
&- \; \frac{\lambda_1}{4}\left\{ 1 - \frac{9\lambda_1}{16\pi^2}\left(\frac{3}{2} - \log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{\mu^2}\right) + \frac{1}{64\pi^2}\frac{\lambda_3^2}{\lambda_1}\log\frac{m_2^2}{\mu^2}\right\}\bar{\rho}_1^4 \\
&- \; \frac{\lambda_3}{4}\left\{ 1 + \frac{3\lambda_2}{16\pi^2}\left(2 - \log\frac{m_2^2}{\mu^2}\right) - \frac{\lambda_3}{4\pi^2}\left(\frac{25}{16} - \log\frac{m_2^2}{\mu^2}\right)\right. \\
&+ \; \left.\frac{3\lambda_1}{16\pi^2}\left(\log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{\mu^2} + 5\log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{m_2^2} - 6\frac{\lambda_1}{\lambda_3}\log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{m_2^2}\right)\right\}\epsilon^2\bar{\rho}_1^4 \\
&+ \; \left\{ 1 + \frac{3\lambda_2 - \lambda_3}{16\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right)\right\} y\epsilon\bar{n}n\bar{\rho}_1 + \frac{\lambda_3^2}{8m_2^2}\epsilon^2\bar{\rho}_1^6 - \frac{\lambda_3 y}{2m_2^2}\epsilon\bar{n}n\bar{\rho}_1^3 + \frac{y^2}{2m_2^2}(\bar{n}n)^2 \Bigg], \qquad (5.11)
\end{aligned}
$$

where we took the limit $d \to 4$. With the VEV v_1 for $\bar{\rho}_1$, the effective potential is given as follows,

$$\begin{aligned} V_{\text{eff}}(v_1) &= V_{\text{cosmo}} + \frac{m^2_{1\text{eff}}}{2} v_1^2 - \frac{m^2_{12\text{eff}}}{2} \epsilon v_1^2 + \frac{\lambda_{1\text{eff}}}{4} v_1^4 + \frac{\lambda_{3\text{eff}}}{4} \epsilon^2 v_1^4 - y_{\text{eff}} \epsilon \bar{n} n v_1 \\ & \quad - \frac{\lambda_3^2}{8m_2^2} \epsilon^2 v_1^6 + \frac{\lambda_3}{2m_2^2} \epsilon (\bar{n}n) v_1^3 - \frac{y^2}{2m_2^2} (\bar{n}n)^2, \end{aligned} \tag{5.12}$$

where the cosmological constant, the effective masses and couplings are defined as,

$$V_{\text{cosmo}} = -h_{1\text{eff}} {m_1}^4 - h_{2\text{eff}} {m_2}^4 - h_{12\text{eff}} {m_{12}}^4 - 2h_{3\text{eff}} {m_1}^2 {m_2}^2, \tag{5.13}$$

$$m^2_{1\text{eff}} = m_1^2 \left(1 - \frac{3\lambda_1}{16\pi^2} \left(\frac{3}{2} - \log \frac{m_1^2 + 3\lambda_1 v_1^2}{\mu^2}\right)\right) - \frac{\lambda_3 m_2^2}{32\pi^2} \left(1 - \log \frac{m_2^2}{\mu^2}\right), \tag{5.14}$$

$$m^2_{12\text{eff}} = m^2_{12} \left\{1 + \frac{3\lambda_1}{16\pi^2} \log \frac{m_1^2 + 3\lambda_1 v_1^2}{m_2^2} + \frac{3\lambda_2}{16\pi^2} \left(1 - \log \frac{m_2^2}{\mu^2}\right) - \frac{\lambda_3}{8\pi^2} \left(\frac{5}{4} - \log \frac{m_2^2}{\mu^2}\right)\right\}, \tag{5.15}$$

$$\frac{\lambda_{1\text{eff}}}{4} = \frac{\lambda_1}{4} \left\{1 - \frac{9\lambda_1}{16\pi^2} \left(\frac{3}{2} - \log \frac{m_1^2 + 3\lambda_1 v_1^2}{\mu^2}\right) + \frac{1}{64\pi^2} \frac{\lambda_3^2}{\lambda_1} \log \frac{m_2^2}{\mu^2}\right\}, \tag{5.16}$$

$$\begin{aligned} \frac{\lambda_{3\text{eff}}}{4} &= \frac{\lambda_3}{4} \left\{1 + \frac{3\lambda_2}{16\pi^2} \left(2 - \log \frac{m_2^2}{\mu^2}\right) - \frac{\lambda_3}{4\pi^2} \left(\frac{25}{16} - \log \frac{m_2^2}{\mu^2}\right) \right. \\ & \left. + \frac{3\lambda_1}{16\pi^2} \left(\log \frac{m_1^2 + 3\lambda_1 v_1^2}{\mu^2} + 5 \log \frac{m_1^2 + 3\lambda_1 v_1^2}{m_2^2} - 6\frac{\lambda_1}{\lambda_3} \log \frac{m_1^2 + 3\lambda_1 v_1^2}{m_2^2}\right)\right\}, \end{aligned} \tag{5.17}$$

$$h_{1\text{eff}} = h_1 + \frac{1}{64\pi^2} \left(\frac{3}{2} - \log \frac{m_1^2 + 3\lambda_1 v_1^2}{\mu^2}\right), \tag{5.18}$$

$$h_{2\text{eff}} = h_2 + \frac{1}{64\pi^2} \left(\frac{3}{2} - \log \frac{m_2^2}{\mu^2}\right), \tag{5.19}$$

$$h_{3\text{eff}} = h_3 + \frac{\epsilon^2}{32\pi^2} \log \frac{m_1^2 + 3\lambda_1 \bar{\rho}_1^2}{\mu^2}, \tag{5.20}$$

$$h_{12\text{eff}} = h_{12} + \frac{1}{32\pi^2} \left(1 - \log \frac{m_2^2}{\mu^2}\right), \tag{5.21}$$

$$y_{\text{eff}} = y \left(1 + \frac{3\lambda_2 - \lambda_3}{16\pi^2} \left(1 - \log \frac{m_2^2}{\mu^2}\right)\right). \tag{5.22}$$

The renormalization scale μ independence of the effective mass, coupling constant and cosmological constant is discussed in the appendix A. We have shown that the following parameters are scale independent.

$$\mu \frac{dm^2_{1\,\text{eff}}}{d\mu} = 0, \quad \mu \frac{d\lambda_{1\,\text{eff}}}{d\mu} = 0, \quad \mu \frac{dV_{\text{cosmo}}}{d\mu} = 0. \tag{5.23}$$

For the other parameters are scale dependent as follows,

$$\mu \frac{d(m^2_{12\,\mathrm{eff}}\epsilon)}{d\mu} = m^2_{12}\epsilon O\left(\frac{m_1^2}{m_2^2}\right) \simeq 0, \tag{5.24}$$

$$\mu \frac{d(\lambda_{3\,\mathrm{eff}}\epsilon^2)}{d\mu} = \epsilon^2 O\left(\frac{m_1^2}{m_2^2}\right) \simeq 0, \tag{5.25}$$

$$\mu \frac{d(y_{\mathrm{eff}}\epsilon)}{d\mu} = \epsilon y O\left(\frac{m_1^2}{m_2^2}\right) \simeq 0, \tag{5.26}$$

If we consider the leading order of the expansion with respect to $\frac{m_1^2}{m_2^2}$. Eqs.(5.24)-(5.26) imply that they are approximately scale independent.

Chapter 6

Renormalization group improvement

In this section, we discuss the RG improvement of the effective potential of Eqs.(5.12)-(5.22). The RG improved effective potential for the models with two scalars is studied in Refs.[17, 18]. The RG improved effective potential with multi-scale is also studied in Ref.[19].

By setting the renormalization scale μ equal to the heavy scalar mass m_2, the effective couplings and masses obtained with Eqs.(5.14)-(5.18) include the large logarithmic correction proportional to $\log \frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}$. In this section, we study the resummation of this type of logarithmic corrections. Their origin is the loop correction of the light scalar, whose virtual momentum ranges from the heavy scalar mass m_2 down to the low energy, so the correction can be calculated using the low-energy effective theory without the heavy scalar. One can derive the low energy effective Lagrangian by integrating the tree-level contribution of the heavy scalar in Eq.(4.27),

$$\begin{aligned} S^{\rm Low}[\bar{\rho}_1, n] &= S[\bar{\rho}_1, 0, 0] + \bar{W}_2^{c\ \rm tree}[\bar{\rho}_1, n] \\ &= \int d^dx \left\{ \frac{1}{2}\partial_\mu\bar{\rho}_1\partial^\mu\bar{\rho}_1 - \frac{m_1^2}{2}\bar{\rho}_1^2 - \frac{\lambda_1}{4}\bar{\rho}_1^4 + h_1 {m_1}^4 + h_2 {m_2}^4 + 2h_3 {m_1}^2 {m_2}^2 + h_{12} m_{12}^4 \right\} \\ &\quad + \frac{i}{2}\int d^dx d^dy O(x) D_{F22}(x,y) O(y), \end{aligned} \tag{6.1}$$

where $D_{F22}(x,y)$ has the following form of the low energy expansion,

$$D_{F22}(x,y) = \frac{1}{i}\left(\frac{1}{m_2^2} - \frac{\Box_x + \frac{\lambda_3}{2}{\bar{\rho}_1}^2}{m_2^4} + \frac{(\Box_x + \frac{\lambda_3}{2}{\bar{\rho}_1}^2)^2}{m_2^6} \right) \delta^d(x-y). \tag{6.2}$$

Keeping the terms up to the operators of dimension six, the effective action is given by,

$$\begin{aligned} S^{\rm Low}[\bar{\rho}_1, n] = \int d^dx \Bigg\{ & \frac{1}{2}\partial_\mu\bar{\rho}_1\partial^\mu\bar{\rho}_1 - \frac{m_1^2}{2}\bar{\rho}_1^2 - \frac{\lambda_1}{4}\bar{\rho}_1^4 + \frac{m_{12}^4}{2m_2^2}\bar{\rho}_1^2 + y\frac{m_{12}^2}{m_2^2}\overline{n}n\bar{\rho}_1 \\ & + h_1 {m_1}^4 + h_2 {m_2}^4 + 2h_3 {m_1}^2 {m_2}^2 + h_{12} m_{12}^4 + \frac{m_{12}^4}{2m_2^4}\partial_\mu\bar{\rho}_1\partial^\mu\bar{\rho}_1 - \frac{\lambda_3}{4}\frac{m_{12}^4}{m_2^4}\bar{\rho}_1^4 \\ & + \frac{y^2}{2m_2^2}(\overline{n}n)^2 - y\frac{m_{12}^2}{m_2^2}\frac{\Box\bar{\rho}_1 + \frac{\lambda_3\bar{\rho}_1^3}{2}}{m_2^2}(\overline{n}n) + \frac{m_{12}^4(\Box\bar{\rho}_1 + \frac{\lambda_3\bar{\rho}_1^3}{2})^2}{2m_2^6} \Bigg\}. \end{aligned} \tag{6.3}$$

Rewriting the action with $\epsilon = \frac{m_{12}^2}{m_2^2}$ and the rescaled field $\rho_1' = \sqrt{1+\epsilon^2}\rho_1$ gives,

$$\begin{aligned} S^{\mathrm{Low}}[\bar\rho_1, n] &= \int d^d x \Bigg\{ \frac{1}{2}\partial_\mu \overline{\rho}_1' \partial^\mu \overline{\rho}_1' - \frac{m_1^2 - \epsilon m_{12}^2}{2}\overline{\rho}_1'^{\,2} - \frac{\lambda_1 + \epsilon^2(\lambda_3 - 2\lambda_1)}{4}\overline{\rho}_1'^{\,4} + \epsilon y \bar n n \overline{\rho}_1' \\ & -\epsilon y \bar n n \frac{\Box \overline{\rho}_1'}{m_2^2} + \frac{1}{2m_2^2}\left(\frac{\epsilon\lambda_3 \overline{\rho}_1'^{\,3}}{2} - y\bar n n\right)^2 + \frac{(\epsilon\Box\overline{\rho}_1')^2}{2m_2^2} + \frac{(\epsilon^2\Box\overline{\rho}_1')}{2m_2^2}\lambda_3\overline{\rho}_1'^{\,3} \\ & + h_1 {m_1}^4 + h_2 {m_2}^4 + 2h_3 {m_1}^2 {m_2}^2 + h_{12} m_{12}^4 \Bigg\}. \end{aligned} \tag{6.4}$$

Next we derive the effective potential containing one-loop effects of the light scalar and improve it with the RG equation. Concerning $d = 6$ operators, we examine the one-loop contribution to the renormalizable terms in the effective potential. As the result of this estimation, we found that the contribution is suppressed by the higher powers of ϵ and $\frac{m_1^2}{m_2^2}$. Therefore we ignore these contributions and obtain the effective potential within the following accuracy.

- Concerning higher dimensional ($d = 6$) terms, we calculate the contribution within the tree-level approximation.
- Concerning the renormalizable part of the effective potential, the one-loop contributions are contained.

By substituting the constant expectation value $(1+\epsilon^2)v_1^2$ into $\bar\rho_1'^2$ of Eq.(6.4), the effective potential at the tree-level approximation is given by,

$$\begin{aligned} V_{\mathrm{eff}}^{\mathrm{tree(Low)}} &= \frac{m_1^2 - \epsilon m_{12}^2}{2}{v_1}^2 + \frac{\lambda_1 + \epsilon^2\lambda_3}{4}{v_1}^4 - \epsilon y \bar n n v_1 - h_1 {m_1}^4 - h_2 {m_2}^4 \\ & -2h_3 {m_1}^2 {m_2}^2 - h_{12} m_{12}^4 - \frac{1}{2m_2^2}\left(\frac{\epsilon\lambda_3 {v_1}^3}{2} - y\bar n n\right)^2 . \end{aligned} \tag{6.5}$$

The total contribution which contains the one-loop corrections and its counter terms is given as follows,

$$
\begin{aligned}
V_{\rm eff}^{\rm Low} &= V_{\rm eff}^{\rm tree(Low)} + V_{\rm eff}^{\rm 1loop(Low)} + V_{\rm eff}^{c({\rm Low})} \\
&= \frac{m_1^2}{2}\left\{1 - \frac{3\lambda_1}{16\pi^2}\left(\frac{3}{2} - \log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\} v_1^2 \\
&\quad -\frac{m_{12}^2}{2}\left\{1 - \frac{3\lambda_1}{16\pi^2}\left(1 - \log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\}\epsilon v_1^2 - \epsilon y \bar{n} n v_1 \\
&\quad +\frac{\lambda_1}{4}\left\{1 - \frac{9\lambda_1}{16\pi^2}\left(\frac{3}{2} - \log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\} v_1^4 \\
&\quad +\frac{\lambda_3}{4}\left\{1 + \frac{9}{8\pi^2}\frac{\lambda_1(\lambda_1-\lambda_3)}{\lambda_3} + \frac{9}{8\pi^2}\frac{\lambda_1(\lambda_3-\lambda_1)}{\lambda_3}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right\}\epsilon^2 v_1^4 \\
&\quad -m_1^4\left\{h_1 + \frac{1}{64\pi^2}\left(\frac{3}{2} - \log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\} - h_2 {m_2}^4 \\
&\quad +2\epsilon m_1^2 m_{12}^2\left\{-\frac{h_3}{\epsilon^2} + \frac{1}{64\pi^2}\left(1 - \log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\} \\
&\quad -\epsilon^2 m_{12}^4\left\{\frac{h_{12}}{\epsilon^2} - \frac{1}{64\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right\} - \frac{1}{2m_2^2}\left(\frac{\lambda_3\epsilon}{2}v_1^3 - y\bar{n}n\right)^2 .
\end{aligned} \tag{6.6}
$$

The derivation of Eq.(6.6) is shown in the appendix B. $V_{\rm eff}^{\rm 1loop\,(Low)}$ and $V_{\rm eff}^{c({\rm Low})}$ correspond to Eq.(B1) and Eq.(B4), respectively. In order to obtain the effective potential Eq.(6.6) from Eq.(B7), we keep the terms such as $m_1^2 v_1^2 \epsilon^0$, $m_{12}^2 v_1^2 \epsilon$, v_1^4 and $\epsilon^2 v_1^4$. Concerning the cosmological constant terms, we keep the terms of the forms $m_1^4\epsilon^0$, $m_1^2 m_{12}^2\epsilon$ and $m_{12}^4\epsilon^2$. We drop the other terms which have the extra suppression factors.

Next we compare the loop effect of the light scalar for the two effective potential in Eq.(6.6) and Eq.(5.12). Note that the effective potential in Eq.(5.12) contains both heavy and light scalar loop effect. In order to compare the loop effect of the light scalar, we set the renormalization scale μ equal to m_2 in both effective potentials. When we set $\mu = m_2$, the effective potential in Eq.(5.12) becomes,

$$
\begin{aligned}
V_{\text{eff}} &= \frac{m_1^2}{2}\left[1-\frac{3\lambda_1}{16\pi^2}\frac{3}{2}-\frac{\lambda_3}{32\pi^2}\frac{m_2^2}{m_1^2}+\frac{3\lambda_1}{16\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right]v_1^2 \\
&-\frac{m_{12}^2}{2}\left[1-\frac{5\lambda_3}{32\pi^2}+\frac{3\lambda_2}{16\pi^2}+\frac{3\lambda_1}{16\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right]\epsilon v_1^2 \\
&+\frac{\lambda_1}{4}\left[1-\frac{9\lambda_1}{16\pi^2}\frac{3}{2}+\frac{9\lambda_1}{16\pi^2}\log\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right]v_1^4 \\
&+\frac{\lambda_3}{4}\left[1+\frac{6\lambda_2}{16\pi^2}-\frac{25\lambda_3}{64\pi^2}\frac{9\lambda_1}{8\pi^2}\left(1-\frac{\lambda_1}{\lambda_3}\right)\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right]\epsilon^2 v_1^4 \\
&-\left(1+\frac{3\lambda_2-\lambda_3}{16\pi^2}\right)y\epsilon\bar{n}nv_1 - m_1^4\left\{h_1+\frac{1}{64\pi^2}\left(\frac{3}{2}-\log\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right\} \\
&-m_2^4\left(h_2+\frac{3}{128\pi^2}\right)-m_{12}^4\left(h_{12}+\frac{1}{32\pi^2}\right) \\
&-2m_1^2m_2^2\left\{h_3+\frac{\epsilon^2}{64\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right\}-\frac{1}{2m_2^2}\left(\frac{\lambda_3}{2}\epsilon v_1^3-y\bar{n}n\right)^2. \qquad (6.7)
\end{aligned}
$$

Comparing Eq.(6.7) and Eq.(6.6), we can find that the coefficients of all the logarithmic term $\log\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}$ are identical to each other. This implies that the low-energy effective potential of Eq.(6.6) properly includes the loop effect of the light scalar. Since the low-energy effective potential can be improved by RG, we define the RG improved effective potential as,

$$
V_{\text{eff}}^{\text{Improved}} = V_{\text{eff}} - V_{\text{eff}}^{\text{Low}} + V_{\text{eff}}^{\text{Low RGimproved}}, \qquad (6.8)
$$

where we set the renormalization scale μ equal to m_2 in V_{eff} and $V_{\text{eff}}^{\text{Low}}$. The first two terms $V_{\text{eff}} - V_{\text{eff}}^{\text{Low}}$ on the right-hand side cancel the loop effect of the lighter scalar in V_{eff} and $V_{\text{eff}}^{\text{Low}}$. Note that this terms includes the loop effect of the heavy scalar. As shown in Eq.(6.8), $V_{\text{eff}}^{\text{Improved}}$, which properly contains the loop effect of the light and heavy scalars, can be obtained by adding the effective potential improved by RG at low energy. The solutions of RG equation at low-energy are given by,

$$
\begin{aligned}
\lambda_1'\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right) &= \frac{\lambda_1'(m_2)}{1-\frac{9\lambda_1'(m_2)}{16\pi^2}\log\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}}, \qquad (6.9) \\
m_1'^2\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right) &= \frac{m_1'^2(m_2)}{\left(1-\frac{9\lambda_1'(m_2)}{16\pi^2}\log\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)^{\frac{1}{3}}}, \qquad (6.10) \\
\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right) &= \bar{h}(m_2)-\frac{1}{64\pi^2}\log\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}, \qquad (6.11)
\end{aligned}
$$

where m_1' and λ_1' are defined in Eq.(B2). $\bar{h}$ is introduced in Eq.(B4). The derivation of Eqs.(6.9)-(6.11) is shown in appendix C. Using the solutions of RG equation of Eqs.(C6)-

(C8), the RG improved effective potential at low energy is obtained as,

$$
\begin{aligned}
V_{\rm eff}^{\rm Low\ RGimproved} &= \frac{1}{2}\frac{m_1^2(m_2)}{\left(1+\frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)\right)^{\frac{1}{3}}}\left(1-\frac{9\lambda_1(m_2)}{32\pi^2}\right)v_1^2 \\
&-\frac{1}{2}\frac{m_{12}^2(m_2)}{\left(1+\frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)\right)^{\frac{1}{3}}}\left(1-\frac{3\lambda_1(m_2)}{16\pi^2}\right)\epsilon v_1^2 \\
&+\frac{1}{4}\frac{\lambda_1(m_2)}{1+\frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)}\left(1-\frac{27\lambda_1(m_2)}{32\pi^2}\right)v_1^4 \\
&+\frac{1}{4}\frac{\lambda_3(m_2)}{1+\frac{9\lambda_1(m_2)}{8\pi^2}\left(1-\frac{\lambda_1(m_2)}{\lambda_3(m_2)}\right)\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)}\left(1-\frac{9\lambda_1(m_2)}{8\pi^2}\left(1-\frac{\lambda_1(m_2)}{\lambda_3(m_2)}\right)\right)\epsilon^2 v_1^4 \\
&-m_1^4\left\{h_1(m_2)+\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)-\bar{h}(m_2)+\frac{3}{128\pi^2}\right\} \\
&+2\epsilon m_1^2 m_{12}^2\left\{-\frac{h_3(m_2)}{\epsilon^2}+\frac{1}{64\pi^2}+\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)-\bar{h}(m_2)\right\} \\
&-\epsilon^2 m_{12}^4\left\{\frac{h_{12}(m_2)}{\epsilon^2}+\bar{h}(m_2)-\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)\right\} \\
&-\epsilon y\bar{n}n v_1-\frac{1}{2m_2^2}\left(\frac{\lambda_3\epsilon}{2}v_1^3-y\bar{n}n\right)^2. \qquad (6.12)
\end{aligned}
$$

In this form, we are able to resum the leading logarithmic corrections. Note that the large large logarithmic correction proportional to $\log\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}$ in the cosmological constant can be interpreted as the running of the coefficient for the cosmological constant of the low energy effective theory. By adding Eq.(6.12) to $V_{\rm eff}-V_{\rm eff}^{\rm Low}$, we finally obtains the renormalization group improved effective potential as,

$$
\begin{aligned}
V_{\rm eff}^{\rm Improved} &= \frac{m_1^2(m_2)}{2}\left[-\frac{\lambda_3}{32\pi^2}\frac{m_2^2}{m_1^2}+\frac{1-\frac{9\lambda_1(m_2)}{32\pi^2}}{\left(1+\frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)\right)^{\frac{1}{3}}}\right]v_1^2 \\
&-\frac{m_{12}^2(m_2)}{2}\left[-\frac{5\lambda_3}{32\pi^2}+\frac{3\lambda_2}{16\pi^2}+\frac{3\lambda_1}{16\pi^2}+\frac{1-\frac{3\lambda_1(m_2)}{16\pi^2}}{\left(1+\frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)\right)^{\frac{1}{3}}}\right]\epsilon v_1^2 \\
&+\frac{\lambda_1(m_2)}{4}\left[\frac{1-\frac{27\lambda_1(m_2)}{32\pi^2}}{1+\frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)}\right]v_1^4 \\
&+\frac{\lambda_3(m_2)}{4}\left[\frac{6\lambda_2}{16\pi^2}-\frac{25\lambda_3}{64\pi^2}+\frac{9\lambda_1}{8\pi^2}\left(1-\frac{\lambda_1}{\lambda_3}\right)+\frac{1-\frac{9\lambda_1(m_2)}{8\pi^2}\left(1-\frac{\lambda_1(m_2)}{\lambda_3(m_2)}\right)}{1+\frac{9\lambda_1(m_2)}{8\pi^2}\left(1-\frac{\lambda_1(m_2)}{\lambda_3(m_2)}\right)\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)}\right]\epsilon^2 v_1^4 \\
&-\left(1+\frac{3\lambda_2-\lambda_3}{16\pi^2}\right)y\epsilon\bar{n}nv_1 - m_1^4\left\{h_1(m_2)+\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)-\bar{h}(m_2)+\frac{3}{128\pi^2}\right\} \\
&-m_{12}^4\left\{h_{12}(m_2)+\frac{1}{32\pi^2}+\epsilon^2\left(\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)-\bar{h}(m_2)+\frac{1}{64\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right)\right\} \\
&-2m_1^2m_2^2\left\{h_3(m_2)+\frac{\epsilon^2}{64\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right\} \\
&+2\epsilon m_1^2 m_{12}^2\left\{\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)-\bar{h}(m_2)+\frac{1}{64\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right\} \\
&-m_2^4\left(h_2(m_2)+\frac{3}{128\pi^2}\right)-\frac{1}{2m_2^2}\left(\frac{\lambda_3}{2}\epsilon v_1^3-y\bar{n}n\right)^2. \qquad (6.13)
\end{aligned}
$$

In Eq.(6.13), the large logarithmic correction to the cosmological constant terms which are proportional to m_{12}^4 and $m_1^2m_2^2$, $m_1^2m_{12}^2$ remains. Concerning the cosmological constant term which are proportional to m_{12}^4, the large logarithmic correction can be canceled by the running of the coefficient for the cosmological constant of the low energy effective theory. Using Eq.(6.11), we can show the relation,

$$
\begin{aligned}
&-m_{12}^4\left\{h_{12}(m_2)+\frac{1}{32\pi^2}+\epsilon^2\left(\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)-\bar{h}(m_2)+\frac{1}{64\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right)\right\} \\
= &-m_{12}^4\left\{h_{12}(m_2)+\frac{1}{32\pi^2}\right\}. \qquad (6.14)
\end{aligned}
$$

Concerning the cosmological constant terms which are proportional to $m_1^2m_2^2$ and $m_1^2m_{12}^2$, these terms can be calculated as,

$$
\begin{aligned}
&-2m_1^2m_2^2\left\{h_3(m_2)+\frac{\epsilon^2}{64\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right\}+2\epsilon m_1^2m_{12}^2\left\{\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)-\bar{h}(m_2)+\frac{1}{64\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right\} \\
= &-2m_1^2m_{12}^2\epsilon\left\{\frac{1}{\epsilon^2}h_3(m_2)+\bar{h}(m_2)-\bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)\right\}. \qquad (6.15)
\end{aligned}
$$

In this calculation, we write $m_1^2 m_2^2$ as $\frac{1}{\epsilon} m_1^2 m_{12}^2$. Using Eqs.(6.14)-(6.15), we rewrite the renormalization improved effective potential of Eq.(6.13) as,

$$
\begin{aligned}
&V_{\mathrm{eff}}^{\mathrm{Improved}} \\
&= \frac{m_1^2(m_2)}{2}\left[-\frac{\lambda_3}{32\pi^2}\frac{m_2^2}{m_1^2} + \frac{1-\frac{9\lambda_1(m_2)}{32\pi^2}}{\left(1+\frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)\right)^{\frac{1}{3}}}\right] v_1^2 \\
&-\frac{m_{12}^2(m_2)}{2}\left[-\frac{5\lambda_3}{32\pi^2} + \frac{3\lambda_2}{16\pi^2} + \frac{3\lambda_1}{16\pi^2} + \frac{1-\frac{3\lambda_1(m_2)}{16\pi^2}}{\left(1+\frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)\right)^{\frac{1}{3}}}\right]\epsilon v_1^2 \\
&+\frac{\lambda_1(m_2)}{4}\left[\frac{1-\frac{27\lambda_1(m_2)}{32\pi^2}}{1+\frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)}\right] v_1^4 \\
&+\frac{\lambda_3(m_2)}{4}\left[\frac{6\lambda_2}{16\pi^2} - \frac{25\lambda_3}{64\pi^2} + \frac{9\lambda_1}{8\pi^2}\left(1-\frac{\lambda_1}{\lambda_3}\right) + \frac{1-\frac{9\lambda_1(m_2)}{8\pi^2}\left(1-\frac{\lambda_1(m_2)}{\lambda_3(m_2)}\right)}{1+\frac{9\lambda_1(m_2)}{8\pi^2}\left(1-\frac{\lambda_1(m_2)}{\lambda_3(m_2)}\right)\log\left(\frac{m_2^2}{m_1^2+3\lambda_1 v_1^2}\right)}\right]\epsilon^2 v_1^4 \\
&-\left(1+\frac{3\lambda_2-\lambda_3}{16\pi^2}\right) y\epsilon\bar{n}n v_1 - m_1^4\left\{h_1(m_2) + \frac{3}{128\pi^2} - \bar{h}(m_2) + \bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)\right\} \\
&-m_2^4\left(h_2(m_2) + \frac{3}{128\pi^2}\right) - m_{12}^4\left(h_{12}(m_2) + \frac{1}{32\pi^2}\right) \\
&-2m_1^2 m_{12}^2\epsilon\left\{\frac{1}{\epsilon^2}h_3(m_2) + \bar{h}(m_2) - \bar{h}\left(\sqrt{m_1^2+3\lambda_1 v_1^2}\right)\right\} - \frac{1}{2m_2^2}\left(\frac{\lambda_3}{2}\epsilon v_1^3 - y\bar{n}n\right)^2. \qquad (6.16)
\end{aligned}
$$

This is the final formula of the RG improved effective potential.

Chapter 7

Numerical analysis

In this section, we study the RG improved effective potential and the VEV of the light scalar by numerical analysis.

7.1 The VEV of the light scalar

To explain how VEV depends on heavy scalar mass, we study the stationary condition of the effective potential,

$$\frac{\partial V_{\text{eff}}^{\text{Improved}}}{\partial v_1} = 0. \tag{7.1}$$

When we keep the leading logarithmic correction and the correction proportional to the mass squared of the heavy scalar, the solution of stationary condition is given as,

$$\begin{aligned} \frac{v_1}{v_{10}} &= \sqrt{\left[-\frac{\lambda_3 m_2^2}{32\pi^2 m_1^2} + \frac{1}{\left\{1 - \frac{9\lambda_1}{16\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right\}^{\frac{1}{3}}}\right]\left[1 - \frac{9\lambda_1}{16\pi^2}\log\left(\frac{m_1^2 + 3\lambda_1 v_1^2}{m_2^2}\right)\right]}, \\ v_{10} &= \sqrt{-\frac{m_1^2}{\lambda_1}}, \end{aligned} \tag{7.2}$$

where v_{10} corresponds to the solution of the level effective potential. The other corrections suppresed by ϵ to the power of $n(n = 1, 2...)$ are ignored. In Fig.7.1, the ratio $\frac{v_1}{v_{10}}$ in Eq.(7.2) is plotted as the function of the heavy scalar mass m_2. The parameters are fixed as $m_1^2 = -(100)^2(\text{GeV})^2$ and $\lambda_1 = \lambda_3 = 1$. This corresponds to $v_{10} = 100$ (GeV). As the heavy scalar mass increases, the correction to VEV increases. By requiring the VEV does not exceed 120 % compared to its tree level value, the upper limit for the heavy scalar mass is about 1000 (GeV). In the region where the heavy scalar mass is larger than 500 (GeV), the condition $m_2^2 \gg |m_1^2|, m_{12}^2$ is easily fulfilled.

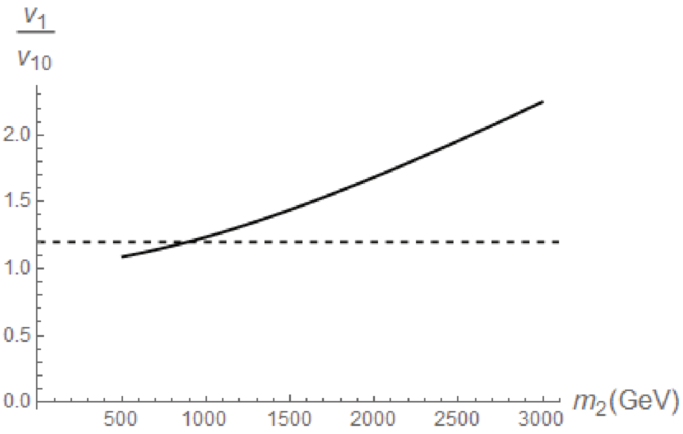

Figure 7.1: The VEV's ratio $\frac{v_1}{v_{10}}$ in Eq.(7.2) is plotted as the function of the heavy scalar mass m_2. The dashed line corresponds to $\frac{v_1}{v_{10}} = 1.2$.

Next we study how the upper limit for the heavy scalar mass changes by varying the coupling constant λ_3. In Fig.7.2, the ratio $\frac{v_1}{v_{10}}$ for $\lambda_3 = 1, 0.1, 0.01$ is shown. For the same reason as in the Fig.7.1, the ratio for the region $m_2 > 500$ GeV is drawn. By requiring the VEV does not exceed 120 % compared to its tree level value, the upper limit for the heavy scalar mass is determined for a fixed value of λ_3. The upper limit for the heavy scalar mass are shown in Table7.1. In Fig.7.3, it is plotted as the function of the coupling λ_3. It turns out that it increases as λ_3 decreases.

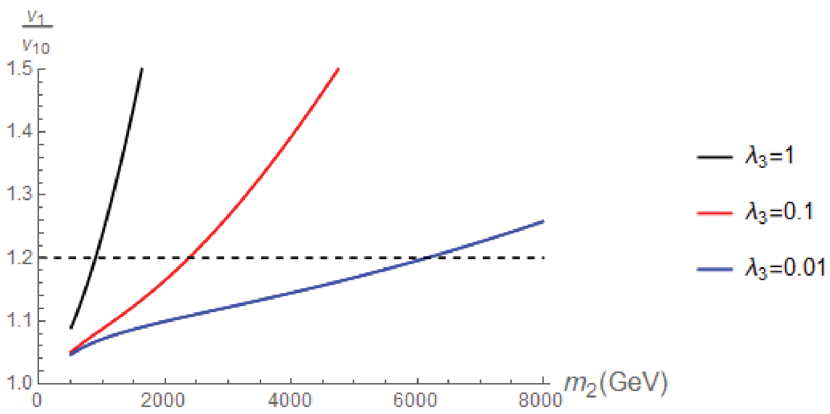

Figure 7.2: The VEV's ratios $\frac{v_1}{v_{10}}$ for $\lambda_3 = 1, 0.1, 0.01$ are plotted as the function of the heavy scalar mass m_2. In this figure, we fixed $m_1^2 = -(100)^2(\text{GeV})^2$. The dashed line corresponds to $\frac{v_1}{v_{10}} = 1.2$.

λ_3	1	0.1	0.01
m_2 (GeV)	1000	2500	6000

Table 7.1: The upper limits for the heavy scalar mass with the different λ_3.

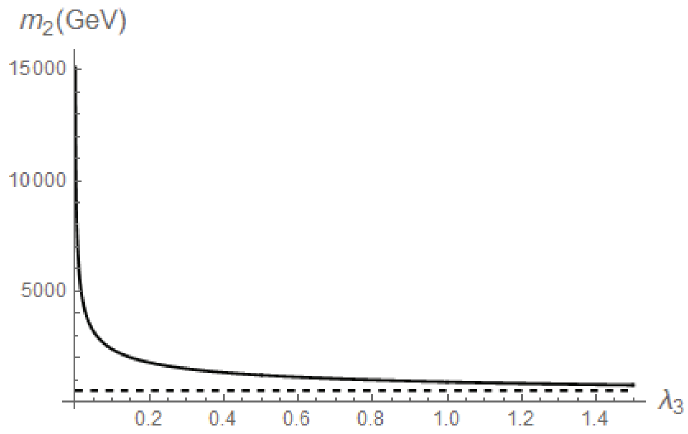

Figure 7.3: The heavy scalar mass is plotted as the function of the coupling λ_3. In this figure, we fixed $m_1^2 = -(100)^2(\text{GeV})^2$. The dashed line corresponds to $m_2 = 500$ (GeV).

7.2 The RG improved effective potential

In this subsection, we show the RG improved effective potential as a function of the VEV of the light scalar numerically. There are two possibilities for the mass hierarchy that satisfies Eq.(4.1) as follows,

$$\text{(i)} \quad m_2^2 \gg -|m_1|^2 \gg m_{12}^2 > 0, \quad \text{(ii)} \quad m_2^2 \gg -|m_1|^2 \simeq m_{12}^2 > 0,$$

In order to estimate the effect of the difference of mass hierarchies, we show the effective potential with m_{12} varied. In the following effective potential, we fixed $m_1^2 = -100$ (GeV)^2 and $m_2 = 1000$ GeV. In Fig.7.4, we show the effective potential with $m_{12} = 10$ GeV, 50 GeV and 100 GeV, respectively. The height of the effective potential with a fixed value of the VEV decreases as m_{12}^2 increases. For the small m_{12} which satisfies the condition (1), the change of the height is small. However, for the larger m_{12} which satisfies the condition (2), the change is visible as drawn with the blue colored line in Fig7.4. We can also see that v_1 which gives the minimum of the potential is about 120 GeV and it does not depend on the value of m_{12}.

In Fig.7.5, we show the effective potential with two different values for m_2. The left panel of Fig.7.5 corresponds to $m_2 = 1000$ GeV and the right panel corresponds to $m_2 = 3000$ GeV. The VEV which gives the minimum of the potential for the latter case is twice larger than that of the former case. This is consistent with the result of subsection 7.1, where the VEV of the light scalar is shown to be an increasing function with respect to m_2. The height of the potential at the fixed value of VEV goes down significantly as m_2 becomes larger.

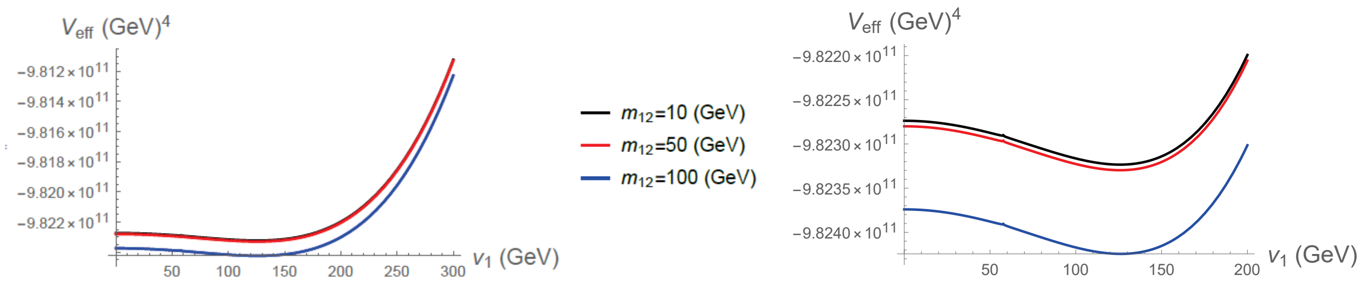

Figure 7.4: In the left panel, the RG improved effective potential on the VEV of the light scalar are shown. We fixed $m_2 = 1000$ GeV and $m_1^2 = -(100)^2$ (GeV)^2. Concerning m_{12} we show the three case, 10 GeV, 50 GeV, 100 GeV. Concerning the couplings, we fixed $\lambda_1 = \lambda_2 = \lambda_3 = 1$. Concerning the cosmological constant, we choose $h_1(m_2) = h_2(m_2) = h_3(m_2) = h_{12}(m_2) = 1$ for simplicity. In the right panel, we focus on the region v_1 is from 0 to 200 GeV. The paramaters are fixed the same value to the left panel.

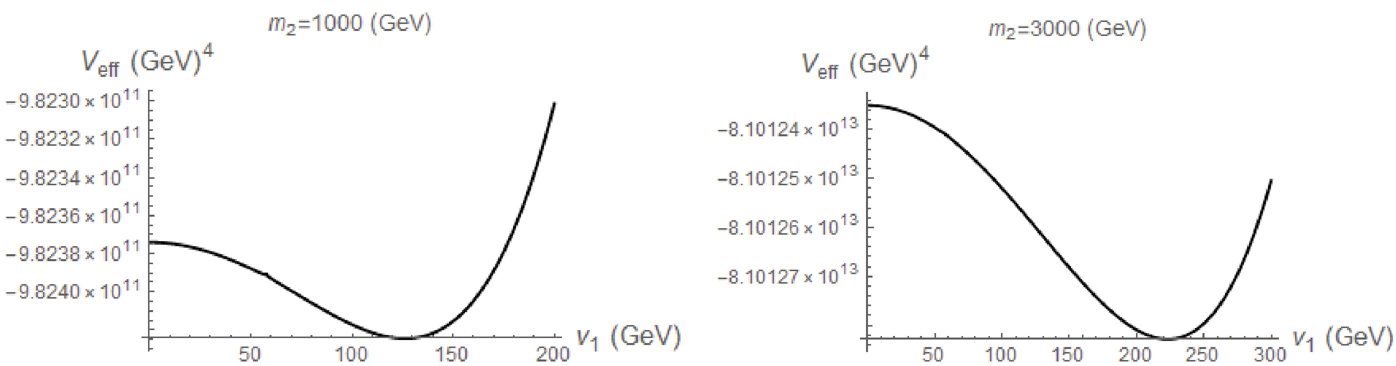

Figure 7.5: The RG improved effective potential with different m_2 is plotted as the function of the VEV. m_1^2 is fixed $-(100)^2$ (GeV)^2. Concerning the couplings, we fixed $\lambda_1 = \lambda_2 = \lambda_3 = 1$. Concerning the cosmological constant, we choose $h_1(m_2) = h_2(m_2) = h_3(m_2) = h_{12}(m_2) = 1$ for simplicity.

7.3 The estimation of the effective Yukawa coupling of the neutrino

In the present model, even if the Yukawa coupling between the neutrino and the second scalar denoted as y is $\mathcal{O}(1)$, the effective Yukawa coupling between the neutrnino and the light scalar denoted as $y\epsilon$ is naturally suppressed by a factor of $\epsilon = \frac{m_{12}^2}{m_2^2}$. Conversely, the order of the effective Yukawa coupling of the neutrino can be used to limit the ratio, ϵ. We consider the case where the effective Yukawa coupling ($y\epsilon$) is the order of 10^{-10} and the Yukawa coupling (y) is unity. We show the three sets of the parameters (m_2, m_{12}) which lead to $y\epsilon = \epsilon = 10^{-10}$ in Table.7.2

m_2	500 (GeV)	1 (TeV)	10 (TeV)
m_{12}	5 (MeV)	10 (MeV)	100 (MeV)

Table 7.2: The sets of the parameters (m_2, m_{12}) which leads to $\epsilon = 10^{-10}$ is shown.

Chapter 8

Summary and discussion

In this work, we studied a model with the light scalar and the heavy scalar. In the model, the Yukawa interaction between the neutrino and the heavy scalar is introdued. The cosmological constant terms are added. They are related to the mass parameters in the model. By integrating both light and heavy scalars out, we derived the low-energy effective potential. The effective potential is given as the function of the VEV of the light scalar. This is achieved by introducing the generating function with the source only for the light scalar. In this way, the one particle irreducibility for the light scalar is maintained, and the diagrams in which the heavy scalar is exchanged are included.

It turns out that the effective potential is independent of the renormalization scale approximately. We set the renormalization scale equal to the mass of the heavy scalar. With this choice of the renormalization scale, the large logarithmic corrections come from the loop effect of the light scalar. We resum the large logarithmic corrections with the RG equation.

The effective Yukawa coupling $Y_{\rm eff}$ between the light scalar and the neutrino, was found to be inversely proportional to the mass squared of the heavy scalar as $Y_{\rm eff} = y_{\rm eff}\frac{m_{12}^2}{m_2^2}$ and is naturally suppressed. The dimension six operators such as the six-point interaction for the light scalar and the four-Fermi interaction for the neutrino are also generated in the tree level. As for the light scalar mass, one finds the large radiative correction proportional to the heavy scalar mass squared. The cosmological constant also suffers from the large contribution proportional to the fourth power of the heavy scalar mass. Concerning the renormalizable coupling such as the quartic interaction of the light scalar, the contribution suppressed by a factor of ϵ^2 is contained. This implies the effective coupling constants and masses of the low-energy effective potential are sensitive to the couplings and the mass which are related to the heavy scalar.

We also numerically study the effect of the heavy scalar on VEV using the stationary condition of the RG improved effective potential. The VEV depends on the heavy scalar mass and we can set the upper limit of the heavy scalar mass by requiring that the radiative correction to the VEV should be limited within a certain range. Combined with the suppression factor of the Yukawa coupling for the neutrino, one can also obtain the limit for the mixing mass. Those limits increase as the coupling constant among the light and the heavy scalars becomes small. We also numerically study how the shape and the height of potential vary by changing the heavy scalar mass and the mixing mass. The height of the effective potential depends on the heavy scalar mass and the mixing mass. This is because the RG improved effective potential contains the term proportional to m_2^4 and m_{12}^4 with the negative sign. As the result, the height of the potential decreases as m_2 (m_{12}) increases.

Acknowledgement

I would like to thank my supervisor, Takuya Morozumi for the valuable discussions and advice through this work and research activities. His research attitude was a great guide to my research. I would like to thank my collaborator Apriadi Salim Adam for all the useful discussions and advice. I am thankful to Yusuke Shimizu and Kei Yamamoto for many useful discussions. The discussion with Takuya Morozumi, Apriadi Salim Adam, Yusuke Shimizu and Kei Yamamoto was very helpful for me to extend my knowledge about theory and phenomenology of the particle physics. I am thankful to staff members in the Theoretical Particle and Hadron Physics Group at Hiroshima University: Tomohiro Inagaki, Chiho Nonaka and Ken-Ichi Ishikawa. I would also like to thank Professor Emeritus Masanori Okawa. They always provided me useful advices and comments on my research. I am also grateful to students in the Theoretical Particle and Hadron Physics Group at Hiroshima University for lots of advice and discussion. Finally, I would like to express my appreciation for my parents. They always encouraged me and supported me from many aspect.

Appendix A

The counter terms for the full theory

In this appendix, we derive the counter terms for the full theory. We also derive the RG equation and study the renormalization point independence of the effective masses and couplings. Since there is no wave function renormalization from one-loop contribution of scalar fields, it is sufficient to study the counter terms for the effective potential. The Lagrangian density for the full theory in terms of the bare masses, couplings and fields is given by

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2}\left(\sum_{i=1}^{2}\rho_{0i}\Box\rho_{0i}+\sum_{i=1}^{2}\rho_{0i}^{2}m_{0i}^{2}+\frac{1}{2}\left(\sum_{i=1}^{2}\rho_{0i}^{4}\lambda_{i}+\rho_{01}^{2}\rho_{02}^{2}\lambda_{03}\right)\right)\\
& -(y_{0}\bar{n}_{0}n_{0}+m_{012}^{2}\rho_{01})\rho_{02}+\bar{n}_{0}i\not{\partial}n_{0}+\mu^{d-4}Z_{h_1}h_{1}m_{1}^{4}+\mu^{d-4}Z_{h_2}h_{2}m_{2}^{4}+\mu^{d-4}Z_{h_{12}}h_{12}m_{12}^{4}.
\end{aligned}
\tag{A1}
$$

It in terms of the renormalized quantities can be written as follows,

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2}\left(\sum_{i=1}^{2}Z_{i}\rho_{i}\Box\rho_{i}+\sum_{i,j=1}^{2}\rho_{i}^{2}Z_{\mathrm{mij}}m_{j}^{2}+\frac{\mu^{2\eta}}{2}\sum_{I=1}^{3}\left(\sum_{i=1}^{2}\rho_{i}^{4}Z_{\lambda\,\mathrm{iI}}\lambda_{I}+\rho_{1}^{2}\rho_{2}^{2}Z_{\lambda 3I}\lambda_{I}\right)\right)\\
& -(Z_{y}y\mu^{\eta}\bar{n}n+Z_{12}m_{12}^{2}\rho_{1})\rho_{2}+i\bar{n}\not{\partial}nZ_{n}+\mu^{d-4}Z_{h_1}h_{1}m_{1}^{4}+\mu^{d-4}Z_{h_2}h_{2}m_{2}^{4}+\mu^{d-4}Z_{h_{12}}h_{12}m_{12}^{4}.
\end{aligned}
\tag{A2}
$$

The counter terms are given by,

$$
\begin{aligned}
\mathcal{L}_{c} \;=\; & -\frac{1}{2}\sum_{i=1}^{2}(Z_{i}-1)\rho_{i}\Box\rho_{i}-\frac{1}{2}\sum_{i,j=1}^{2}\rho_{i}^{2}(Z_{\mathrm{mij}}-\delta_{\mathrm{ij}})m_{j}^{2}-((Z_{y}-1)y\mu^{\eta}\bar{n}n+(Z_{12}-1)m_{12}^{2}\rho_{1})\rho_{2}\\
& -\frac{\mu^{2\eta}}{4}\sum_{I=1}^{3}\left(\sum_{i=1}^{2}\rho_{i}^{4}(Z_{\lambda\,\mathrm{iI}}-\delta_{\mathrm{iI}})\lambda_{I}+\rho_{1}^{2}\rho_{2}^{2}(Z_{\lambda 3I}-\delta_{3I})\lambda_{I}\right)\\
& +i\bar{n}\not{\partial}n(Z_{n}-1)-\mu^{d-4}(Z_{h_1}-1)h_{1}m_{1}^{4}-\mu^{d-4}(Z_{h_2}-1)h_{2}m_{2}^{4}-\mu^{d-4}(Z_{h_{12}}-1)h_{12}m_{12}^{4}.
\end{aligned}
$$

The one-loop effective potential is computed as,

$$\begin{aligned} V_{\rm eff}^{1\,\rm loop} &= -i\frac{1}{2V^{d-1}T}\log\det\left(\frac{-\delta^2 S_{\rm tree}}{\delta\rho_i(x)\delta\rho_j(y)}\bigg|_{\rho_i=v_i\mu^{-\eta}}\right)+V_c \\ &= \frac{1}{2}\int\frac{d^dk}{(2\pi)^d i}\log\Big[\left(m_1^2+\left(3\lambda_1 v_1^2+\frac{\lambda_3}{2}v_2^2\right)-k^2\right)\left(m_2^2+\left(3\lambda_2 v_2^2+\frac{\lambda_3}{2}v_1^2\right)-k^2\right) \\ &\quad -(m_{12}^2+v_1v_2\lambda_3)^2\Big]+V_c. \end{aligned} \tag{A3}$$

where V_c is the counter tems for effective potential and it is given by,

$$\begin{aligned} V_c &= \frac{\mu^{-2\eta}}{2}\Big[\{(Z_{m11}-1)m_1^2+Z_{m12}m_2^2\}v_1^2+\{(Z_{m22}-1)m_2^2+Z_{m21}m_1^2\}v_2^2+2(Z_{12}-1)m_{12}^2v_1v_2\Big] \\ &+\mu^{-2\eta}\frac{(Z_{\lambda11}-1)\lambda_1+Z_{\lambda12}\lambda_2+Z_{\lambda13}\lambda_3}{4}v_1^4+\mu^{-2\eta}\frac{Z_{\lambda21}\lambda_1+(Z_{\lambda22}-1)\lambda_2+Z_{\lambda23}\lambda_3}{4}v_2^4 \\ &+\mu^{-2\eta}\frac{Z_{\lambda31}\lambda_1+Z_{\lambda32}\lambda_2+(Z_{\lambda33}-1)\lambda_3}{4}v_1^2v_2^2+(Z_y-1)y\bar{n}nv_2 \\ &-\mu^{-2\eta}\{(Z_{h_1}-1)h_1m_1^4+(Z_{h_2}-1)h_2m_2^4+(Z_{h_{12}}-1)h_{12}m_{12}^4\}. \end{aligned} \tag{A4}$$

The one-loop effective potential can be written as follows by expanding the logarithmic terms up to $(m_{12}^2+v_1v_2\lambda_3)^2$.

$$\begin{aligned} V_{\rm eff}^{1\,\rm loop} &\simeq \frac{1}{2}\int\frac{d^dk}{(2\pi)^d i}\left\{\log(m_1^2+3\lambda_1v_1^2+\frac{\lambda_3}{2}v_2^2-k^2)+\log(m_2^2+3\lambda_2v_2^2+\frac{\lambda_3}{2}v_1^2-k^2)\right\} \\ &\quad -\frac{1}{2}\int\frac{d^dk}{(2\pi)^d i}\frac{(m_{12}^2+v_1v_2\lambda_3)^2}{(m_1^2+3\lambda_1v_1^2+\frac{\lambda_3}{2}v_2^2-k^2)(m_2^2+3\lambda_2v_2^2+\frac{\lambda_3}{2}v_1^2-k^2)}+V_c+\ldots \end{aligned} \tag{A5}$$

This contribution can be calculated by using the following general d-dimentional integration formulas and Feynman parameters formula,

$$\begin{aligned} \int\frac{d^dk}{i(2\pi)^d}\ln(m^2-k^2) &= -\frac{\Gamma[-\frac{d}{2}]}{(4\pi)^{\frac{d}{2}}}\left(m^2\right)^{\frac{d}{2}}, \\ \int\frac{d^dk}{i(2\pi)^d}\frac{1}{(m^2-k^2)^\alpha} &= \frac{\Gamma[\alpha-\frac{d}{2}]}{(4\pi)^{\frac{d}{2}}\Gamma[\alpha]}\frac{1}{(m^2)^{\alpha-\frac{d}{2}}}, \\ \frac{1}{ab} &= \int_0^1 dx\frac{1}{[ax+b(1-x)]^2}. \end{aligned} \tag{A6}$$

We collect only the one-loop divergence parts and the counter terms as follows,

$$
\begin{aligned}
& V_{\text{eff}}^{1\,\text{loop div}} + V_c \\
= \; & -\frac{C_{\text{UV}}}{64\pi^2}\{m_1^4 + m_2^4 + 2m_{12}^4\} - \frac{C_{\text{UV}}}{32\pi^2}\left(3m_1^2\lambda_1 + m_2^2\frac{\lambda_3}{2}\right)v_1^2 \\
& -\frac{C_{\text{UV}}}{32\pi^2}\left(3m_2^2\lambda_2 + m_1^2\frac{\lambda_3}{2}\right)v_2^2 - \frac{C_{\text{UV}}}{16\pi^2}m_{12}^2\lambda_3 v_1 v_2 \\
& -\frac{C_{\text{UV}}}{64\pi^2}\left\{\left(9\lambda_1^2 + \frac{\lambda_3^2}{4}\right)v_1^4 + \left(9\lambda_2^2 + \frac{\lambda_3^2}{4}\right)v_2^4 + 3(\lambda_1+\lambda_2)\lambda_3 v_1^2 v_2^2 + 2\lambda_3^2 v_1^2 v_2^2\right\} \\
& +\frac{\mu^{-2\eta}}{2}\left[\{(Z_{m11}-1)m_1^2 + Z_{m12}m_2^2\}v_1^2 + \{(Z_{m22}-1)m_2^2 + Z_{m21}m_1^2\}v_2^2 + 2(Z_{12}-1)m_{12}^2 v_1 v_2\right] \\
& +\mu^{-2\eta}\frac{(Z_{\lambda 11}-1)\lambda_1 + Z_{\lambda 12}\lambda_2 + Z_{\lambda 13}\lambda_3}{4}v_1^4 + \mu^{-2\eta}\frac{Z_{\lambda 21}\lambda_1 + (Z_{\lambda 22}-1)\lambda_2 + Z_{\lambda 23}\lambda_3}{4}v_2^4 \\
& +\mu^{-2\eta}\frac{Z_{\lambda 31}\lambda_1 + Z_{\lambda 32}\lambda_2 + (Z_{\lambda 33}-1)\lambda_3}{4}v_1^2 v_2^2 + (Z_y - 1)y\bar{n}n v_2 \\
& -\mu^{-2\eta}\{(Z_{h_1}-1)h_1 m_1^4 + (Z_{h_2}-1)h_2 m_2^4 + (Z_{h_{12}}-1)h_{12}m_{12}^4\},
\end{aligned}
\tag{A7}
$$

where the divergence C_{UV} is defined by,

$$
C_{\text{UV}} = \frac{1}{\eta} - \gamma + \log 4\pi. \tag{A8}
$$

We require that the counter terms eliminate all the divergences as,

$$
V_c \;=\; -V_{\text{eff}}^{1\,\text{loop div}}. \tag{A9}
$$

This condition can determine the Z factors of the counter terms as follows,

$$
\begin{aligned}
Z_{m\,\text{ii}} &= 1 + \frac{3C_{\text{UV}}}{16\pi^2}\lambda_i, \quad Z_{m12} = Z_{m21} = \frac{C_{\text{UV}}}{32\pi^2}\lambda_3, && \text{(A10)} \\
Z_{12} &= 1 + \frac{C_{\text{UV}}}{16\pi^2}\lambda_3, \quad Z_{\lambda\,\text{ii}} = 1 + \frac{C_{\text{UV}}}{16\pi^2}9\lambda_i, && \text{(A11)} \\
Z_{\lambda i3} &= \frac{C_{\text{UV}}}{64\pi^2}\lambda_3, \quad Z_{\lambda 3i} = \frac{C_{\text{UV}}}{16\pi^2}3\lambda_3, \quad Z_{\lambda 33} = 1 + \frac{C_{\text{UV}}}{8\pi^2}\lambda_3, && \text{(A12)} \\
Z_y &= 1, \quad Z_{\lambda 12} = Z_{\lambda 21} = 0, && \text{(A13)} \\
(Z_{h_i}-1)h_i &= -\frac{C_{\text{UV}}}{64\pi^2}, \quad (Z_{h_{12}}-1)h_{12} = -\frac{C_{\text{UV}}}{32\pi^2}, && \text{(A14)}
\end{aligned}
$$

where $i = 1, 2$. This completes the derivation of the Z factors in the counter terms for the full theory.

Next we derive the RG equations for the parameters of theory and study the renormalization point independence of their parameters. The RG equations for the coupling constants can be derived as,

$$
\begin{aligned}
\mu\frac{d\lambda_3}{d\mu} &= \frac{1}{8\pi^2}(2\lambda_3 + 3\lambda_2 + 3\lambda_1)\lambda_3, && \text{(A15)} \\
\mu\frac{d\lambda_i}{d\mu} &= \frac{1}{8\pi^2}9\lambda_i^2 + \frac{1}{32\pi^2}\lambda_3^2, \quad (i = 1, 2). && \text{(A16)}
\end{aligned}
$$

The RG equations for the mass parameters is given by,

$$\mu\frac{dm_1^2}{d\mu} = \frac{3}{8\pi^2}\lambda_1 m_1^2 + \frac{1}{16\pi^2}\lambda_3 m_2^2, \tag{A17}$$

$$\mu\frac{dm_2^2}{d\mu} = \frac{3}{8\pi^2}\lambda_2 m_2^2 + \frac{1}{16\pi^2}\lambda_3 m_1^2, \tag{A18}$$

$$\mu\frac{dm_{12}^2}{d\mu} = \frac{\lambda_3}{8\pi^2}m_{12}^2. \tag{A19}$$

The coefficients of the cosmological constants satisfy the following RG equations,

$$\mu\frac{dh_1}{d\mu} = \frac{-1}{16\pi^2}\left(\frac{1}{2} + 12\lambda_1 h_1 + 2\lambda_3 h_3\right), \tag{A20}$$

$$\mu\frac{dh_2}{d\mu} = \frac{-1}{16\pi^2}\left(\frac{1}{2} + 12\lambda_2 h_2 + 2\lambda_3 h_3\right), \tag{A21}$$

$$\mu\frac{dh_3}{d\mu} = \frac{-1}{16\pi^2}(\lambda_3(h_1+h_2) + 6(\lambda_2+\lambda_1)h_3), \tag{A22}$$

$$\mu\frac{dh_{12}}{d\mu} = -\frac{1}{16\pi^2}(1 + 4\lambda_3 h_{12}). \tag{A23}$$

Using the RG equations of Eqs.(A15)-(A23), we can examine the independence of the renormalization points of the effective masses and the effective coupling constants in one loop effective potential of the Eq.(5.12). The independence of the renormalization points of the these parameters of Eqs.(5.14)-(5.17) and Eq.(5.22) are given by,

$$\mu\frac{dm_{1\,\mathrm{eff}}^2}{d\mu} = m_1^2\frac{3\lambda_1}{8\pi^2} + m_2^2\frac{\lambda_3}{16\pi^2} - m_1^2\frac{3\lambda_1}{8\pi^2} - \frac{\lambda_3 m_2^2}{16\pi^2} = 0, \tag{A24}$$

$$\mu\frac{d(m_{12\,\mathrm{eff}}^2\epsilon)}{d\mu} = \frac{m_{12}^4}{m_2^2}\left\{\frac{\lambda_3}{4\pi^2} - \frac{3\lambda_2}{8\pi^2} + \frac{3\lambda_2}{8\pi^2} - \frac{\lambda_3}{4\pi^2} + \frac{\lambda_3}{16\pi^2}\frac{m_1^2}{m_2^2}\right\} = \frac{\lambda_3 m_{12}^2\epsilon}{16\pi^2}\left(\frac{m_1^2}{m_2^2}\right), \tag{A25}$$

$$\mu\frac{d\lambda_{1\,\mathrm{eff}}}{d\mu} = \frac{9\lambda_1^2}{8\pi^2} + \frac{\lambda_3^2}{32\pi^2} + \lambda_1\left\{-\frac{9\lambda_1}{8\pi^2} - \frac{1}{32\pi^2}\frac{\lambda_3^2}{\lambda_1}\right\} = 0, \tag{A26}$$

$$\begin{aligned}\mu\frac{d(\lambda_{3\,\mathrm{eff}}\epsilon^2)}{d\mu} &= \epsilon^2\left\{\frac{\lambda_3(2\lambda_3 + 3\lambda_2 + 3\lambda_1 + 3\lambda_2 - 4\lambda_3 - 3\lambda_1 + 2\lambda_3 - 6\lambda_2)}{8\pi^2}\right\} - \frac{\lambda_3\epsilon^2}{8\pi^2}\frac{m_1^2}{m_2^2} \\ &= -\frac{\lambda_3\epsilon^2}{8\pi^2}\left(\frac{m_1^2}{m_2^2}\right),\end{aligned} \tag{A27}$$

$$\mu\frac{d(y_{\mathrm{eff}}\epsilon)}{d\mu} = \epsilon y\left\{\frac{3\lambda_2 - \lambda_3 + \lambda_3 - 3\lambda_2}{8\pi^2}\right\} - \frac{\epsilon y\lambda_3}{16\pi^2}\frac{m_1^2}{m_2^2} = -\frac{\lambda_3 y\epsilon}{16\pi^2}\left(\frac{m_1^2}{m_2^2}\right). \tag{A28}$$

By ignoring the sub-reading correction $O(\frac{m_1^2}{m_2^2})$, we can find that all effective masses and couplings do not depend on the renormalization point. The remaining dependency is due to the truncation of these suppressed contributions in deriving the effective potential. The cosmological constants of Eq.(5.13) is written as follows,

$$
\begin{aligned}
V_{\text{cosmo}} &= -\frac{m_{12}^4}{32\pi^2}\left(1-\log\frac{m_2^2}{\mu^2}\right)-\frac{m_1^4}{64\pi^2}\left(\frac{3}{2}-\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\\
&\quad -\frac{m_2^4}{64\pi^2}\left(\frac{3}{2}-\log\frac{m_2^2}{\mu^2}\right)-h_1 m_1^4-h_2 m_2^4-h_{12}m_{12}^4-2h_3 m_1^2 m_2^2.
\end{aligned}
\tag{A29}
$$

The renormalization point independence of V_{cosmo} is explicitly shown as,

$$
\begin{aligned}
\mu\frac{dV_{\text{cosmo}}}{d\mu} &= \frac{2\lambda_3 h_3}{16\pi^2}(m_1^4+m_2^4)-2(h_1+h_2)m_1^2\frac{1}{16\pi^2}\lambda_3 m_2^2\\
&\quad +\frac{2}{16\pi^2}(\lambda_3(h_1+h_2))m_1^2 m_2^2-h_3\frac{2}{16\pi^2}\lambda_3(m_2^4+m_1^4)\\
&= 0.
\end{aligned}
\tag{A30}
$$

Appendix B

Derivation of $V_{\rm eff}^{\rm Low}$ in Eq.(6.6)

In this appendix, we show the outline of the derivation of Eq.(6.6). In Eq.(6.6), $V_{\rm eff}^{\rm 1\,loop}$ is given by,

$$\begin{aligned} V_{\rm eff}^{\rm 1\,loop(Low)} &= \frac{1}{2}\int \frac{d^dk}{(2\pi)^d i}\log(m_1'^2 + 3\lambda_1' v_1^2 - k^2) \\ &= -\frac{(m_1'^2+3\lambda_1'v_1^2)^2}{64\pi^2}\left(C_{UV} + \frac{3}{2} - \log(m_1'^2 + 3\lambda_1'v_1^2)\right), \end{aligned} \quad \text{(B1)}$$

where the divergence $C_{\rm UV}$ is given by Eq.(A8). The coupling constant and the mass are defined as,

$$m_1'^2 = m_1^2 - \epsilon m_{12}^2, \quad \lambda_1' = \lambda_1 + \epsilon^2(\lambda_3 - \lambda_1). \quad \text{(B2)}$$

If only the divergent parts is extracted from the Eq.(B1), it is derived as,

$$\begin{aligned} V_{\rm eff}^{\rm 1\,loop(Low)}\Big|_{\rm div.} &= -\frac{(m_1'^2+3\lambda_1'v_1^2)^2}{64\pi^2}C_{UV} \\ &= -\frac{(m_1^2-\epsilon m_{12}^2)^2 + 6(m_1^2-\epsilon m_{12}^2)(\lambda_1+\epsilon^2(\lambda_3-\lambda_1))v_1^2 + 9(\lambda_1+\epsilon^2(\lambda_3-\lambda_1))^2v_1^4}{64\pi^2}C_{UV}. \end{aligned} \quad \text{(B3)}$$

The counter term in order to subtract this divergence is given by,

$$V_{\rm eff}^{c\rm (Low)} = \frac{1}{2}(Z_{m_1'}-1)(m_1^2-\epsilon m_{12}^2)\mu^{-2\eta}v_1^2 + \frac{\lambda_1'}{4}(Z_{\lambda_1'}-1)v_1^4\mu^{-2\eta} - (Z_{\bar{h}}-1)\mu^{-2\eta}\bar{h}(m_1^2-\epsilon m_{12}^2)^2. \quad \text{(B4)}$$

where the Z factors are determined so that the counter terms satisfy $V_{\rm eff}^{c\rm (Low)} = -\,V_{\rm eff}^{\rm 1\,loop(Low)}\Big|_{\rm div.}$.

$$Z_{m_1'} - 1 = \frac{3\lambda_1' C_{\rm UV}}{16\pi^2}, \quad Z_{\lambda_1'} - 1 = \frac{9C_{\rm UV}}{16\pi^2}\lambda_1'. \quad \text{(B5)}$$

$$(Z_{\bar{h}} - 1)\bar{h} = -\frac{1}{64\pi^2}C_{UV}. \quad \text{(B6)}$$

By adding the counter terms to the tree and one-loop contribution, we can obtain the finite effective potential as,

$$
\begin{aligned}
& V_{\rm eff}^{\rm tree(Low)} + V_{\rm eff}^{\rm 1loop(Low)} + V_{\rm eff}^{c\rm(Low)} \\
= \;& -h_1 {m_1}^4 - h_2 {m_2}^4 - 2h_3 {m_1}^2 {m_2}^2 - h_{12} m_{12}^4 - \frac{1}{2m_2^2}\left(\frac{\epsilon\lambda_3 {v_1}^3}{2} - y\bar{n}n\right)^2 \\
& + \frac{m_1^2 - \epsilon m_{12}^2}{2}{v_1}^2 + \frac{\lambda_1 + \epsilon^2\lambda_3}{4}{v_1}^4 - \epsilon y\bar{n}n v_1 \\
& + \frac{(m_1^2 - \epsilon m_{12}^2 + 3(\lambda_1 + \epsilon^2(\lambda_3 - \lambda_1))v_1^2)^2}{64\pi^2}\left(\log\frac{m_1^2 - \epsilon m_{12}^2 + 3(\lambda_1 + \epsilon^2(\lambda_3 - \lambda_1))v_1^2}{\mu^2} - \frac{3}{2}\right).
\end{aligned}
\tag{B7}
$$

We keep the terms such as $m_1^2 v_1^2 \epsilon^0$, $m_{12}^2 v_1^2 \epsilon$, v_1^4 and $\epsilon^2 v_1^4$ to get the final form of the effective potential Eq.(6.6) from Eq.(B7). Concerning the cosmological constant terms, we keep the terms of the forms $m_1^4\epsilon^0$, $m_1^2 m_{12}^2 \epsilon$ and $m_{12}^4\epsilon^2$.

Appendix C

RG equation and its solutions for low energy effective theory

In this appendix, we study the RG equations for the coupling, mass, and cosmological constant of the low-energy effective action in Eq.(6.4). We also derive the solutions for these equations in the leading logarithmic approximation. At low-energy effective theory, the relations between the renormalized quantities and the bare ones are given as,

$$\lambda'_{10} = Z_{\lambda'_1}\lambda'_1\mu^{2\eta}, \quad m'^2_{10} = Z_{m'_1}m'^2_1, \quad \bar{h}_0 m'^4_{10} = Z_{\bar{h}}\bar{h}m'^4_1\mu^{-2\eta}. \tag{C1}$$

where Z factors are given by,

$$Z_{\lambda'_1} = 1 + \frac{9C_{\rm UV}}{16\pi^2}\lambda'_1, \quad Z_{m'_1} = 1 + \frac{3\lambda'_1 C_{\rm UV}}{16\pi^2}, \quad (Z_{\bar{h}} - 1)\bar{h} = -\frac{C_{\rm UV}}{64\pi^2}. \tag{C2}$$

Using the relations of Eqs.(C1)-(C2), we can derive the RG equations as,

$$\mu\frac{d\lambda'_1(\mu)}{d\mu} - \frac{9\lambda'^2_1}{8\pi^2} = 0, \tag{C3}$$

$$\mu\frac{dm'^2_1(\mu)}{d\mu} - m'^2_1(\mu)\frac{3\lambda'_1}{8\pi^2} = 0, \tag{C4}$$

$$\mu\frac{d\bar{h}(\mu)}{d\mu} + \frac{1}{32\pi^2} = 0. \tag{C5}$$

By solving the RG equations, we obtain the RG improved couplings and masses, cosmological constants as,

$$\lambda'_1(\mu_0) = \frac{\lambda'_1(\mu)}{1 + \frac{9\lambda'_1(\mu)}{16\pi^2}\log\frac{\mu^2}{\mu_0^2}}, \tag{C6}$$

$$m'^2_1(\mu_0) = \frac{m'^2_1(\mu)}{\left(1 + \frac{9\lambda'_1(\mu)}{16\pi^2}\log\frac{\mu^2}{\mu_0^2}\right)^{\frac{1}{3}}}, \tag{C7}$$

$$\bar{h}(\mu_0) = \bar{h}(\mu) - \frac{1}{64\pi^2}\log\frac{\mu_0^2}{\mu^2}. \tag{C8}$$

Bibliography

[1] Y. Fukuda *et al.* [Super-Kamiokande], Phys. Rev. Lett. **81**, 1562-1567 (1998) doi:10.1103/PhysRevLett.81.1562 [arXiv:hep-ex/9807003 [hep-ex]].

[2] V. N. Aseev *et al.* [Troitsk], Phys. Rev. D **84**, 112003 (2011) doi:10.1103/PhysRevD.84.112003 [arXiv:1108.5034 [hep-ex]].

[3] C. Kraus, B. Bornschein, L. Bornschein, J. Bonn, B. Flatt, A. Kovalik, B. Ostrick, E. W. Otten, J. P. Schall and T. Thummler, *et al.* Eur. Phys. J. C **40**, 447-468 (2005) doi:10.1140/epjc/s2005-02139-7 [arXiv:hep-ex/0412056 [hep-ex]].

[4] G. Hinshaw *et al.* [WMAP], Astrophys. J. Suppl. **208**, 19 (2013) doi:10.1088/0067-0049/208/2/19 [arXiv:1212.5226 [astro-ph.CO]].

[5] P. A. R. Ade *et al.* [Planck], Astron. Astrophys. **594**, A13 (2016) doi:10.1051/0004-6361/201525830 [arXiv:1502.01589 [astro-ph.CO]].

[6] S. M. Davidson and H. E. Logan, Phys. Rev. D **80**, 095008 (2009) doi:10.1103/PhysRevD.80.095008 [arXiv:0906.3335 [hep-ph]].

[7] S. Gabriel and S. Nandi, Phys. Lett. B **655**, 141-147 (2007) doi:10.1016/j.physletb.2007.04.062 [arXiv:hep-ph/0610253 [hep-ph]].

[8] N. Haba and K. Tsumura, JHEP **06**, 068 (2011) doi:10.1007/JHEP06(2011)068 [arXiv:1105.1409 [hep-ph]].

[9] P. A. N. Machado, Y. F. Perez, O. Sumensari, Z. Tabrizi and R. Z. Funchal, JHEP **12**, 160 (2015) doi:10.1007/JHEP12(2015)160 [arXiv:1507.07550 [hep-ph]].

[10] A. S. Adam, Y. Kawamura and T. Morozumi, doi:10.1093/ptep/ptab129 [arXiv:2108.03639 [hep-ph]].

[11] P. A. Zyla *et al.* [Particle Data Group], PTEP **2020**, no.8, 083C01 (2020) doi:10.1093/ptep/ptaa104

[12] R. Penco, [arXiv:2006.16285 [hep-th]].

[13] A. V. Manohar, doi:10.1093/oso/9780198855743.003.0002 [arXiv:1804.05863 [hep-ph]].

[14] J. Haller, A. Hoecker, R. Kogler, K. Mönig, T. Peiffer and J. Stelzer, Eur. Phys. J. C **78**, no.8, 675 (2018) doi:10.1140/epjc/s10052-018-6131-3 [arXiv:1803.01853 [hep-ph]].

[15] E. Ma, Phys. Rev. Lett. **86**, 2502-2504 (2001) doi:10.1103/PhysRevLett.86.2502 [arXiv:hep-ph/0011121 [hep-ph]].

[16] M. Bando, T. Kugo, N. Maekawa and H. Nakano, Prog. Theor. Phys. **90**, 405-418 (1993) doi:10.1143/PTP.90.405 [arXiv:hep-ph/9210229 [hep-ph]].

[17] A. V. Manohar and E. Nardoni, JHEP **04**, 093 (2021) doi:10.1007/JHEP04(2021)093 [arXiv:2010.15806 [hep-ph]].

[18] H. Okane, PTEP **2019**, no.4, 043B03 (2019) doi:10.1093/ptep/ptz022 [arXiv:1901.05200 [hep-ph]].

[19] L. Chataignier, T. Prokopec, M. G. Schmidt and B. Swiezewska, JHEP **03**, 014 (2018) doi:10.1007/JHEP03(2018)014 [arXiv:1801.05258 [hep-ph]].