
Research Article

INTERSECTION OF CRITICAL MATHEMATICS EDUCATION AND ETHNOMATHEMATICS

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ABSTRACT

Ethnomathematics has gained a certain level of momentum in the developing countries, and some developed countries with multicultural backgrounds. On the other hand, critical mathematics education that originated in developed countries, has highlighted critical points of ethnomathematical research.

In this research the complementary relation between critical mathematics education and ethnomathematics has been deliberated in order to connect ethnomathematics to school mathematics as a part of educational endeavor. Subsequently a foundational framework has been proposed to strengthen the critical nature by means of reciprocity and develop mathematics education based on ethnomathematics.

Key words: ethnomathematics, critical mathematics education, complimentary relationship, critical reflection

I INTRODUCTION

Mathematics and science education, which is regarded as a key to development in the developing countries, is facing various problems. Some of these are quantitative and physical problems, which require financial input, such as buildings and equipments, while others are qualitative, which require innovation in curriculum, teaching method and management. It is this qualitative aspect of education that is deeply related to attitudinal aspect in a whole system of education. Thus its importance is cited in a new trend of educational cooperation (JICA, 1994), which has a symbolic meaning leading to ownership. In this respect, the quality of students' learning activity should be a central concern because they are the final beneficiaries, and especially in mathematics education many researchers have argued that learning is hindered by a gap between the mathematical activities with which students are familiar in the environment and the mathematics which they learn in classroom (Gay & Cole, 1967; Carraher & Carraher, 1985; Gerdes, 1990).

The concept of ethnomathematics was introduced by D'Ambrosio (1985) in an effort to call attention to and thus to alleviate this gap. It has resonated with a sense of problem among researchers who have tried to tackle the cultural aspect of mathematics education. The examples as shown in Ascher (1991) and Gerdes (1988, 1990) have gradually added concrete images to its general characteristics and given

material form to its abstract existence. However, the clearer its characteristics become, the more criticisms are made against its potentiality and applicability to mathematics education, as can be seen in the following quotation :

Ethnomathematics does not amount to a set of general thinking tools since the mathematical activity is 'locked' into this practice, of which it is part, and it cannot function as a tool or basis to criticize that practice itself. Being critical towards the use of mathematics in the context of practice requires viewing that practice from an external perspective in a way that allows the mathematics to be distinguished in some way from the remaining aspects. It is this complete integration of mathematical activity with practice which marks it as distinct from the mathematics of the classroom. (Keitel, 1997, p. 19)

In other words, her criticism is equivalent to the observation that ethnomathematical activities are closely linked with practice and there is no necessity for the practitioners to explore the implication and method of their activities. Furthermore, general thinking, which forms a core of school mathematics, demands a completely opposite direction, that is consideration of the reason and method of activity. Thus, the critical points in her words can be summarized into the following three points.

- (1) Ethnomathematics cannot express itself in its own words.
- (2) The practitioner of ethnomathematics is not necessarily conscious of its implication.
- (3) The objectives of ethnomathematics and school mathematics are different in nature.

This means that after bringing ethnomathematics into the classroom for resolution of the gap, *a new kind of gap* between ethnomathematics and students should be re-created so that the latter reflects critically the former, which is a common and trivial activity at the beginning, by shedding a new light. Since these points are extracted from the perspective of critical mathematics education proposed by Keitel, the first objective of this research is :

To consider the relationship between ethnomathematics and critical mathematics education after their individual examination.

Applying this result, the second objective of this research is :

To consider the preconditions such that ethnomathematics may provide mathematics education with a new perspective for the sake of foundational reconsideration.

II CRITICAL MATHEMATICS EDUCATION (hereafter cited as CME)

Keitel has referred to Skovsmose from various angles in CME and Skovsmose has developed this theory more overtly and radically. In this research, therefore, the focus of consideration is based upon the theory by Skovsmose.

CME is an educational theory that reflects the critical theory of the Frankfurt School and develops educational practice for the formation of critical citizenship. Its systematization is being made in northern Europe and Germany. The subject, mathematics, had a tendency of being excluded from the practice because of its formal and objective nature, but Skovsmose pointed out the importance of incorporating a critical aspect into mathematics education, as follows :

It is necessary to increase the interaction between ME [Mathematics Education] and CE [Critical Education], if ME is not degenerated into one of the most important ways of socializing students (to be understood as students or pupils) into the technological society and at the same time destroying the possibilities of developing a critical attitude towards precisely this technological society. (1985, p.

338)

Here, we would like to clarify the objectives of CME through the verification of its three key terms, instead of trying to define directly the meaning of 'critical'. These three key terms, which are listed below, play a central role in fulfilling the objectives of CME.

- (1) Critical competence
- (2) Critical distance
- (3) Critical engagement

More concretely, (1) Critical competence means that students need the capability to think and judge by themselves what is important and they are presumed to have it to participate actively in the educational process. This is a human element of the educational process. (2) Critical distance is to keep a distance from the given subject or curriculum, and the teacher and students are not supposed to take it for granted. This regards an element of curriculum in the process. (3) Critical engagement means to direct the educational interests of the classroom to outside school; this is an element of objective of the educational process.

These three terms could be regarded as pillars, which constitute critical mind to be nurtured in the CME. The first term represents the intellectual base for independent thought, the second term the attitude by which to bridge subject and object, and the third a perspective of social practice. In other words, its educational

process takes the stepwise approach in expanding children's views about society and concretizing critical thinking towards that end, as shown in the above Fig. 1.

Employing linguistic metaphor, CME develops syntactics, semantics and pragmatics in its theory. This expansive process doesn't allow it to remain within the boundary of existing subjects or concentrate only on development of certain psychological abilities, but it unavoidably promotes as a teaching approach the integration of topics and subjects, and enhances critical thinking in totality. Skovsmose (1994) calls this approach thematization or projectization of learning and explains the lessons in Denmark as follows.

(Example of Thematization)

Skovsmose and two teachers conducted lessons with thematization strategy six hours every week for two months. The main theme was "Economic Relationships in the World of the Child" with the sub-themes "Pocket Money", "The Child Benefit Allowance" and "Money Needed for Equipment of Youth Club". Three concentric circles describe these sub-themes. The child itself is placed in the inner most circle, the child as part of the family in the middle circle and the child as part of society in the outer most circle.

These sub-themes are further divided into units. For example, the first sub-theme has three units "Pocket money", "Saving and shopping" and "Salary". Through discussion or diaries about these units the students are required to express their own opinions on whether it is reasonable to do some things in order to get pocket money, how much work is reasonable, what to buy with pocket money and etc.

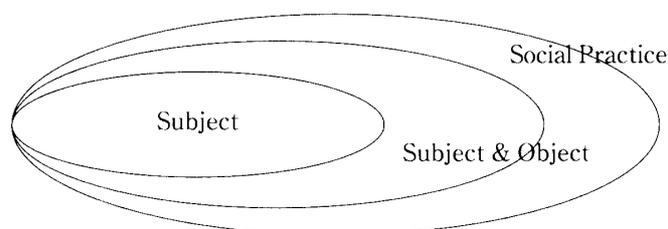


Fig. 1 Expansion of children's views

As observed in this example, through virtual reality the CME draw attention of children to the social meanings of mathematics in the complex relationship between the problems and themselves. The value peculiar to mathematics is not sought there. Mathematics serves as a tool to clarify “Economic Relationships in the World of Child” and therefore it cannot be critical to itself.

The critical perspective in mathematics education, however, has become increasingly important in modern society. During the annual conference of the Japan Society of Mathematics Education, Keitel characterized modern society by saying “Mathematics becomes implicit and “invisible”: an increasing mathematization of our society is complemented by an increasing demathematization of its individual members” (1997, p. 2) and underlined mathematics education from that angle.

It is difficult to grow and deepen insights about the social meaning of mathematics through a traditional way of teaching difficult equations. In other words, children have to reconstruct the meaning of mathematics by themselves through the critical verification of its social implication. It is, therefore, important to have not just the mathematics in textbooks but the mathematics embedded in every day life. Ethnomathematics has opened up a new possibility in this direction and will be able to bring a new dimension to the CME.

III ETHNOMATHEMATICS

The term, ethnomathematics, sometimes brings confusion into the definition whether to refer to a single mathematical activity, a set of mathematical activities in each culture, the research to analyze these activities or any others. We think it is not very productive to try to be precise in the definition, but awareness of a multi-faceted definition arouses caution for discussion.

‘ethnomathematics’ can refer to a certain practice as well as to the study of this practice. In what follows we use ‘ethnomathematics’ in both senses, although we primarily think of ethnomathematics as including certain educational ideas and a research perspective. (Vithal and Skovsmose, 1997, p. 133)

Even in this research, we will not differentiate these two ‘ethnomathematics’, as practice and analysis. The more explanatory words, mathematical activities and ethnomathematical research, will be employed to make a clear distinction only when necessity arises. While many mathematical activities have been identified and unified into ethnomathematics, some researchers have recently tried to clarify the structural relationship among some strands of the ethnomathematical theme.

(1) Bishop (1994, p. 15)

- a) research on mathematical knowledge in traditional cultures
- b) research on mathematical knowledge in non-Western societies
- c) research on mathematical knowledge of different groups in society

(2) Vithal and Skovsmose (1997, p. 134-135)

- a) research to challenge the traditional history of mathematics
- b) research to analyze the mathematics of traditional cultures
- c) research to explore the mathematics in every day settings that mathematical knowledge is generated in a wide variety of contexts
- d) research to focus on the relationship between ethnomathematics and mathematics education

Except for (2) d), these classifications (1) and (2) have similarities from the historical and socio-cultural

points of view. Of course Bishop as a mathematics educator knew the importance of (2) d) but Vithal and Skovsmose (1997) pointed out clearly that this has a potentiality to integrate the other strands.

The intention of this paper belongs to this very (2) d) and is to lay a foundation for the relationship between mathematics education and ethnomathematics, which appeared first as objection from the developing countries. The fulfillment of this goal requires ethnomathematics to be self-referential, which will be discussed in the next chapter.

Here we would like to take one example (Gerdes, 1990) for further discussion. Gerdes deliberated on the application of sand drawing, Sona, to the mathematics curriculum as a teaching material.

A Sona drawer traditionally has to draw efficiently and beautifully by any means. The method was invented to mark a set of equidistant points with a fingertip on the swept ground and make a drawing by use of these points as reference. The theme determines how many rows and columns of points are to be plotted. For example, the Fig. 2 has 5×6 points and represents the theme 'the marks left on the ground by a chased chicken'.

Many Sona drawings are done under the restriction of symmetry and in one stroke. And the laws such as symmetry and repetition incarnate in the drawing, can represent arithmetic relation, sequence, symmetry and similarity in mathematics education. Gerdes attempts to resurrect a cultural value with introduction of this Sona. And this creates a new research field by means of applying mathematical activities, immanent within African culture, to the curriculum development.

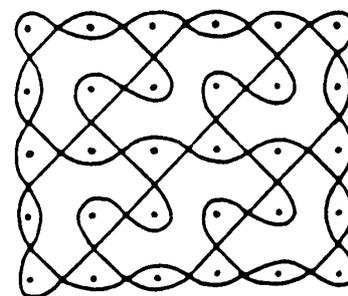


Fig. 2 Sona

IV BILATERAL CONSIDERATION OF CME AND ETHNOMATHEMATICS

First of all, we would like to take up the following sentences in order to start consideration of the relationship between ethnomathematics and CME :

Ethnomathematics can be interpreted as a reaction to the cultural imperialism which is built into modernization theory. . . . Critical Mathematics Education can also be described as a reaction to modernization theory, but in this case as a reaction from within a highly technological society. (Vithal and Skovsmose, 1997, p. 132)

This means that both CME and ethnomathematics have a common critical stance to modernization theory, but they also differ in such points as society, mathematics and process considered in their analysis. While the CME in the developed countries has a political background and view critically the application aspect of formalized mathematics into society, the ethnomathematics in developing countries has a cultural background and pay attention to practicing aspects of mathematics embedded in day-to-day activities.

This simple distinction may be useful at the beginning, but any society is not stationary, and the developing countries are also undergoing urbanization. Each country has a mixture of both characteristics. So besides the reason to overcome criticism of the educational application of ethnomathematics, it is necessary to consider both theories simultaneously because of this societal change. Now we would like to consider the relationship between ethnomathematics and CME in both directions and to find a complementary role to strengthen each other structurally.

1. Consideration of CME by use of ethnomathematics

The three key terms of CME, critical capability, critical distance and critical engagement, are reviewed in relation with ethnomathematics. As for the first term, it is necessary to retain an inner standard for the criticism, and this standard and critical capability are fortified by the reflection of their own mathematical activities.

The second term implies the need to keep a distance from the curriculum and it requires another kind of mathematics different from the one under discussion. Ethnomathematics substantiates this mathematics in a practical way.

... whenever we increase our understanding of other cultures, we increase understanding of our own by seeing what is or is not distinctive about us and by shedding more light on assumptions that we make which could, in fact, be otherwise. Our concepts of space and time are, after all, only our ideas and not objective truth. And, there is no single correct way to depict objects in space, nor one correct way to orient a picture in order to comprehend its contents. (Ascher, 1991, pp. 186-187)

And Ascher has shown the following picture as an example. This picture stimulates us to imagine and investigate an alternative mathematics beyond the horizon of our culture.

The third term critical engagement means to direct students' interests toward the world outside the classroom. It naturally brings about such awareness as the need to keep stay oneself within the situation of ethnomathematical practice, because the ethnomathematics is unavoidably intertwined with other activities in the society.

The above consideration has revealed that ethnomathematics concretizes the key terms of CME by provision of practical examples.



Fig. 3 Musk-ox hunting on North Somerset Island

2. Consideration of Ethnomathematics from the Perspective of CME

Now we would like to consider ethnomathematics from the perspective of CME. Criticism by CME has prompted us to reconsider the educational implication of ethnomathematics. Although it takes a critical stance against Western mathematics from its original background, the application of ethnomathematics to the classroom requires it to verify itself critically. Gerdes showed a kind of solution in his educational consideration in this direction.

The artisan, who imitates a known production technique, is, generally, not doing mathematics. But the artisan(s) who discovered the technique, *did* mathematics, was/were thinking mathematically. When pupils are stimulated to *reinvent* such a production technique, they are doing and learning mathematics. (1988, pp. 140-141)

This means that the artisan of mathematical activities, with little mathematical consciousness, practices his own activity as a part of culture. It is however this mathematical consciousness, and in other words the objectization of practice from a mathematical perspective, that can enable

ethnomathematics to be incorporated into educational practice. And this is not the viewpoint of the practitioner but that of the creator.

In the example of the Sona drawing, there exists a specific method 'to plot a matrix of points on the sand beforehand' in order to make a perfect drawing without hesitation. This method transforms 'to draw Sona' to 'to plot a matrix of points' and 'to follow an algorithm to travel through these points'. Only the repetition of drawing Sona a thousand times will not give birth to this method, but this invention was enabled only through analytical consideration of the process. This transformation provides justification for its applicability to education. Therefore, the critical verification of ethnomathematics has a close relationship with the possibility that ethnomathematics may be applied to mathematics education and we think that the last point by Keitel can provide a solution in this connection.

Next, we would like to examine the self-critical nature of ethnomathematics in this example by use of the three key terms of CME. The Gerdes' educational stance in Sona assumes that students basically have critical competence and encourages them to do mathematics from the viewpoint of creator. Here implicitly a critical distance from ethnomathematics is required. In other words, this example already includes some perspectives of CME. However, the structural and theoretical strength of ethnomathematical research should be increased through the perspective of CME in order that the research may be effectively and consistently applied to the teaching-learning process.

3. Measuring activity

Looking at ethnomathematics from the viewpoint of the creator suggests that it is to be interpreted as a generative process of mathematical activity. Constructivism is a theory of mathematics education with a focus on children's activity from this generative aspect. According to the theory, the operation is an internalized activity of children as a result of action towards an object and mathematics is regarded as a set of such operations. On the other hand, Bishop (1991) grasps the activity from the cultural point and claims that there are six universalities among various activities despite their different appearances. Here we would like to take one of them, measuring, to consider application of above discussion to ethnomathematics, which is regarded as generative and cultural.

Measuring activity of beans with an improvised tin container is quite prevalent in the Kenyan markets and a seller uses various sizes of containers to measure the quantity requested by a customer. The containers are semi standardized in such a sense that the original contents weigh, for a example, 500g or 250g. Just to imitate this activity in the classroom is not enough for a mathematics class, because the scene is too familiar for the children to have a viewpoint of creator. Questions like "which of the two boxes contains more beans", "How can we measure exactly the quantity of beans which somebody wants?" create *a new gap* which was mentioned at the beginning and serve as a starting point to rethink the reason and the method of this trivial activity.

While measuring beans with a can, the children can imagine and play many situations. Looking at the measuring activity from this angle, three key terms, which are critical competence, critical distance and critical engagement, correspond with "to be able to measure beans with a can", "to consider how effectively or exactly to measure beans" and "for whom or why to measure beans". This indicates how the learning process in this measuring activity develops in terms of the three terms of CME.

As the learning activity progresses, they experience the generative process of measuring activity from an arbitrary unit through common unit to standardized unit and grasp the property of unit. Within

this activity, children also may have an opportunity of being exposed to other measuring activities. Different units of the same quantity (e.g. meter and feet) have a cross-cultural connection, units of different quantities are combined to create a new concept (e.g. meter/second) and a new unit can be created to measure new objects (e.g. sweetness of fruit).

Among these activities, there are two important sides, commonness and difference. The former is expressed as the universality of mathematical activities (Bishop, 1991) that penetrates all of them. As mentioned in the above, children develop the concept of quantity and unit through the measuring activities in terms of CME. On the other hand, the latter is a reflection of cultural difference. The comparative thinking stimulates and enhances judging ability that is critical to live in a society that operates with myriad of information. The terms of CME envisages this growth of children's view towards society through mathematics education. For example, it is important for children to have the competence to look critically at data provided by the modern machine, which is assumed always to give the correct answer, and to judge if it is really correct, using the simple method a employed before the introduction of sophisticated machinery.

V CONCLUSION

The first objective of this paper, to consider the relationship between ethnomathematics and CME in both directions, has been done so far. It has been shown that mathematical activities substantiate three key terms in CME, and in return these terms can provide a structure and thus a rationale for the application of ethnomathematics to mathematics education. On the other hand, the second objective, that is consideration of a prerequisite for ethnomathematics to contribute positively to mathematics education, has also been achieved simultaneously in this two-way consideration because of their complementary relationship. In other words, ethnomathematics requires the objectization of itself in the educational process from the perspective of CME.

So far, ethnomathematics has been developed critical of school mathematics, but its educational application necessitates it to be viewed critically as well from the perspective of CME. This means that the integration of 'ethnomathematics as method' with 'ethnomathematics as object' forms a foundation for education based on ethnomathematics from the perspective of CME. The following framework is proposed for an integrated approach in ethnomathematical research.

- (1) To reflect critically mathematics education through mathematical activities and ethnomathematical research :
 - a) Mathematical implication
 - b) Social implication
- (2) To reflect critically ethnomathematics from the perspectives of critical mathematics education :
 - a) Mathematical implication
 - b) Social implication

The distinction between mathematical implication and social implication is important in considering the characteristics of the subject, mathematics. The mathematical implication here concerns the generative aspect of a mathematical concept peculiar to the subject and, on the other hand, the social implication concerns the critical view towards the application of mathematical concepts in the social

context. Since mathematics education deals with acquisition of concepts, the mathematical implication has been given more consideration, but societal change also necessitates consideration of the social aspect of mathematics education.

In this framework, the first component (1) has potential to reveal the uncritical nature of the present mathematics education. Students are frequently required to develop mathematical thinking as an objective tool with which they find little meaning and are seldom encouraged to consider the relation between this objective tool and their own mathematical activities. Here the integration of CME and ethnomathematics has to play an important role in uncovering what has been taken for granted. The second component (2) will invite the students to reflect on their activities and to develop their thinking from there. Their own activities are the target of reflexive thinking and at the same time the source for a higher level of reflection. For this development the three key terms of CME will provide a direction how to develop critical thinking on their mathematical activities.

This integrated approach, using the above framework, plays a pivotal role in the practical and theoretical development of the ethnomathematical program so that ethnomathematics research will eventually produce a fruitful alternative to the present school mathematics.

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